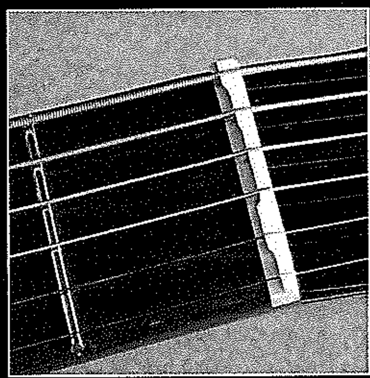
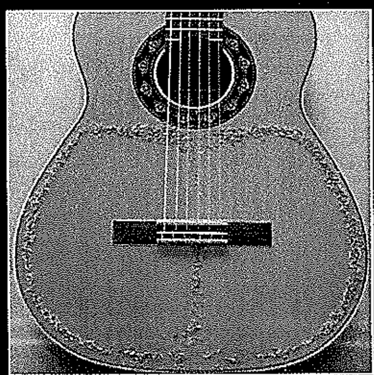
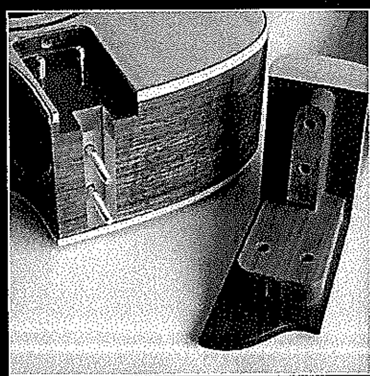
Contemporary Acoustic Guitar Design and Build

2nd Edition

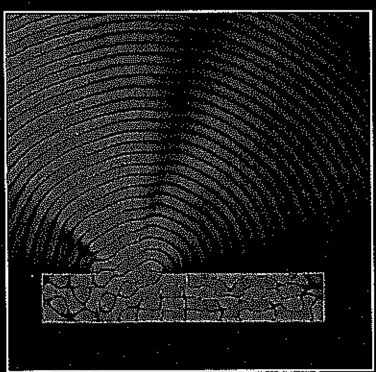


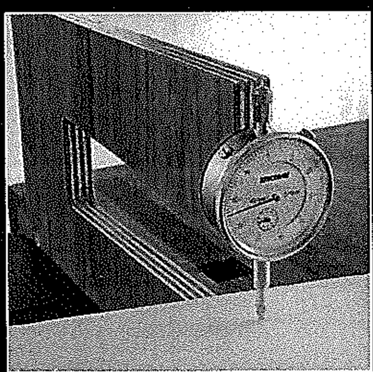
Trevor Gore
with
Gerard Gilet

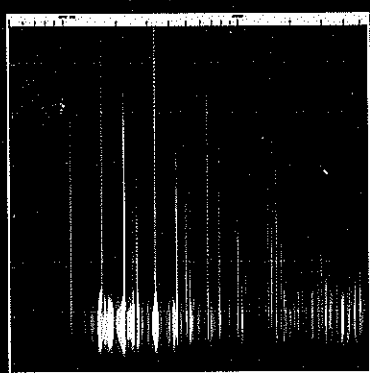




Volume 1; Design







4.5.2. The tap tone method of establishing wood properties

For over thirty years Australian physicist and luthier Graham Caldersmith has been publishing papers on the physics of stringed instruments and the materials they are made from. His paper entitled "Vibrations of Orthotropic Rectangular Plates II" published in Acustica 73, 1991 is the basis of the method used here. We have modified his nomenclature in order to be consistent with ours.

The principle is straight forward: as plates vibrate according to their material properties, if you can measure the frequency of certain modes of plate vibration you can derive the material properties of the plate.

Three natural modes of vibration of a free rectangular plate are illustrated in Fig. 4.5-2.

