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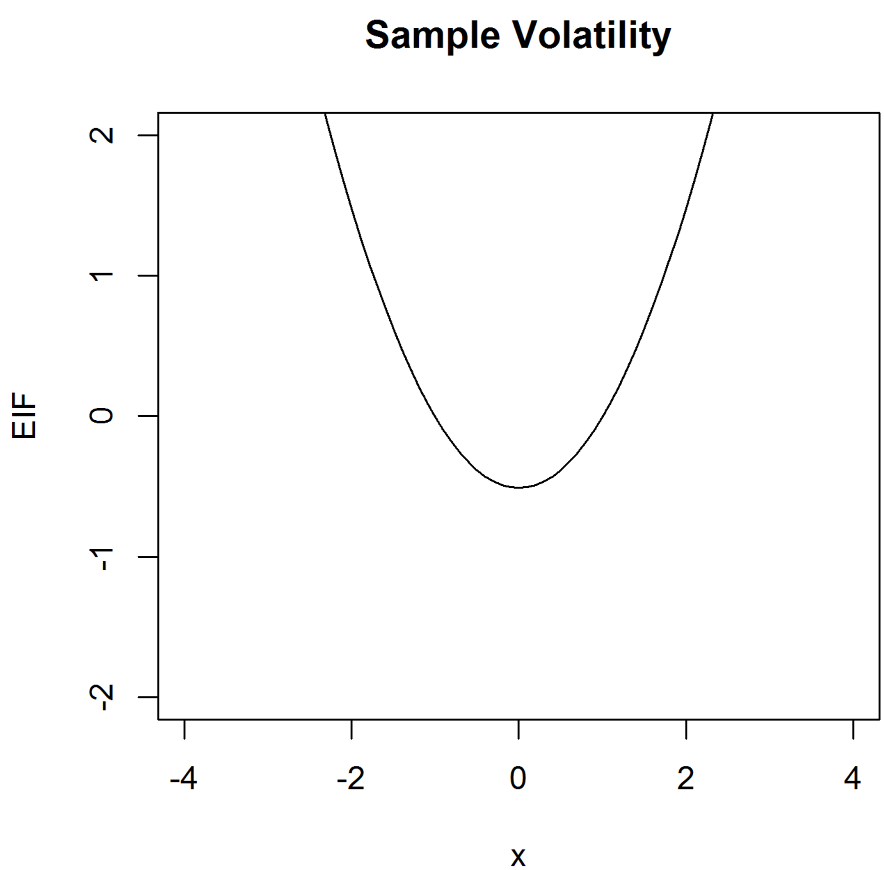
CFRM 543

Homework 3

<u>Problem 2.</u>
Modify eifPlot.R

It is intuitive for EIF takes on negative as well as positive values as in EIF we are trying to capture the influence of the outliers as well as the influence of the inliers. Negative influence corresponds to negative bias in the estimates of volatility. Sample volatility is not a robust estimator of volatility as it is very sensitive to outliers in the sample. This is because the influence of the outliers is unbounded. This can be seen in the plot.




```
n = 60
# Copmpute n standard normal quantiles
probs = (1:n - .5)/n
xn = qnorm(probs)
x = seq(-4,4,.1)
k=length(x)
```

```
eif=rep(0,k)
par(mfrow = c(2,2))
par(pty = "s")

# EIF sd
for(i in 1:k) {
  eif[i]=(n+1)*(sd(c(x[i],xn))-sd(xn))
}
plot(x,eif,type = "l",ylim = c(-2,2), ylab = "EIF",
     main = "SAMPLE MEAN")
```

Problem 3

Part a)

Modify compareETLvsVol.r

```
etl <- function(r, alpha = 0.05) {
  r <- sort(r)
  mean <- mean(r)
  n.tail <- ifelse( alpha == 0, 1, ceiling(alpha*length(r)))
  ETL <- -1/n.tail * sum(r[which((1:length(r)) <= n.tail)])
  (alpha/(dnorm(qnorm(alpha))) ) * (ETL + mean)
}

library(mpo)
ret = smallcapM[,1:8]
plot.zoo(ret)
alpha = .05
etl = apply(ret,2,etl)
stdev = apply(ret,2,sd)
ratio = etl/stdev
out = data.frame(rbind(etl,stdev,ratio))
etlname = paste(100*alpha,"% ETL",sep = "")
rownames(out) = c(etlname,"STD.DEV","ETL/S.D.")
round(out,2)
```

Output:

```
          PLXS BWINB HGIC WTS HTLD  PLAB OFG  WSBC
5% ETL    0.18  0.08 0.09 0.08 0.08 0.20 0.16 0.09
STD.DEV   0.19  0.08 0.09 0.09 0.09 0.33 0.20 0.09
ETL/S.D. 0.94  0.98 1.04 0.87 0.97 0.61 0.81 0.97
```

By comparing the above table with the plots of the returns on the eight stocks, one can see that the stocks with no abnormal spikes tend to have NETL comparable to their volatility estimates. I.e. the closer the distribution of the returns is to NORMAL distribution, the closer is NETL to volatility estimates.

