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Homework 2

Problem 1

Under full investment and no shorting constraint, HHI(w) has minimum value of 1/n and it is achieved when all assets have equal weights. HHI(w) has the maximum value of 1. The minimum value can be proven using Lagrange multiplier as the function has only one minimum since there is only one way to arrange the assets in such a way so that all weights are equal (i.e. 1/n). The maximum occurs when all investment is concentrated in one of the assets with all other assets' weights set to 0. However, for N > 1 assets there is N ways of arranging the weights and they will all yield the same result: HHI(w) = 1. Hence, the function has more than one maximum and the Lagrange multiplier approach will not work in this case.

Problem 2

Under full investment and no shorting constraint, the maximum value of turnover is 2. This happens when the weights at time T are the exact opposite of the portfolio weights at time T-1. Since we are assuming full investment and no shorting constraint this means that the sum of the weights at time T has to be equal to 1 and the sum of the weights at time T-1 is also equal to one but with the exact opposite concentration in weights, as follows **wt** = [w1, w2, …, wn] and **wt-1**=[1-wt1, 1-wt2, …, 1-wtn]. Then $TO(wt, wt-1) = \sum_{i=1}^{n} |\mathrm{wt,i} - \mathrm{wt} - 1, \mathrm{i}|$ can be rewritten as $\sum_{i=0}^{n} |\mathrm{wt,i}| = \sum_{i=0}^{n} |\mathrm{wt} - 1, \mathrm{i}|$ so 1 = 1. For every non-zero value in wt vector we will have a zero value in wt-1 vector and vice versa with both vectors still summing up to 1. This leads to $TO(wt, wt-1)$ = 1 + 1=2.

Problem 3

As seen below, we are able to achieve the lowest variance portfolio with long only constraint. This is, also, the portfolio with the highest Sharpe Ratio. The general rule is that the more constraint are added to the portfolio, the lower the Sharp ratio. We can also see another rule at work when comparing the three portfolios which is that we can achieve a higher portfolio return but not without taking on more risk. The higher the mean return, the higher the deviation.

	MEAN	SD	SharpeRatio
gmv.lo	0.0061	0.0159	0.3854
gmv.box	0.0103	0.0454	0.2271
gmv.shbox	0.0108	0.0393	0.2745

The variances of the individual assets are as follows:

MODI	MGF	MEE	FCEL	OII	SEB	RML	AEOS	BRC	CTC
0.0107	0.0003	0.0217	0.0732	0.0269	0.0156	0.0128	0.0574	0.0120	0.0146

The expected returns of the assets are as follows:

