GEE Comparison

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Simulation Setup

	iter	n	p	QELR Param
Sim1	500	300	3	V1
Sim2	500	300	3	V2
Sim3	500	300	5	V3
Sim4	500	300	10	V4
Sim5	500	300	12	V5

Remarks:

- 1. I've excluded the "unstructured" correlation structure from simulation.
- 2. In terms of gee parameters:
 - 1. Maxiter: Manually increased maxiter to a large number (eg. 200), when working correlation is not independent.
 - 2. Scale. fixed: Sometimes solutions would converge when scale. fixed = TRUE (default = FALSE). Currently set as FALSE
- 3. GEE package: only used the sandwich estimator's result (Robust variance)

```
allout$V1 %>% round(.,2)
```

```
## [,1] [,2] [,3]
## [1,] -1.2 -0.2 0.2
## [2,] -0.2 -0.1 -0.2
## [3,] 0.2 -0.2 1.0
```

allout\$V2 %>% round(.,2)

```
## [,1] [,2] [,3]
## [1,] -1.2 0.2 0.2
## [2,] 0.2 -0.1 0.2
## [3,] 0.2 0.2 1.0
```

```
allout$V3 %>% round(.,2)
##
       [,1]
            [,2] [,3]
                       [,4] [,5]
## [1,] -1.5 -0.20 0.2 -0.20 0.2
## [2,] -0.2 -0.75 -0.2 0.20 -0.2
## [3,] 0.2 -0.20 0.0 -0.20 0.2
## [4,] -0.2 0.20 -0.2 0.75 -0.2
## [5,] 0.2 -0.20 0.2 -0.20
allout$V4 %>% round(.,2)
##
        [,1]
              [,2]
                    [,3] [,4]
                               [,5]
                                    [,6] [,7]
                                               [,8]
                                                     [,9] [,10]
##
   [1,] -1.5 -0.20
                    0.20 -0.2 0.20 -0.20
                                         0.2 - 0.20
                                                     0.20
                                               0.20 -0.20
                                                            0.2
   [2,] -0.2 -1.17 -0.20 0.2 -0.20 0.20 -0.2
    [3,] 0.2 -0.20 -0.83 -0.2
                               0.20 -0.20
                                          0.2 - 0.20
                                                    0.20
                                                           -0.2
##
   [4,] -0.2 0.20 -0.20 -0.5 -0.20 0.20 -0.2 0.20 -0.20
                                                            0.2
   [5,] 0.2 -0.20 0.20 -0.2 -0.17 -0.20 0.2 -0.20
                                                           -0.2
   [6,] -0.2 0.20 -0.20 0.2 -0.20 0.17 -0.2
                                              0.20 - 0.20
                                                            0.2
   [7,] 0.2 -0.20
                   0.20 -0.2 0.20 -0.20 0.5 -0.20
                                                           -0.2
   [8,] -0.2 0.20 -0.20 0.2 -0.20 0.20 -0.2 0.83 -0.20
                                                            0.2
  [9,] 0.2 -0.20 0.20 -0.2 0.20 -0.20 0.2 -0.20
                                                           -0.2
## [10,] -0.2 0.20 -0.20 0.2 -0.20 0.20 -0.2 0.20 -0.20
                                                            1.5
```

Differences Between GEE and GEEPACK: Do both packages generate the same estimates?

Short answer: YES for independence correlation.

GEE stops and returns an error message when solution diverges, however GEEPACK would still produce (erroneous, some extremely large values) estimates.