Part b)

```
etl <- function(r, alpha = 0.05) {
  r <- sort(r); mean <- mean(r)
  n.tail <- ifelse( alpha == 0, 1, ceiling(alpha*length(r)))
  -1/n.tail * sum(r[which((1:length(r)) <= n.tail)])
}

netl <- function(r, alpha = 0.05) {
  r <- sort(r)
  mean <- mean(r)
  n.tail <- ifelse( alpha == 0, 1, ceiling(alpha*length(r)))
  ETL <- -1/n.tail * sum(r[which((1:length(r)) <= n.tail)])
  (alpha/(dnorm(qnorm(alpha))) ) * (ETL + mean)
}

library(mpo)
## assume risk free rate for the period is 0
rf = rep(0, 8)
ret = smallcapM[,1:8]
plot.zoo(ret)
alpha = .05
etl = apply(ret,2,etl)
netl = apply(ret,2,netl)
stdev = apply(ret,2,sd)
ratio = netl/stdev
out = data.frame(rbind(netl,stdev,ratio))
etlname = paste(100*alpha,"% NETL",sep = "")
rownames(out) = c(etlname,"STD.DEV","NETL/S.D.")
round(out,2)

starr <- (apply(ret, 2, mean)-rf)/etl
nstarr <- (apply(ret, 2, mean)-rf)/netl
sharpe <- (apply(ret, 2, mean)-rf)/stdev
ratio1 <- starr/sharpe
ratio2 <- nstarr/sharpe

out1 = data.frame(rbind(starr,sharpe,ratio1))
names = paste(100*alpha,"% STARR",sep = "")
rownames(out1) = c(names,"Sharpe","STARR/Sharpe")
round(out1,2)

out2 = data.frame(rbind(nstarr,sharpe,ratio2))
names = paste(100*alpha,"% NSTARR",sep = "")
rownames(out2) = c(names,"Sharpe","NSTARR/Sharpe")
round(out2,2)
```

Output:

```
                PLXS BWINB HGIC WTS  HTLD  PLAB OFG  WSBC
5% STARR        0.09  0.06 0.07 0.06 0.07 0.07 0.07 0.04
Sharpe          0.15  0.12 0.13 0.11 0.12 0.09 0.11 0.07
```

STARR/Sharpe	0.56	0.52	0.50	0.59	0.53	0.85	0.64	0.52

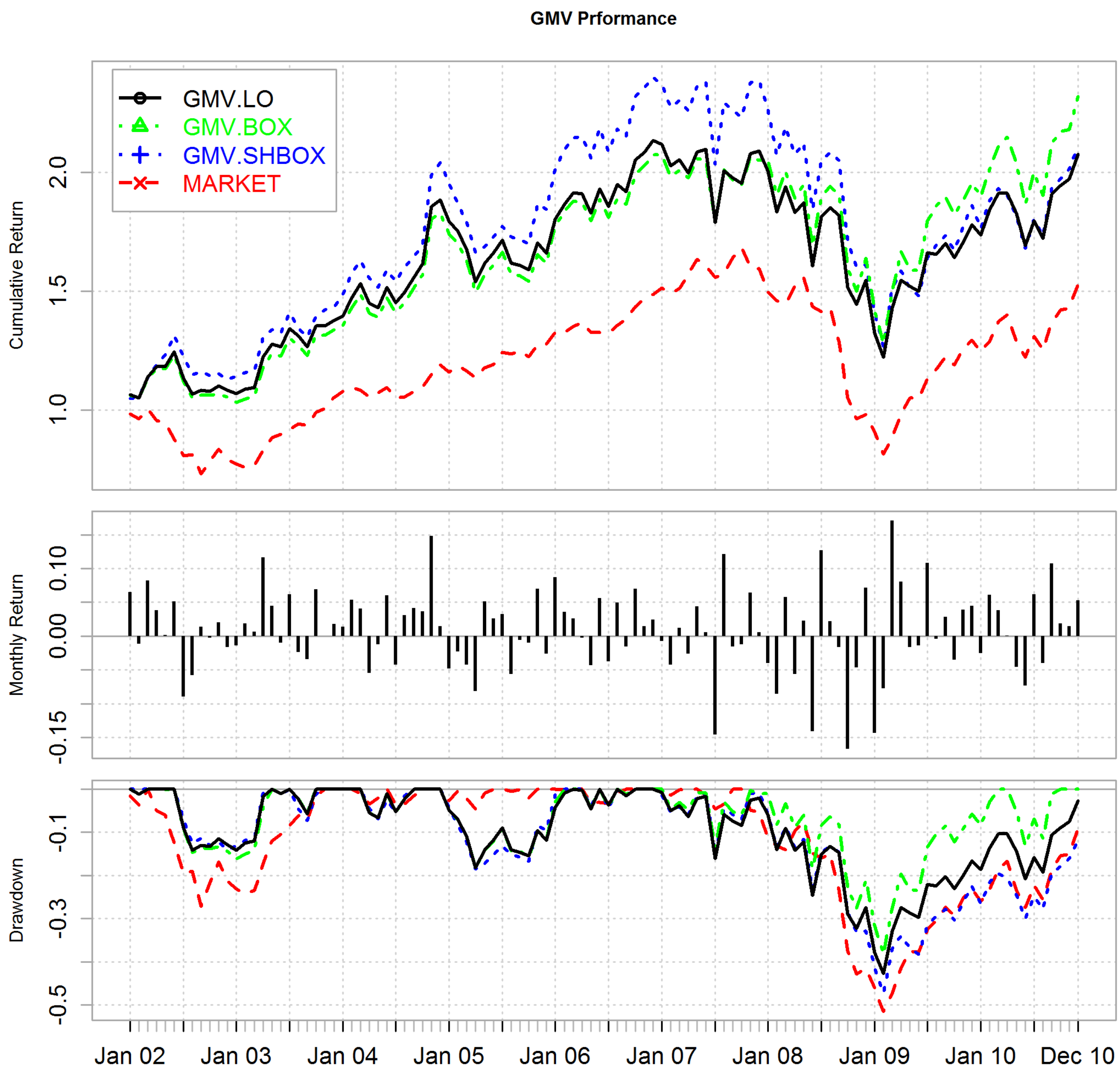
	PLXS	BWINB	HGIC	WTS	HTLD	PLAB	OFG	WSBC
5% NSTARR	0.16	0.12	0.13	0.12	0.13	0.14	0.13	0.07
Sharpe	0.15	0.12	0.13	0.11	0.12	0.09	0.11	0.07
NSTARR/Sharpe	1.06	1.02	0.96	1.14	1.03	1.64	1.24	1.03