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MODI      MGF      MEE    FCEL      OII      SEB      RML    AEOS      BRC      CTC
 0.006   0.005   0.017   0.069   0.018   0.010  -0.003   0.079   0.014  -0.001
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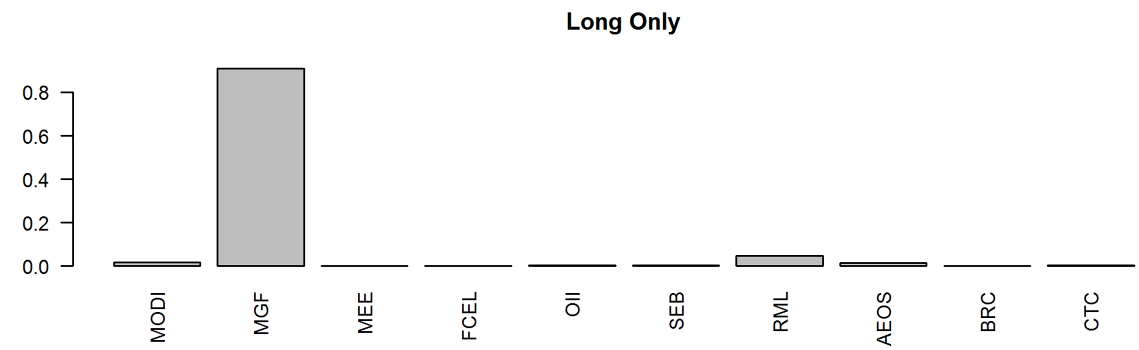
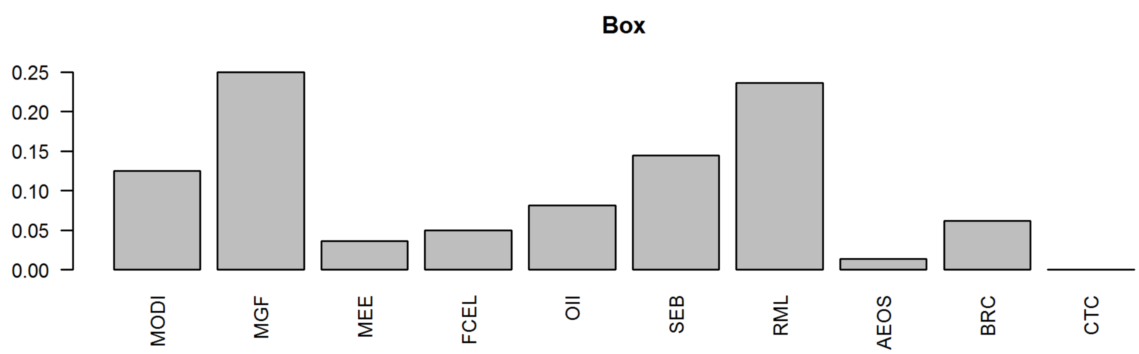
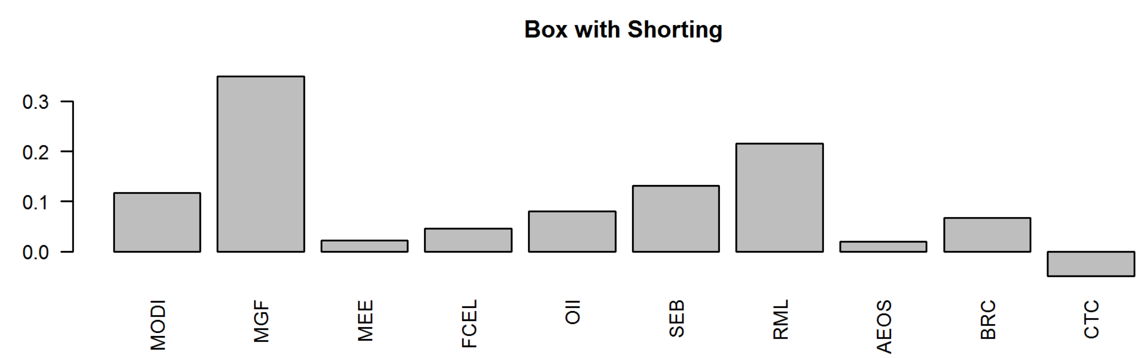
Being that ticker MGF has the lowest variance, we expect to see the highest concentration in that asset when constructing and optimum min variance portfolio. This is in fact what we are seeing in all three portfolios in the below 3 plots. Next, if no other constraints are present, the assets with the least correlation with MGF and one another are assigned weights so that a total asset weights add up to 1. As expected, we see that the weight of MGF is the max allowed weight in the next two portfolios. And, CTC with its low variance and low return is the first stock to be shorted when shorting is allowed.