The below analysis answers a few questions to address the comparison/differences:

1. Out of 500 simulations, how many converged solutions do we get?

```
rbind(sim1= sapply(allout$sim1$convergecase, sum),
    sim2= sapply(allout$sim2$convergecase, sum),
    sim3= sapply(allout$sim3$convergecase, sum)) %>% t()
```

```
##
               sim1 sim2 sim3
## logit
                500 500
                          500
                500
                     500
                          500
## geepack_ind
                     500 500
## geepack_exc
                497
## geepack_ar1
                500
                     500 500
## gee_ind
                500
                     500
                          500
                     490
## gee_exc
                 72
                           93
## gee_ar1
                  1
                       0
                            0
## mle
                500
                     500
                          500
```

1. Distribution of the differences between GEE and GEEPACK estimates (among the converged cases)?

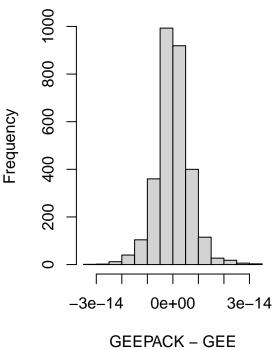
Sim1

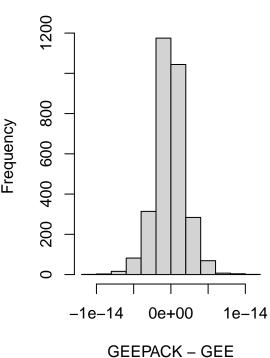
- 1. Independence correlation shows negligible differences.
- 2. Exchangeable correlation shows noticeable differences. This refers to the 72 converged cases from "GEE". Could it be that GEEPACK and GEE yield different divergence/ convergence cases? Further deep dive can be done here.
- 3. AR-1: only 1 converged case here, no comment.

checkdiff(allout\$sim1\$OUT\$geepack_ind, allout\$sim1\$OUT\$gee_ind)

Difference in mean estimates

Difference in se estimates



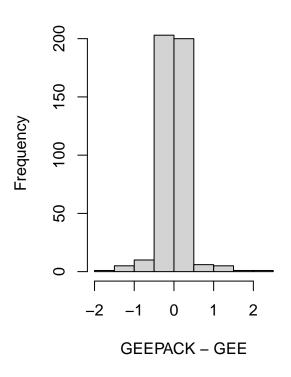


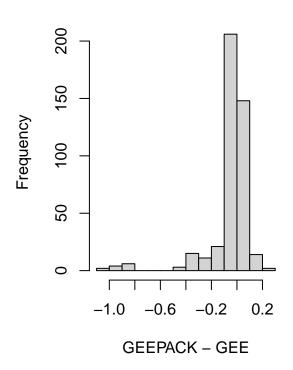
number of convergence: 500 out of 500

##	Esti	imate	Std.	Error
##	Min.	:-3.020e-14	Min.	:-1.005e-14
##	1st Qu	.:-3.608e-15	1st Qu	.:-1.221e-15
##	Median	: 0.000e+00	${\tt Median}$:-1.110e-16
##	Mean	: 8.142e-17	Mean	:-1.129e-16
##	3rd Qu	.: 3.719e-15	3rd Qu	.: 1.055e-15
##	Max.	: 3.353e-14	Max.	: 1.005e-14

checkdiff(allout\$sim1\$OUT\$geepack_exc, allout\$sim1\$OUT\$gee_exc)

Difference in se estimates



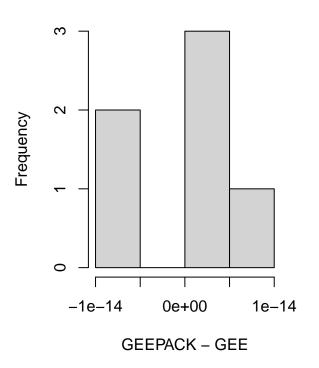


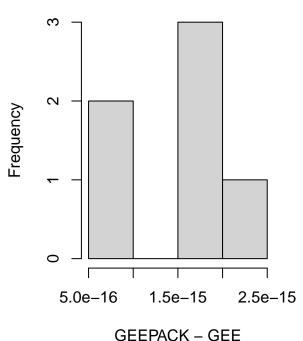
number of convergence: 72 out of 500

##	Estimate	Std. Error
##	Min. :-1.6935958	Min. :-1.0862177
##	1st Qu.:-0.0118562	1st Qu.:-0.0268160
##	Median :-0.0000346	Median :-0.0006945
##	Mean : 0.0008746	Mean :-0.0491836
##	3rd Qu.: 0.0128033	3rd Qu.: 0.0028516
##	Max. : 2.0350040	Max. : 0.2490535

checkdiff(allout\$sim1\$OUT\$geepack_ar1, allout\$sim1\$OUT\$gee_ar1)