Similar to NETL/sample volatility, the ration of STARR to Sharpe is closer to 1 for the tickers with returns distributions similar to NORMAL dist. Also, similar to the previous case tickers PLAB and OFG show NSTARR to Sharpe ration farthest away from 1 as these are the tickers showing most volatility, i.e. large spikes for year 2008-2009.

Problem 5.

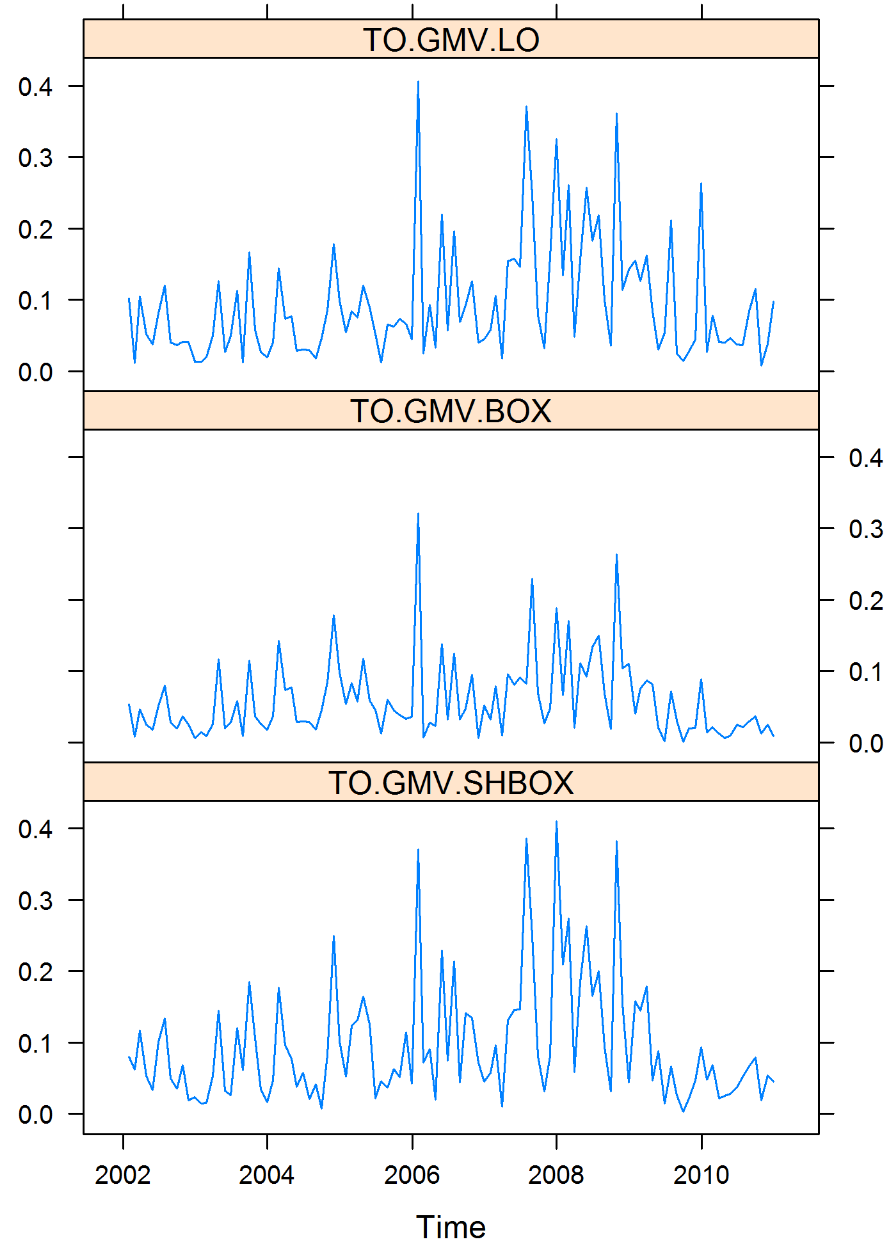
Part a)

Box portfolio has the highest diversification score as well as the lowest turnover score. It, also has better Sharpe ratio than the other two portfolios. However, its drawdown score is higher than that of, for example short box constraint portfolio. Nevertheless, I believe that box portfolio will be the most popular choice, especially among risk averse investors.