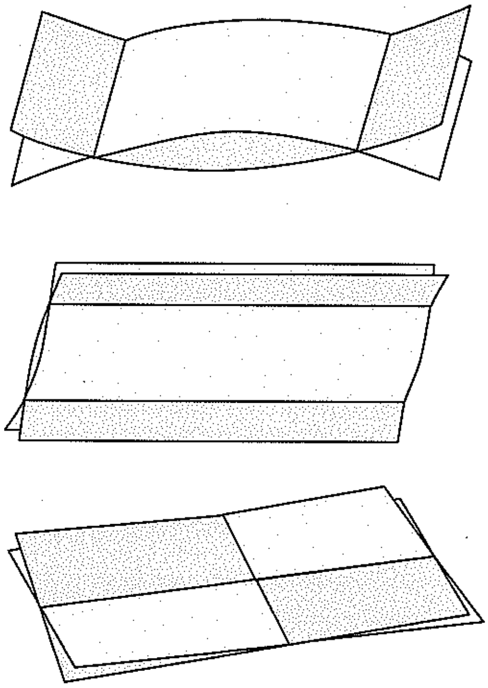


Fig. 4.5-2 *Three of the vibration modes of a free rectangular plate*

If the frequencies of these vibration modes are measured, the values of Young's Modulus along the grain E_L, Young's Modulus across the grain E_C and the shear modulus G_{LC} can be derived. The measurement technique is as follows:

The wood must be flat (no twist, cupping or bowing) of uniform thickness to better than 0.1mm and of rectangular profile. If any one of these conditions has not been met you will not generate accurate results from your tap tests. The half plate must be held at a node (green dot, at ~22.4% of length for the bending modes, in the centre for the twisting mode) and tapped at an anti-node (brown star), Fig. 4.5-3.

To help in producing pure tones, the hold point should be chosen to be on a node of the vibration mode under test whilst at the same time being in an anti-nodal position of other modes in order to suppress them. The frequency of vibration is measured using a microphone positioned close to an anti-node of the mode being measured. The microphone signal is fed into a frequency counter (a good electronic or software tuner will suffice) or a spectrum analyser. Further details are contained in Build Section 6.2.