Difference in se estimates





number of convergence: 1 out of 500

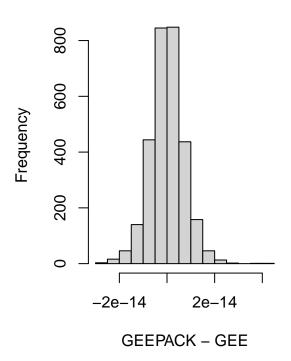
Estimate Std. Error :-7.994e-15 :8.604e-16 ## Min. Min. 1st Qu.:-5.621e-15 1st Qu.:1.152e-15 ## Median : 3.469e-16 Median :1.749e-15 Mean :-4.441e-16 Mean :1.624e-15 3rd Qu.: 2.859e-15 3rd Qu.:1.929e-15 : 8.549e-15 Max. :2.442e-15 Max.

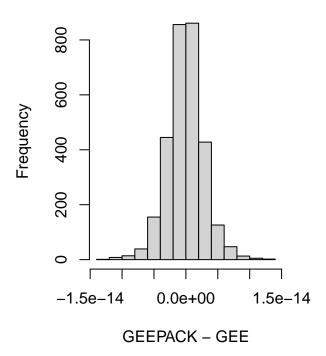
Sim2

- 1. Independence correlation shows negligible differences.
- 2. Exchangeable correlation shows noticeable differences in both packages, using the 490 converged cases from GEE.
- 3. AR-1: 0 converged case here, no comparison/comment.

checkdiff(allout\$sim2\$OUT\$geepack_ind, allout\$sim2\$OUT\$gee_ind)

Difference in se estimates



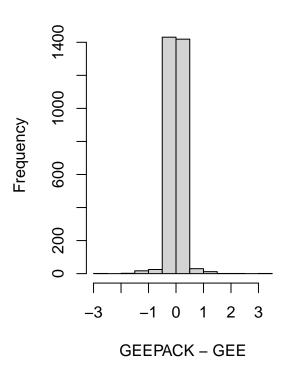


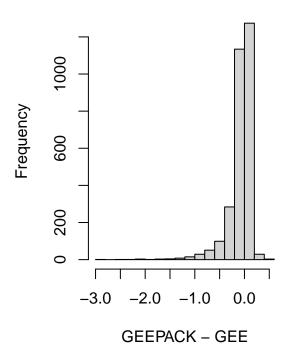
number of convergence: 500 out of 500

##	Estimate	Std. Error
##	Min. $:-2.798e-14$	Min. :-1.227e-14
##	1st Qu.:-4.219e-15	1st Qu.:-1.776e-15
##	Median : 5.551e-17	Median : 0.000e+00
##	Mean : 6.861e-17	Mean :-5.108e-17
##	3rd Qu.: 4.365e-15	3rd Qu.: 1.665e-15
##	Max. : 4.063e-14	Max. : 1.321e-14

checkdiff(allout\$sim2\$OUT\$geepack_exc, allout\$sim2\$OUT\$gee_exc)

Difference in se estimates





number of convergence: 490 out of 500

Estimate Std. Error :-2.859892 :-2.8003126 ## Min. 1st Qu.:-0.1093432 1st Qu.:-0.005257 Median : 0.000000 Median :-0.0002693 ## : 0.000007 :-0.0877836 ## Mean Mean ## 3rd Qu.: 0.005852 3rd Qu.: 0.0046210 : 0.4927737 Max. : 3.394594 Max.