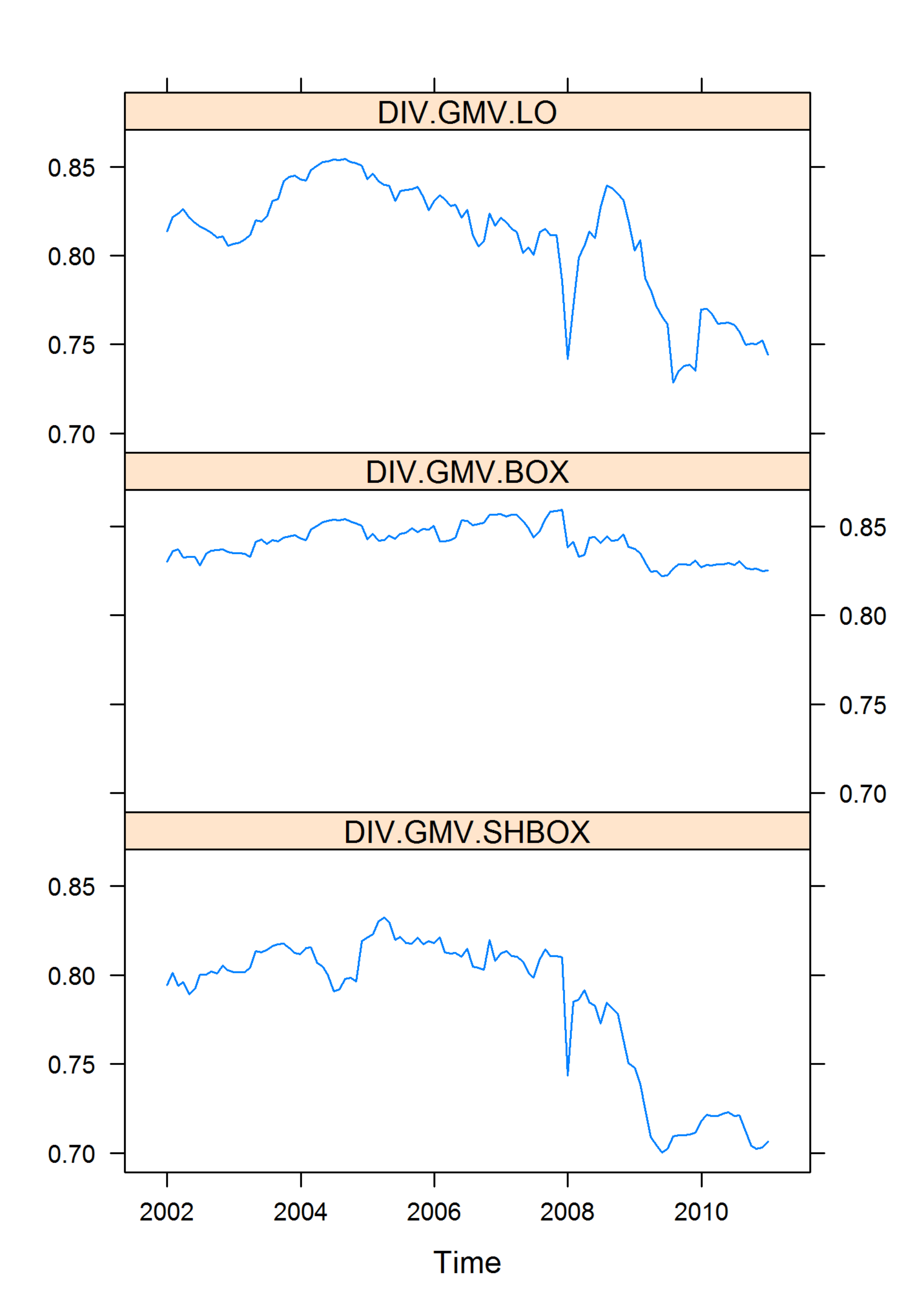
Turnover scores:

```
DIV.GMV.LO      DIV.GMV.BOX     DIV.GMV.SHBOX
   0.8162660       0.8422234       0.8016589
```