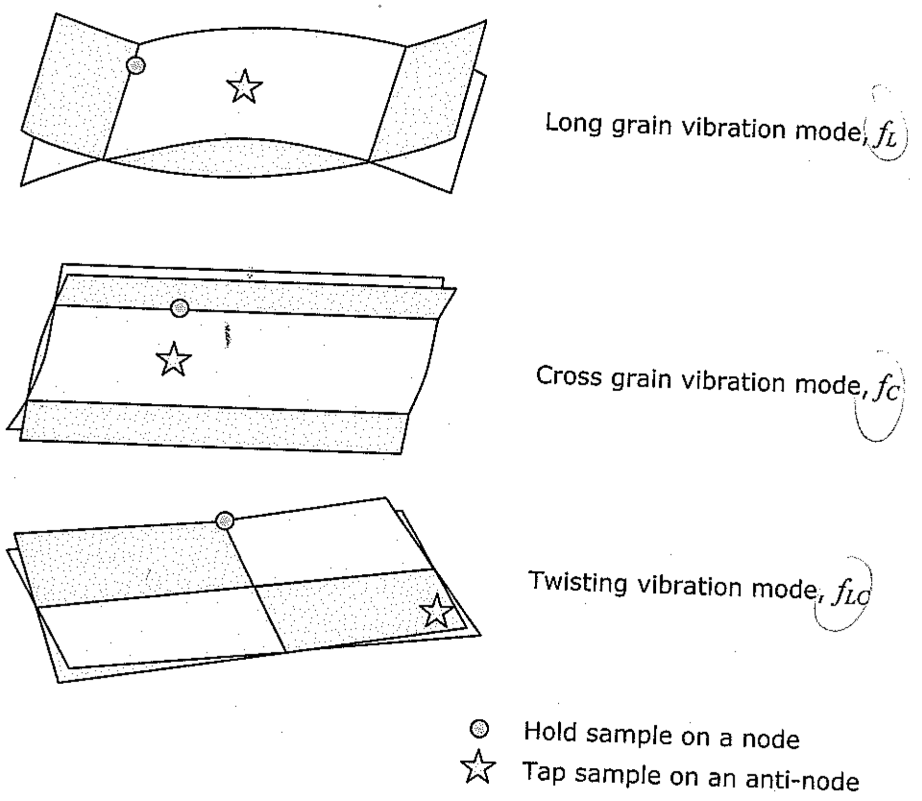


Fig. 4.5-3 *Locations to hold and tap when measuring tap test frequencies*

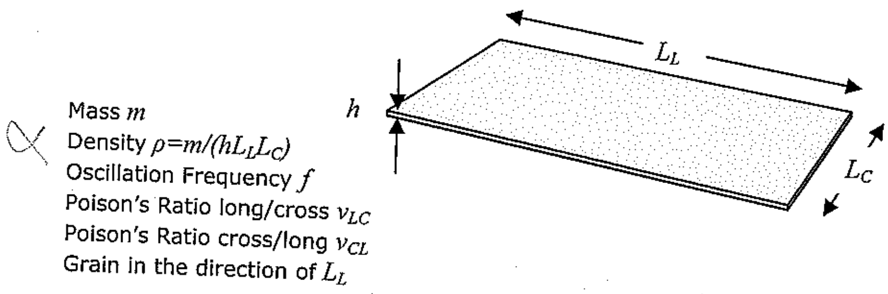


Fig. 4.5-4 *Nomenclature for deriving elastic constants*

Using the nomenclature of Fig. 4.5-4, where Poison's ratio is the ratio of the transverse to longitudinal strains, the expressions for the various moduli are given below.

Young's Modulus along the grain is given by:

$$E_L = \frac{1}{(\pi/2)^2 (3/2)^4}\left[12(1-\nu_{LC}\nu_{CL})\right]\frac{\rho L_L^4 f_L^2}{h^2} \qquad \text{Equ. 4.5-1}$$

Like other wood properties, the values for Poison's ratio, ν, vary by species and for different samples within a species. The values for some common guitar timbers are presented and a rough average for the all species product $\nu_{LC}\nu_{CL}$ is given in Table 4.5-1.

Species		ν_{LC}	ν_{CL}	$\nu_{LC} \times \nu_{CL}$
Mahogany,	African	0.297	0.033	0.0098
Mahogany,	Honduras	0.314	0.033	0.0104
Maple,	Sugar	0.424	0.065	0.0276
Maple,	Red	0.434	0.063	0.0273
Walnut,	Black	0.495	0.052	0.0257
Cedar,	Western red	0.378	—	
Redwood		0.360	—	
Spruce,	Sitka	0.372	0.040	0.0149
Spruce,	Engelmann	0.422	0.083	0.0350
	Average	**0.3884**	**0.0527**	**0.0215**

Table 4.5-1 *Poison's Ratio for various guitar woods*[30]

Using an average figure of 0.02 for the product $\nu_{LC}\nu_{CL}$ (which results in numerical errors of less than ~1% across the various species) Equ. 4.5-1 reduces to: Fig. from 4.5-3

$$E_L = 0.94146 \frac{\rho L_L^4 f_L^2}{h^2} \qquad \text{Equ. 4.5-2}$$

4.5-7

Young's Modulus across the grain is given by a very similar equation:

$$E_C = 0.94146 \frac{\rho L_C^4 f_C^2}{h^2} \qquad \text{Equ. 4.5-3}$$

The shear modulus is given by:

$$G_{LC} = \frac{3}{(\pi/2)^2} \left[\frac{\rho L_L^2 L_C^2 f_{LC}^2}{h^2} \right]$$

This simplifies to:

$$G_{LC} = 1.21585 \left[\frac{\rho L_L^2 L_C^2 f_{LC}^2}{h^2} \right] \qquad \text{Equ. 4.5-4}$$

The density ρ is calculated by weighing the sample and dividing the mass by the product of the three linear dimensions (L_L, L_C, and h). Knowing the material properties, we can now examine how a plate with these properties will vibrate under particular circumstances.