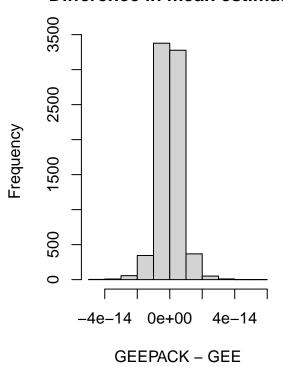
#checkdiff(allout\$sim2\$OUT\$geepack_ar1, allout\$sim2\$OUT\$gee_ar1) # no convergence cases

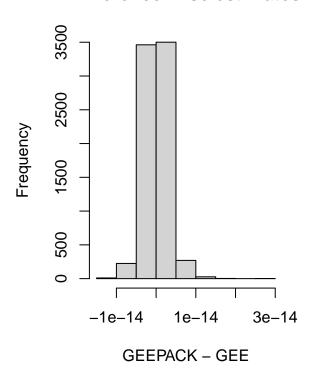
Sim3

- 1. Independence correlation shows negligible differences.
- 2. Exchangeable correlation shows noticeable differences in both packages, using the 93 converged cases from GEE.
- 3. AR-1: 0 converged case here, no comparison/comment.

checkdiff(allout\$sim3\$OUT\$geepack_ind, allout\$sim3\$OUT\$gee_ind)

Difference in se estimates



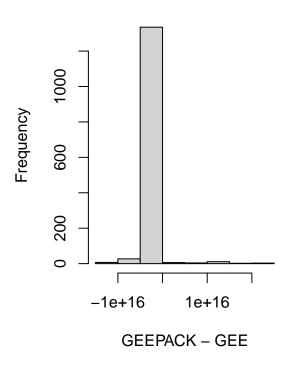


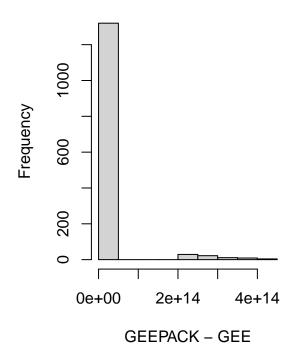
number of convergence: 500 out of 500

Std. Error ## Estimate :-4.818e-14 :-1.471e-14 ## Min. Min. 1st Qu.:-3.109e-15 1st Qu.:-1.166e-15 Median : 0.000e+00 Median : 5.551e-17 Mean : 2.910e-17 Mean : 1.084e-16 3rd Qu.: 3.109e-15 3rd Qu.: 1.277e-15 Max. : 5.018e-14 Max. : 2.653e-14

checkdiff(allout\$sim3\$OUT\$geepack_exc, allout\$sim3\$OUT\$gee_exc)

Difference in se estimates





number of convergence: 93 out of 500

```
##
       Estimate
                            Std. Error
           :-1.497e+16
                                 :0.000e+00
##
    Min.
                          Min.
    1st Qu.: 0.000e+00
                          1st Qu.:0.000e+00
   Median : 0.000e+00
##
                          Median :0.000e+00
           :-1.775e+13
    Mean
                          Mean
                                 :1.533e+13
    3rd Qu.: 0.000e+00
                          3rd Qu.:0.000e+00
##
    Max.
           : 2.108e+16
                          Max.
                                 :4.378e+14
```

#checkdiff(allout\$sim3\$0UT\$geepack_ar1, allout\$sim3\$0UT\$gee_ar1) # no convergence cases

Differences across different working correlations: Do all correlation structures generate the same estimates?

This can be answered by checking the bias and relative s.e:

- 1. Independence correlation: already proved from previous simulation that it guarantees good estimates for parameters.
- 2. Exchangeable correlation: poor estimates based on the converged results from GEE.
- 3. AR1 correlation: almost no convergence, hence no comment.