Diversification scores:

```
DIV.GMV.LO    DIV.GMV.BOX       DIV.GMV.SHBOX
    0.8162660      0.8422234      0.8016589
```




```
library(mpo)
library(PortfolioAnalytics)
library(ROI)
library(ROI.plugin.quadprog)
library(ROI.plugin.glpk)
library(lattice)
returns = smallcapM[,1:10]
MARKET = smallcapM[,"VWRETD"]
plot.zoo(returns, main = "SMALL-CAPS")
```

```
funds = colnames(returns)
mu <- apply(returns, 2, mean)

pspec = portfolio.spec(assets=funds)
pspec.fi = add.constraint(pspec, type="full_investment")
pspec.lo = add.constraint(pspec.fi, type="long_only")
pspec.gmvLo = add.objective(pspec.lo, type="risk", name="var")
pspec.box = add.constraint(pspec.fi,type="box",min=0,max=.2)
pspec.gmvBox = add.objective(pspec.box, type="risk", name="var")
pspec.shbox = add.constraint(pspec.fi,type="box",min=-0.1,max=.3)
pspec.gmvShbox = add.objective(pspec.shbox, type="risk", name="var")


# Optimize Portfolio at Monthly Rebalancing and 5-Year Training
bt.gmvLo <- optimize.portfolio.rebalancing(returns, pspec.gmvLo,
                         optimize_method="ROI",
                         rebalance_on="months",
                         training_period=60,
                         trailing_periods=60)

bt.gmvBox <- optimize.portfolio.rebalancing(returns, pspec.gmvBox,
                            optimize_method="ROI",
                            rebalance_on="months",
                            training_period=60,
                            trailing_periods=60)

bt.gmvShbox <- optimize.portfolio.rebalancing(returns, pspec.gmvShbox,
                             optimize_method="ROI",
                             rebalance_on="months",
                             training_period=60,
                             trailing_periods=60)
# Extract time series of portfolio weights
wts.gmvLo = extractWeights(bt.gmvLo)
wts.gmvBox = extractWeights(bt.gmvBox)
wts.gmvShbox = extractWeights(bt.gmvShbox)


# Compute cumulative returns of portfolio
GMV.LO = Return.rebalancing(returns, wts.gmvLo)
GMV.BOX = Return.rebalancing(returns, wts.gmvBox)
GMV.SHBOX = Return.rebalancing(returns,wts.gmvShbox)

# Combine GMV.LO, box, shbox and MARKET cumulative return0
ret.comb <- na.omit(merge(GMV.LO, GMV.BOX, GMV.SHBOX, MARKET,all=F))
names(ret.comb) = c("GMV.LO","GMV.BOX", "GMV.SHBOX","MARKET")

# return analysis
png(file="GMVPerform.png", height=7, width= 7, unit="in", res=300)
```

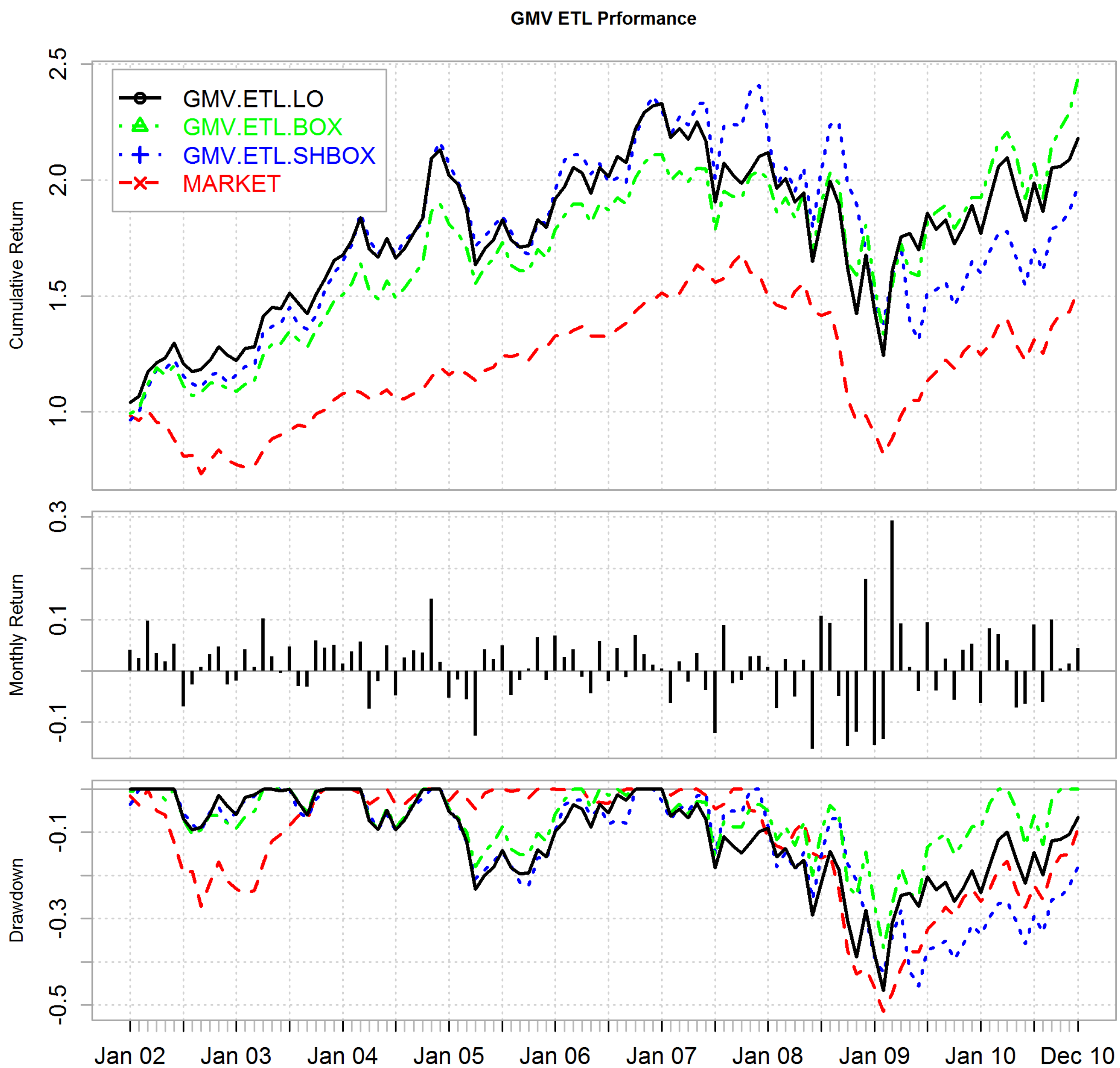
```
charts.PerformanceSummary(ret.comb, wealth.index = T,
      lty = c(1,4,3,2), colorset = c("black", "green", "blue","red"),
      cex.legend = 1.3,cex.axis = 1.3, main="GMV Prformance")
dev.off()

# Calculte the DIV (diversification) values for the time series of the weights
DIV.GMV.LO=DIV(wts.gmvLo)
DIV.GMV.BOX=DIV(wts.gmvBox)
DIV.GMV.SHBOX=DIV(wts.gmvShbox)
DIV.comb=na.omit(merge(DIV.GMV.LO, DIV.GMV.BOX, DIV.GMV.SHBOX,
                all=F))
xyplot(DIV.comb,scales=list(y="same"),main="")
ADIV.comb=sapply(DIV.comb,mean,2)
# Print average diversification values
print(ADIV.comb)

# Calculate the TO (turnover) values for the time series of weights
TO.GMV.LO=TO(wts.gmvLo)
TO.GMV.BOX=TO(wts.gmvBox)
TO.GMV.SHBOX=TO(wts.gmvShbox)

TO.comb=na.omit(merge(TO.GMV.LO, TO.GMV.BOX, TO.GMV.SHBOX,all=F))
xyplot(TO.comb,scales=list(y="same"),main="")
ATO.comb=sapply(TO.comb,mean,2)
# Print average turnover values
print(ATO.comb)
```