[30] Source: USDA Wood Handbook, 1999.

4.5.3. Establishing the target plate thickness

To determine the target thickness for a particular plate (the one that will be on the next guitar) we need a way of ensuring that that plate, with its own particular values of Young's modulus, density etc., will always produce the same vibrational performance when the only variable we can manipulate is the plate thickness[31].

The vibrations of a plate are very dependent on the conditions at its boundaries. Whilst some success has been achieved in relating free plate performance to final instrument performance for violins, this has not been the case for guitars. A guitar has a rather complex outline shape and the edge conditions are difficult to define as they are neither simply supported nor fully constrained in terms of position and angle of rotation. Rather, they could be considered to be mass loaded. The details of how a plate is suspended when attached to a guitar need not concern us however, because we are not going to attempt to predict an absolute frequency of a particular mode of vibration. What we are going to do is to *assume* that the edge is fully constrained and derive a parameter which we call the *vibrational stiffness*, (having the dimensions of frequency and is therefore effectively a target frequency), which combines the thickness of the plate with the other material parameters so that we can always produce a plate that meets the target vibrational stiffness by varying its thickness, irrespective of the value of its other material constants. Consequently when the plate is attached to its braces and the rest of the guitar body, provided how it is attached is repeatable, a consistent target vibrational frequency will be achieved on the actual instrument. This is analogous to the old master's technique of listening for a particular tap tone, or combination of taps tones, along and across the grain and developing an intrinsic knowledge through experience of how they will relate to the performance of the finished instrument.

The method described here is an extension of the work published by R. F. S. Hearmon in 1946[32]. Hearmon was a physicist working at the Forest Products Research laboratory in Princes Risborough, UK. Like a number of other authors, Hearmon uses a value of 0.01 for the Poison's ratio product $\nu_{LC}\nu_{CL}$, whilst we prefer a value of 0.02 which is more representative of guitar woods. This change accounts for the minor variances between Hearmon's original equations and those presented here.

For convenience of comparison with Hearmon's work we will revert to Cartesian coordinates (x,y,z) where x is parallel to the grain (same as L), y is across the grain (same as C) and z is in the direction of the thickness of the plate. For small deflections w in the z direction the equation of motion for a rectangular orthotropic plate is given by:

$$D_1\frac{\partial^4 w}{\partial x^4}+D_2\frac{\partial^4 w}{\partial y^4}+2D_3\frac{\partial^4 w}{\partial x^2\partial y^2}+\rho h\frac{\partial^2 w}{\partial t^2}=0 \qquad \text{Equ. 4.5-5}$$

where

$$D_1=\frac{E_x h^3}{12(1-\nu_{xy}\nu_{yx})}$$

$$D_2=\frac{E_y h^3}{12(1-\nu_{xy}\nu_{yx})}$$

$$D_3=\frac{E_x\nu_{yx}h^3}{12(1-\nu_{xy}\nu_{yx})}+\frac{G_{xy}h^3}{6}$$

[31] Some might argue that you should maintain a constant plate thickness and shave the bracing to suit, or manipulate both the bracing and plate thickness. Our view is that such approaches tend to add more variability rather than reduce it.

[32] *The fundamental frequency of vibration of rectangular wood and plywood plates*, R.F.S. Hearmon, 1946 *Proc. Phys. Soc.* 58 78-92.

t=time and ν_{yx} = 0.05 (same as ν_{CL}, though Hearmon uses 0.025) for typical guitar plate woods.