Sim1

```
## truth true_se estmean_gee_ind bias_gee_ind se_gee_ind re_gee_ind ## [1,] -1.2 0.2972 -1.2105 -0.0105 0.2984 1.0039
```

```
## [2,] -0.4 0.2980
                                                    0.2861
                                                               0.9602
                            -0.4087
                                         -0.0087
## [3,]
        0.4 0.3058
                             0.4064
                                         0.0064
                                                    0.3175
                                                               1.0383
## [4,]
        -0.1 0.2378
                            -0.1238
                                         -0.0238
                                                    0.2293
                                                               0.9643
## [5,]
       -0.4 0.2621
                            -0.3783
                                         0.0217
                                                    0.2659
                                                               1.0145
## [6,]
        1.0 0.1832
                             1.0082
                                          0.0082
                                                    0.1887
                                                               1.0301
##
       truth true_se estmean_gee_exc bias_gee_exc se_gee_exc re_gee_exc
## [1,]
       -1.2 0.2972
                         -4.3963
                                        -3.1963
                                                    0.4869
                                                               1.6382
## [2,] -0.4 0.2980
                            2.1509
                                         2.5509
                                                    0.4250
                                                               1.4262
## [3,]
        0.4 0.3058
                            3.1317
                                         2.7317
                                                    0.3842
                                                               1.2565
## [4,]
       -0.1 0.2378
                            -2.4585
                                         -2.3585
                                                    0.5207
                                                               2.1899
## [5,]
       -0.4 0.2621
                            1.9328
                                         2.3328
                                                    0.4828
                                                               1.8422
## [6,]
        1.0 0.1832
                            -0.3843
                                         -1.3843
                                                    0.1873
                                                               1.0226
```

Sim2

gee_ind
1.0050
0.9814
1.0202
0.9719
0.9971
1.0389
gee_exc
_
2.0663
1.8650
1.5319
2.4265
2.0597
1.4168

Sim3

##		truth	true_se	estmean_gee_ind	bias_gee_ind	se_gee_ind	re_gee_ind
##	[1,]	-1.50	0.4394	-1.5638	-0.0638	0.4595	1.0457
##	[2,]	-0.40	0.3852	-0.4245	-0.0245	0.3704	0.9614
##	[3,]	0.40	0.3166	0.3851	-0.0149	0.2950	0.9318
##	[4,]	-0.40	0.3088	-0.4124	-0.0124	0.2906	0.9410
##	[5,]	0.40	0.3995	0.4539	0.0539	0.4169	1.0435
##	[6,]	-0.75	0.3807	-0.7653	-0.0153	0.3762	0.9881
##	[7,]	-0.40	0.2801	-0.4140	-0.0140	0.2813	1.0044
##	[8,]	0.40	0.2910	0.4112	0.0112	0.2887	0.9919
##	[9,]	-0.40	0.3426	-0.4089	-0.0089	0.3322	0.9699
##	[10,]	0.00	0.3330	0.0005	0.0005	0.3313	0.9949
##	[11,]	-0.40	0.2486	-0.4272	-0.0272	0.2435	0.9793
##	[12,]	0.40	0.3165	0.4270	0.0270	0.3083	0.9743
##	[13,]	0.75	0.3284	0.7684	0.0184	0.3235	0.9852
##	[14,]	-0.40	0.3229	-0.3954	0.0046	0.3176	0.9836
##	[15,]	1.50	0.3176	1.5169	0.0169	0.3189	1.0042