Part b)

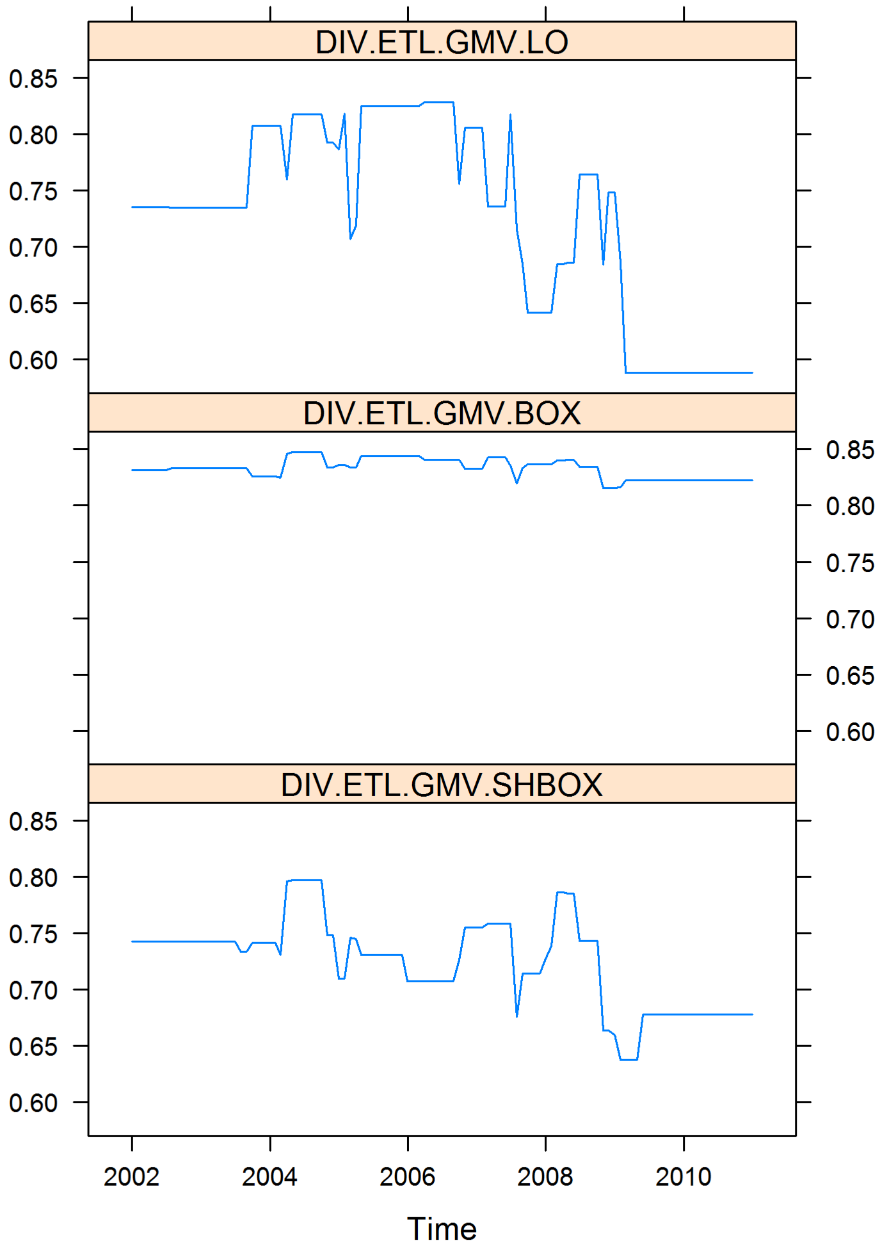
Add GMVETL portfolios with an objective to minimize Expected Tail Loss with a 5% confidence level.

```
pspec = portfolio.spec(assets=funds)
pspec.fi = add.constraint(pspec, type="full_investment")
pspec.lo = add.constraint(pspec.fi, type="long_only")
pspec.gmvLo = add.objective(pspec.lo, type="risk", name="var")
pspec.gmvETL.Lo = add.objective(pspec.lo, type="risk", name="ETL", arguments=list(p=0.95))
pspec.box = add.constraint(pspec.fi,type="box",min=0,max=.2)
pspec.gmvBox = add.objective(pspec.box, type="risk", name="var")
pspec.gmvETL.Box = add.objective(pspec.box, type="risk", name="ETL", arguments=list(p=0.95))
pspec.shbox = add.constraint(pspec.fi,type="box",min=-0.1,max=.3)
pspec.gmvShbox = add.objective(pspec.shbox, type="risk", name="var")
pspec.gmvETL.Shbox = add.objective(pspec.shbox, type="risk", name="ETL", arguments=list(p=0.95))
```