Hearmon provides boundary conditions for a clamped plate and solves for the vibrational frequency using the Rayleigh method with the Ritz modification and goes on to demonstrate the veracity of his solution. For a rectangular plate with all edges clamped (no rotation and no displacement at the edges) the frequency of vibration of the plate in cycles per second (Hz) is given by:

a = overall length
b = overall width lower bout

$$f = \frac{1}{\pi a^2 b^2}\left\{\frac{126}{\rho h}\left[D_1 b^4 + D_2 a^4 + \frac{4}{7} D_3 a^2 b^2\right]\right\}^{0.5} \qquad \text{Equ. 4.5-6}$$

The terms a and b are the characteristic length (x or long grain direction) and characteristic width (y or cross grain direction) respectively of the rectangular plate. For our purposes the "plate" will ultimately be guitar shaped rather than rectangular, so a and b need to be characteristic dimensions of the guitar that the plate will be used on. Keeping things simple, we quite arbitrarily use the overall length and the overall lower bout width of the guitar box as the characteristic dimensions.

The next step is to invert Equ. 4.5-6 so we can compute h, the required thickness of the guitar top. At the same time we will also substitute for D_1, D_2 and D_3. This yields

$$h = \frac{f\pi a^2[12\rho(1-\nu_{xy}\nu_{yx})]^{0.5}}{\left[126\left\{E_x + \left(\frac{a}{b}\right)^4 E_y + \left(\frac{a}{b}\right)^2\left(\frac{4}{7}E_x\nu_{yx} + \frac{4}{42}12(1-\nu_{xy}\nu_{yx})G_{xy}\right)\right\}\right]^{0.5}}$$

We can substitute in the average values that we determined for the Poison's ratio product $\nu_{xy}\nu_{yx}$ (same as $\nu_{LC}\nu_{CL}$ = 0.02) and ν_{yx} (same as ν_{CL} = 0.05) and simplify the equation by evaluating the constants and also revert back to our original nomenclature thus yielding

$$h = \frac{0.95977 f a^2 \rho^{0.5}}{\left\{E_L + \left(\frac{a}{b}\right)^4 E_C + \left(\frac{a}{b}\right)^2 (0.02857E_L + 1.12G_{LC})\right\}^{0.5}} \qquad \text{Equ. 4.5-7}$$

Having measured the material properties (Section 4.5.2), we have appropriate values for all the right hand side variables of Equ. 4.5-7 except for f.

f is the vibrational stiffness parameter that we discussed earlier in the section. We originally determined f by experience – for this piece of top wood which we know is stiff (or soft) how thick will we leave it? If we chose 2.8mm we changed the value of f in Equ. 4.5-7 until h evaluated to 2.8mm. We quite rapidly settled on suitable values for the vibrational stiffness parameter for the tops and backs of steel string and traditionally braced classical guitars for the bracing systems that we use. What this means is that we can rapidly measure the elastic constants of a top (or back) that we want to use using the techniques of Section 4.5.2, feed the numbers into Equ. 4.5-7 with an appropriate value for the vibrational stiffness parameter f, and irrespective of whether that particular piece of wood is softer or stiffer than "normal" we will know how thick to leave it to give exactly the same vibrational performance.

Having used this method for a number of years we can attest to the fact that it has practically eliminated performance variations due to variations in top wood properties and taken the guess work out of thicknessing tops and backs.

The values of vibrational stiffness parameter that we use (f in Equ. 4.5-7) are tabulated in Table 4.5-2 and are well suited to the various body sizes we build when we apply the appropriate values for a and b (the overall length and width respectively of the guitar box). Of course, you can choose any value for f that suits you. Smaller numbers give thinner panels.

Panel Type	**Vibrational Stiffness Value f**
Steel String Top	75.0
Steel String Back	55.0
Traditional Classical Top	60.0
Traditional Classical Back	50.0

Table 4.5-2 *Vibrational Stiffness Parameter values*

As we have now predicted the final thickness of the panel (a top for example), and as we already know the wood's density, we can also predict the mass of the top if we know the guitar's area[33]. Consequently, we have a method of determining the quality of a piece of top wood. The lower the predicted mass, the better the top. The lighter top will yield a slightly louder guitar for a given, consistent bracing system, provided the rest of the guitar is assembled in a controlled way. However, its overall tonality will be consistent with others built to the same vibrational stiffness criteria.