truth true_se estmean_gee_exc bias_gee_exc se_gee_exc re_gee_exc

[1,] -1.50	0.4394	-5.0473	-3.5473	0.8046	1.8309
[2,] -0.40	0.3852	0.4381	0.8381	0.3311	0.8595
[3,] 0.40	0.3166	1.5828	1.1828	0.3338	1.0541
[4,] -0.40	0.3088	0.8646	1.2646	0.3326	1.0769
[5,] 0.40	0.3995	2.6236	2.2236	0.8562	2.1429
[6,] -0.75	0.3807	-3.8809	-3.1309	0.4556	1.1967
[7,] -0.40	0.2801	0.6118	1.0118	0.3315	1.1839
[8,] 0.40	0.2910	1.9740	1.5740	0.3139	1.0785
[9,] -0.40	0.3426	1.3557	1.7557	0.5304	1.5484
[10,] 0.00	0.3330	-1.6660	-1.6660	0.3473	1.0429
[11,] -0.40	0.2486	0.2844	0.6844	0.2976	1.1971
[12,] 0.40	0.3165	1.3952	0.9952	0.3345	1.0571
[13,] 0.75	0.3284	-0.9469	-1.6969	0.3097	0.9432
[14,] -0.40	0.3229	0.5070	0.9070	0.3461	1.0719
[15,] 1.50	0.3176	-0.2062	-1.7062	0.3080	0.9698
	[2,] -0.40 [3,] 0.40 [4,] -0.40 [5,] 0.40 [6,] -0.75 [7,] -0.40 [8,] 0.40 [9,] -0.40 [10,] 0.00 [11,] -0.40 [12,] 0.40 [13,] 0.75 [14,] -0.40	[2,] -0.40 0.3852 [3,] 0.40 0.3166 [4,] -0.40 0.3088 [5,] 0.40 0.3995 [6,] -0.75 0.3807 [7,] -0.40 0.2801 [8,] 0.40 0.2910 [9,] -0.40 0.3426 [10,] 0.00 0.3330 [11,] -0.40 0.2486 [12,] 0.40 0.3165 [13,] 0.75 0.3284 [14,] -0.40 0.3229	[2,] -0.40 0.3852 0.4381 [3,] 0.40 0.3166 1.5828 [4,] -0.40 0.3088 0.8646 [5,] 0.40 0.3995 2.6236 [6,] -0.75 0.3807 -3.8809 [7,] -0.40 0.2801 0.6118 [8,] 0.40 0.2910 1.9740 [9,] -0.40 0.3426 1.3557 [10,] 0.00 0.3330 -1.6660 [11,] -0.40 0.2486 0.2844 [12,] 0.40 0.3165 1.3952 [13,] 0.75 0.3284 -0.9469 [14,] -0.40 0.3229 0.5070	[2,] -0.40 0.3852 0.4381 0.8381 [3,] 0.40 0.3166 1.5828 1.1828 [4,] -0.40 0.3088 0.8646 1.2646 [5,] 0.40 0.3995 2.6236 2.2236 [6,] -0.75 0.3807 -3.8809 -3.1309 [7,] -0.40 0.2801 0.6118 1.0118 [8,] 0.40 0.2910 1.9740 1.5740 [9,] -0.40 0.3426 1.3557 1.7557 [10,] 0.00 0.3330 -1.6660 -1.6660 [11,] -0.40 0.2486 0.2844 0.6844 [12,] 0.40 0.3165 1.3952 0.9952 [13,] 0.75 0.3284 -0.9469 -1.6969 [14,] -0.40 0.3229 0.5070 0.9070	[2,] -0.40 0.3852 0.4381 0.8381 0.3311 [3,] 0.40 0.3166 1.5828 1.1828 0.3338 [4,] -0.40 0.3088 0.8646 1.2646 0.3326 [5,] 0.40 0.3995 2.6236 2.2236 0.8562 [6,] -0.75 0.3807 -3.8809 -3.1309 0.4556 [7,] -0.40 0.2801 0.6118 1.0118 0.3315 [8,] 0.40 0.2910 1.9740 1.5740 0.3139 [9,] -0.40 0.3426 1.3557 1.7557 0.5304 [10,] 0.00 0.3330 -1.6660 -1.6660 0.3473 [11,] -0.40 0.2486 0.2844 0.6844 0.2976 [12,] 0.40 0.3165 1.3952 0.9952 0.3345 [13,] 0.75 0.3284 -0.9469 -1.6969 0.3097 [14,] -0.40 0.3229 0.5070 0.9070 0.3461