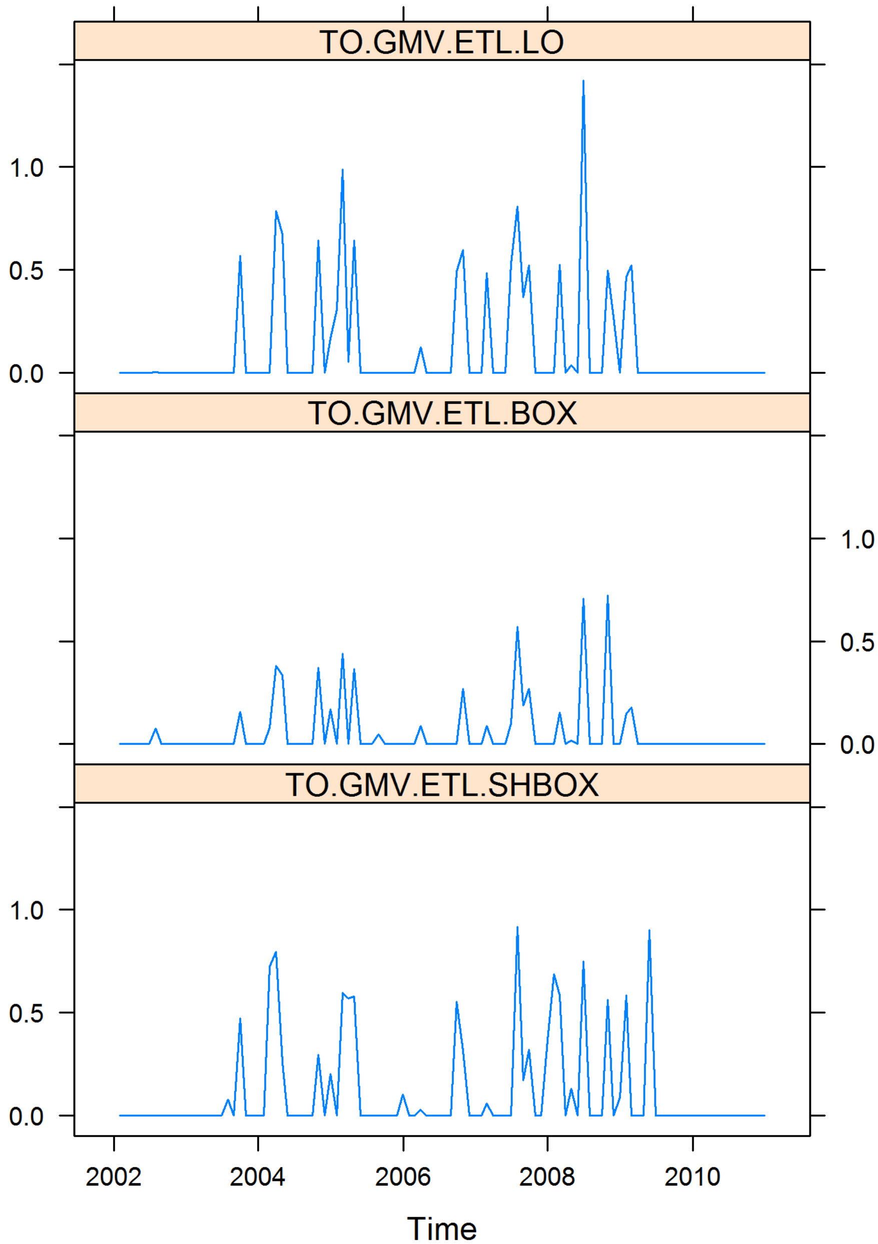
Using ETL as an objective to minimize has put box with shorting portfolio in a much less favorable light than the same constraint portfolio but with variance as an objective. For the first few month of the trading year it appears that box with shorting portfolio performed worse than long-only portfolio. The new objective has considerably lowered turnover score for all three portfolios.