Table 4.5-3 gives the values of the various elastic constants for a variety of guitar woods measured using the method described in Section 4.5.2, and the corresponding target plate thicknesses and masses for a medium sized steel string guitar (box size 490mm long by 390mm wide [a and b], approximately Martin 000 size). When the relevant equations are set up in a spreadsheet, taking the 7 measurements that are required (length, width, original thickness, mass; long, cross and diagonal mode frequencies) the target thickness can be derived in less than 5 minutes for the whole measurement and analysis process. Preparing the samples takes a little longer. The tonewood plates need to be trimmed so they are accurately rectangular and reduced to the maximum uniform thickness that removes all the saw marks. However, this part of the process has to be done anyway. Non-rectangular samples and rough sawn timber of uneven thickness will give inaccurate results. Likewise, timber that has significant variation in how well quartered it is (and consequently a large variation across the width in its cross grain Young's modulus) will also give spurious results. Timber where the grain alternates between on quarter and off quarter a number of times across the width of the sample should be avoided. It will be very difficult to establish a cross grain mode frequency for samples like this and we have also found that it seldom makes a good sounding guitar. We normally measure unjoined top/back plate halves. The theory still works for full sized joined top/back plates, but after the joined halves have been reduced to uniform thickness, the cross and diagonal mode frequencies can be less than 20Hz, which can be difficult to measure accurately with an inexpensive microphone.

It's worth pointing out a few interesting revelations in Table 4.5-3. Engelmann Spruce samples 7 and 8 had low values for E_L and E_C, so they had to be left thicker to result in the same performance. However, because they were also of very low density (for Engelmann spruce) the overall top mass is lower than that of stiffer samples and indeed this wood made excellent guitars.

It is also worth noting that whilst the western redcedar we have used is of lower density (on average) than Engelmann spruce, its lower stiffness does not adequately compensate for this and it generally produces a heavier top. Although not shown in this table, Sitka spruce generally produces a heavier top than either Engelmann spruce or western redcedar, but it is also a harder wood, which is a preferred quality for some guitar designs.

[33] If you designed your guitar outline shape using CAD techniques, the program will likely give you an area for it. If you used more manual methods and made a template out of a uniform sheet material such as acrylic or polycarbonate, you can calculate the area by measuring the density of a rectangular off-cut of the sheet material. The area of the guitar is given by the mass of the template divided by the product of the material's density and its thickness. The area of the guitar used to calculate the values in the rightmost column of Table 4.5-3 was 0.144215m^2.