DIV.ETL.GMV.LO	DIV.ETL.GMV.BOX	DIV.ETL.GMV.SHBOX
0.7351923	0.8330232	0.7336510




```
   TO.GMV.ETL.LO    TO.GMV.ETL.BOX TO.GMV.ETL.SHBOX
    3.582204e-16      2.740863e-16     2.983724e-16
```


TO.GMV.ETL.LO
TO.GMV.ETL.BOX
TO.GMV.ETL.SHBOX
0.0
0.5
1.0
2002
2004
2006
2008
2010
Time

①. SSD fails the Cash Risk Reduction coherence axiom. for sample estimate

$$SSD = \left[\frac{1}{n} \sum_{r_i \le \bar{r}} (r_i - \bar{r})^2\right]^{1/2} \quad * \quad \bar{r} = \frac{1}{n} \sum_{i=1}^{n} r_i$$

$$SSD(r_1 + c, \ldots, \bar{r} + c) = \left[\frac{1}{n} \sum_{r_i \le \bar{r}} (r_i + c - \bar{r} - c)^2\right]^{1/2}$$

$$= \left[\frac{1}{n} \sum_{r_i \le \bar{r}} (r_i - \bar{r})^2\right]^{1/2} = SSD(r_1, r_2 \ldots \bar{r})$$

②. $$LPM(2, MAR) = \int_{-\infty}^{MAR} (MAR - r)^2 \cdot f(r)\,dr$$

$$= \cancel{\left[\int_{-\infty}^{MAR} (MAR - r)^2 \cdot \frac{1}{2s} \exp\left(-\frac{|r - \mu|}{s}\right) dr\right]^{1/2}}$$

LPM(2, MAR) fails Positive Homogeneity Axiom

$$\int_{-\infty}^{MAR} (MAR - pr)^2 \cdot f(pr)\,dr$$

③. let $R = (r_1, r_2, \ldots, r_n)$ $\quad s > 0$

$$E(R) = \int_{-\infty}^{\infty} r \frac{1}{2s} \exp\left(-\frac{|r-\mu|}{s}\right) dr$$

$$\boxed{\begin{array}{l} \text{let } t = \frac{r-\mu}{s} \quad dt = \frac{1}{s} \\ r = ts + \mu \end{array}}$$

$$E(R) = \frac{1}{2} \int_{-\infty}^{\infty} (ts + \mu) \exp(-|t|)\, dt$$

$$= \mu + \frac{s}{2} \int t \exp(-|t|)\, dt = \mu + \frac{s}{2}\left[\int_{-\infty}^{\mu} t \exp(-|t|)\, dt + \int_{\mu}^{+\infty} t \exp(-|t|)\right]$$

$$= \mu + \frac{s}{2}\left[\left[-(t+1)(\exp(-|t|))\right]_{-\infty}^{\mu} + \left[-(t+1)(\exp(-|t|))\right]_{\mu}^{\infty} + c\right] = \mu + \frac{s}{2}(0) = \mu$$

$$= E(R) = \mu$$

$$\sigma = \left[\int_{-\infty}^{\infty} (r-\mu)^2 f_{DE}(r)\right]^{1/2} = \left[\int_{-\infty}^{\infty} (r-\mu)^2 \frac{1}{2s} \exp\left(-\frac{|r-\mu|}{s}\right) dr\right]^{1/2}$$

3) b) $R = (r_1, r_2, \ldots, r_n)$

let $f(R \mid (\mu, s))$ be the density of R which depends on parameters μ and s. Then $\mathcal{L}(\mu, s) = f(R \mid (\mu, s))$ which is a function of μ and s where R is fixed.

$$\log(\mathcal{L}(\mu, s)) = \log(f(R \mid (\mu, s)) = \log\left(\frac{1}{2s} \exp\left(-\frac{|r - \mu|}{s}\right)\right)$$

$$= n(\log(1) - \log(2s)) - \frac{1}{s} \sum_{i=1}^{n} (|R_i - \mu|) = -n \log(2s) - \frac{1}{s} \sum_{i=1}^{n} |r_i - \mu|$$

c) $$\frac{d}{d\mu} \log(\mathcal{L}(\mu)) = \frac{-\sum_{i=1}^{n} 1}{s} = \frac{-n}{s} = \hat{\mu} = \bar{R}$$

d) $$\frac{d}{ds} \log(\mathcal{L}(s)) = \frac{\sum_{i=1}^{n} |r_i - \bar{R}|}{n}$$