Sample #	Species	Length	Width	Thickness	Mass	Density	F long	F cross	F diag	Elong	Ecross	Elong/Ecross	G	Target thickness SS	Panel Mass SS
		mm	mm	mm	grm	kg/m³	Hz	Hz	Hz	GPa	GPa	-	GPa	mm	grm
	Top Woods														
1	Engelmann Spruce	605	238	4.00	228	395.9	61.6	105.0	38.2	11.84	0.82	14.4	0.91	2.72	155
2	Engelmann Spruce	605	238	3.90	224	398.9	60.0	107.8	39.2	11.91	0.92	12.9	1.02	2.68	154
3	Engelmann Spruce	553	216	3.85	169	367.5	67.8	93.5	49.6	10.03	0.44	22.6	1.06	2.86	151
4	Engelmann Spruce	553	216	4.05	178	367.9	70.4	105.8	49.8	9.79	0.51	19.0	0.97	2.88	153
5	Engelmann Spruce	595	230	3.70	191	377.2	58.2	102.0	49.0	11.01	0.76	14.6	1.51	2.65	144
6	Engelmann Spruce	595	230	3.85	201	381.5	66.0	100.0	46.0	13.23	0.68	19.5	1.24	2.54	140
7	Engelmann Spruce	594	221	3.75	156	316.9	52.0	115.7	43.1	7.14	0.68	10.5	0.88	2.97	136
8	Engelmann Spruce	594	221	3.70	152	312.9	50.0	107.6	40.8	6.70	0.59	11.3	0.80	3.07	139
9	Engelmann Spruce	557	224	4.10	190	371.4	70.7	140.8	56.1	10.01	1.04	9.6	1.32	2.69	144
10	Engelmann Spruce	557	224	4.40	203	369.8	75.7	144.9	58.4	9.92	0.95	10.4	1.23	2.72	145
	Average					**366.0**				**10.16**	**0.74**	**14.5**	**1.09**	**2.78**	**146**
11	Port Orford Cedar	602	283	3.80	270	417.1	59.6	57.0	25.7	12.69	0.57	22.4	0.67	2.80	169
12	Port Orford Cedar	602	283	3.80	267	412.4	58.4	57.0	26.6	12.04	0.56	21.5	0.71	2.84	169
	Average					**414.7**				**12.37**	**0.56**	**21.9**	**0.69**	**2.82**	**169**
13	Western Red Cedar	590	236	4.30	194	324.0	56.5	118.6	44.0	6.38	0.72	8.9	0.80	3.13	146
14	Western Red Cedar	590	236	4.50	204	325.6	58.0	127.2	46.0	6.17	0.76	8.1	0.80	3.16	148
15	Western Red Cedar	595	236	4.45	210	336.1	57.2	134.9	32.0	6.55	0.90	7.3	0.42	3.20	155
16	Western Red Cedar	594	236	4.30	203	336.8	54.7	122.7	32.0	6.39	0.80	8.0	0.45	3.26	158
17	Western Red Cedar	560	202	5.00	208	367.8	76.5	162.8	60.4	7.97	0.61	13.0	0.83	3.11	165
18	Western Red Cedar	559	187	5.40	203	359.6	72.5	194.0	75.6	5.96	0.53	11.2	0.94	3.41	177
19	Western Red Cedar	593	223	4.30	192	337.7	56.5	140.1	41.6	6.79	0.83	8.1	0.67	3.12	152
20	Western Red Cedar	593	223	4.25	191	339.8	55.2	139.0	41.2	6.67	0.85	7.9	0.68	3.14	154
	Average					**340.9**				**6.61**	**0.75**	**9.1**	**0.70**	**3.19**	**157**
	Back Woods														
21	East Indian Rosewood	568	218	3.90	398	824.2	48.5	129.0	40.7	12.49	1.92	6.5	1.67	2.52	300
22	East Indian Rosewood	569	226	4.00	414	804.9	51.0	130.9	43.9	12.91	2.12	6.1	1.95	2.41	280
23	East Indian Rosewood	552	221	3.40	302	728.1	43.3	129.1	36.0	10.32	2.36	4.4	1.48	2.46	259
24	East Indian Rosewood	552	221	3.40	299	720.9	43.8	128.0	38.9	10.46	2.29	4.6	1.71	2.43	252
25	East Indian Rosewood	613	221	3.80	409	794.5	50.3	124.2	38.4	18.51	1.91	9.7	1.81	2.16	248
26	East Indian Rosewood	613	221	3.80	406	788.7	42.7	113.0	33.8	13.24	1.57	8.5	1.39	2.50	285
27	East Indian Rosewood	607	218	4.65	486	789.8	54.4	146.6	49.2	13.82	1.67	8.3	1.88	2.41	274
28	East Indian Rosewood	606	218	4.65	489	796.0	55.0	148.3	51.2	14.14	1.72	8.2	2.05	2.37	273
	Average					**780.9**				**13.24**	**1.94**	**7.0**	**1.74**	**2.41**	**271**

Table 4.5-3 *Measured elastic constants and thickness targets*

The table also illustrates the point that, even for very select wood, the material properties are quite variable. Visual appearance (other than how well quartered a sample is) gives few clues about the material properties. We look for good quartering and listen for a clear, sustaining tap tone, which is indicative of low damping. A muddled tap tone is seldom good. Wood that gets over these first two hurdles is then tested as above. All the above wood except the Port Orford cedar and western redcedar samples 17 and 18 made excellent guitars. We rejected the Port Orford cedar, a wood we seldom see in Australia. The grain wandered on and off quarter (being responsible for the low value for E_C), and the predicted top mass was too high for our liking. Its tap tone was unconvincing. Western redcedar sample 17 was high density for cedar, and although of relatively high stiffness, this was not sufficient to deliver a top panel of low enough mass for a master grade guitar. Sample 18 was also of relatively high density but of more typical longitudinal stiffness, with low cross grain stiffness due to being off-quarter. The predicted panel mass was sufficient to disqualify this wood from inclusion in a master grade guitar.

Panels which are out of flat, due to twisting or cupping will rarely have a good, measurable tap tone whilst in that condition. If they can be flattened (back wood especially), well and good, and you should get a result. If they can't be flattened you should be asking yourself if that piece is suitable for use in a guitar. It is possible to compute the elastic constants of warped wood using deflection methods, but it takes considerably more time, though it is just as accurate as the dynamic method. Warped panels can be measured this way provided they are pre-loaded enough to take out the warp and are measured twice with each face on top once and the results averaged, as different sides of the panel will give different results. We have not recorded the mathematics or the procedure here, because its use is only of benefit on warped panels, which probably should not be used anyway. The builder should carefully store wood so that it will dry evenly and so that warping will not occur.

4.5.4. Sensitivity to long and cross grain stiffness variations

Wood is orthotropic in that its Young's modulus along the grain is typically 10 to 20 times its Young's modulus across the grain. Many instrument builders appear to prefer wood with high cross grain stiffness. We thought it worth examining the relative effects of long and cross grain stiffness on the panel's performance as a soundboard. We performed a sensitivity analysis using Hearmon's equations (in particular Equ. 4.5-6) answering the question "what is the effect on performance if the elastic moduli are varied by plus and minus 50%". f, the frequency of vibration of the panel, was evaluated whilst varying one of the elastic constants plus and minus 50% with the other elastic constants held at a constant value. The original ratio E_l/E_C was set at 16.6.

What we found was that:

1) A 50% change in E_L predicted a 23.4% change in vibrational frequency of the panel
2) A 50% change in E_C predicted a 3.02% change in vibrational frequency of the panel
3) A 50% change in G_{LC} predicted a 1.35% change in vibrational frequency of the panel

What this says is that because of the highly orthotropic nature of wood, the cross grain stiffness has very little influence on the low frequency vibrational performance of the panel. The performance is dominated by the long grain stiffness. The corollary of this is that if stiffness across the grain is important to you, then you need to address that in the bracing system. Just a small increase in the inclination of a brace across the grain will be much more effective than seeking top wood with high cross grain stiffness.

In Section 4.4.2, when we analysed the stiffness of the top under the torque produced by string loading, we made the assumption that there was little by way of load shedding across the grain, with the significant part of the load being supported by the longitudinal stiffness of the structure. The analysis above supports that assumption.

4.5.5. Panel design decisions

A critical decision for the designer is to choose how active a role the back panel is to take in the guitar's sound production. For steel string guitars used for finger picking, we generally prefer to have a live back. The advantages we find in this are as follows:

- It lowers the guitar's first resonant frequency (the coupled Helmholtz frequency) giving the impression of having used a larger body size
- It produces a more "breathy", transparent sound, which we happen to prefer
- It reduces the amplitude of the second resonance (the top main resonance or $T(1,1)_2$), which on an efficient guitar can otherwise be too high, thus avoiding both tuning issues (as discussed previously) and "wolf note" issues
- It introduces a third resonance peak, the $T(1,1)_3$, which both spreads the low frequency response to give a greater "gain-bandwidth" product (higher output) and introduces greater tonal colour.