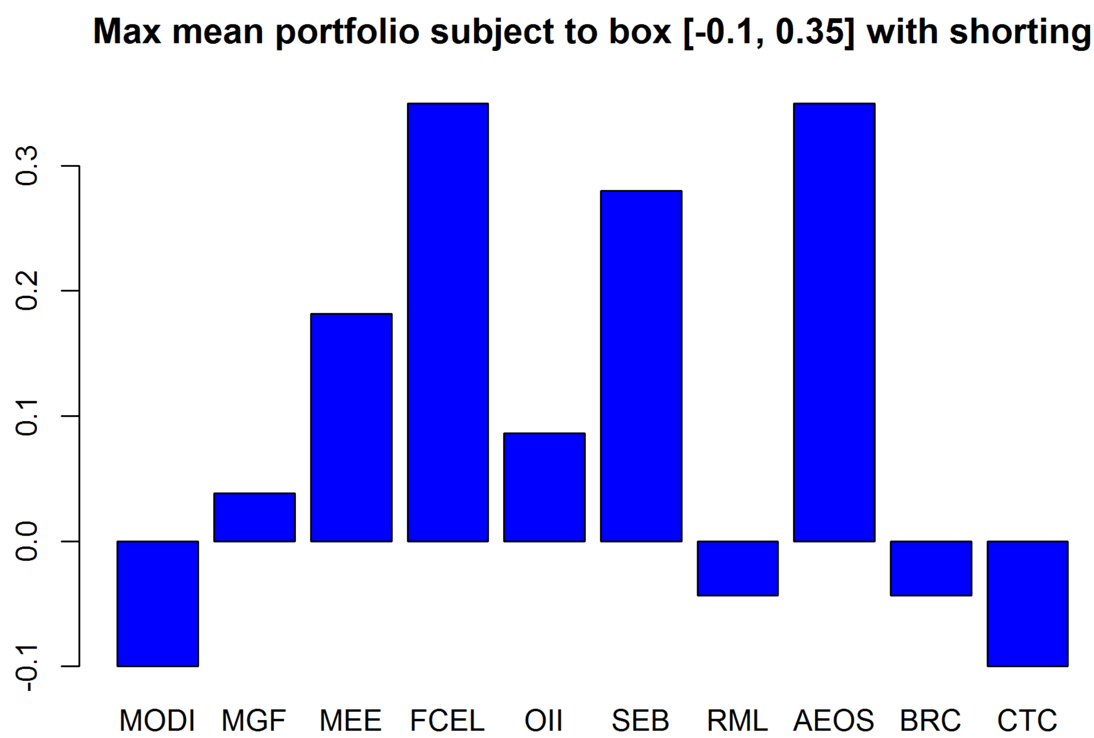






Problem 4

Part a)



Part b)

Target mean = 0.03469789

Target mean portfolio weights:

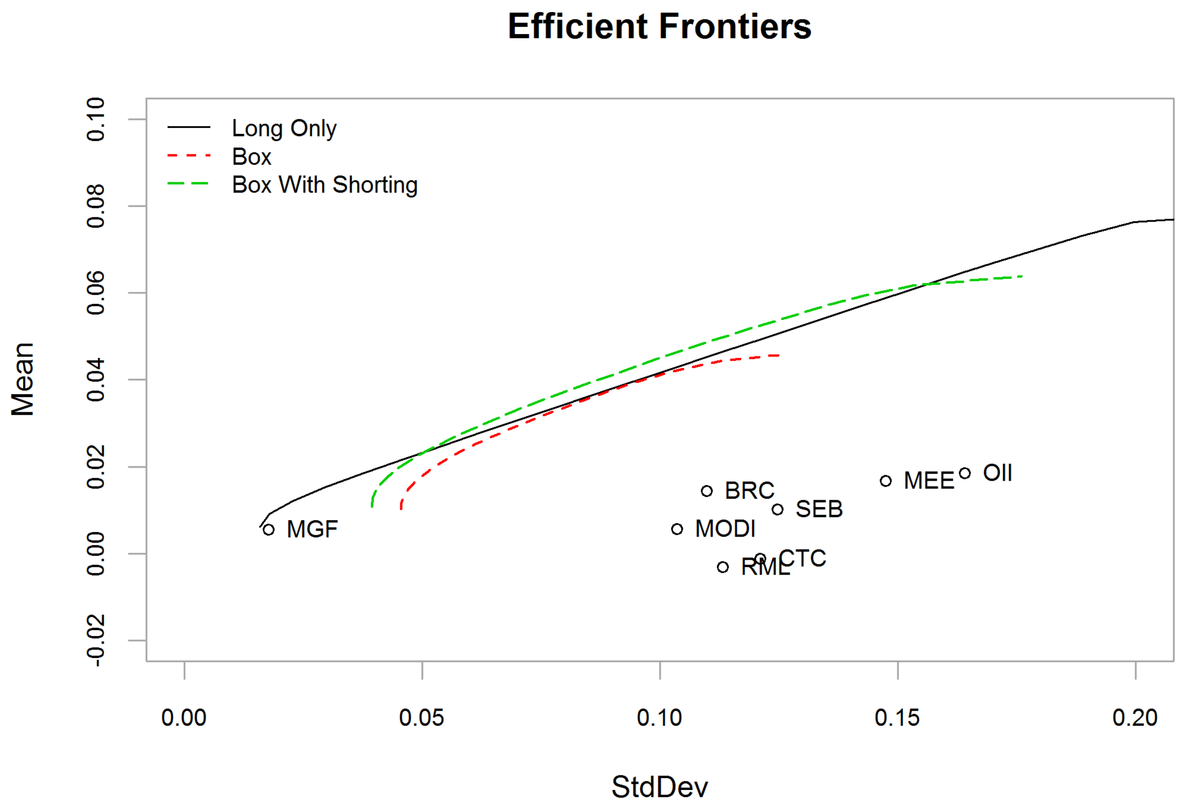
MODI	MGF	MEE	FCEL	OII	SEB	RML	AEOS	BRC	CTC
-0.077	0.647	-0.023	0.142	0.101	0.165	0.126	0.214	0.110	-0.406

<u>Problem 5</u>

lambda = 3

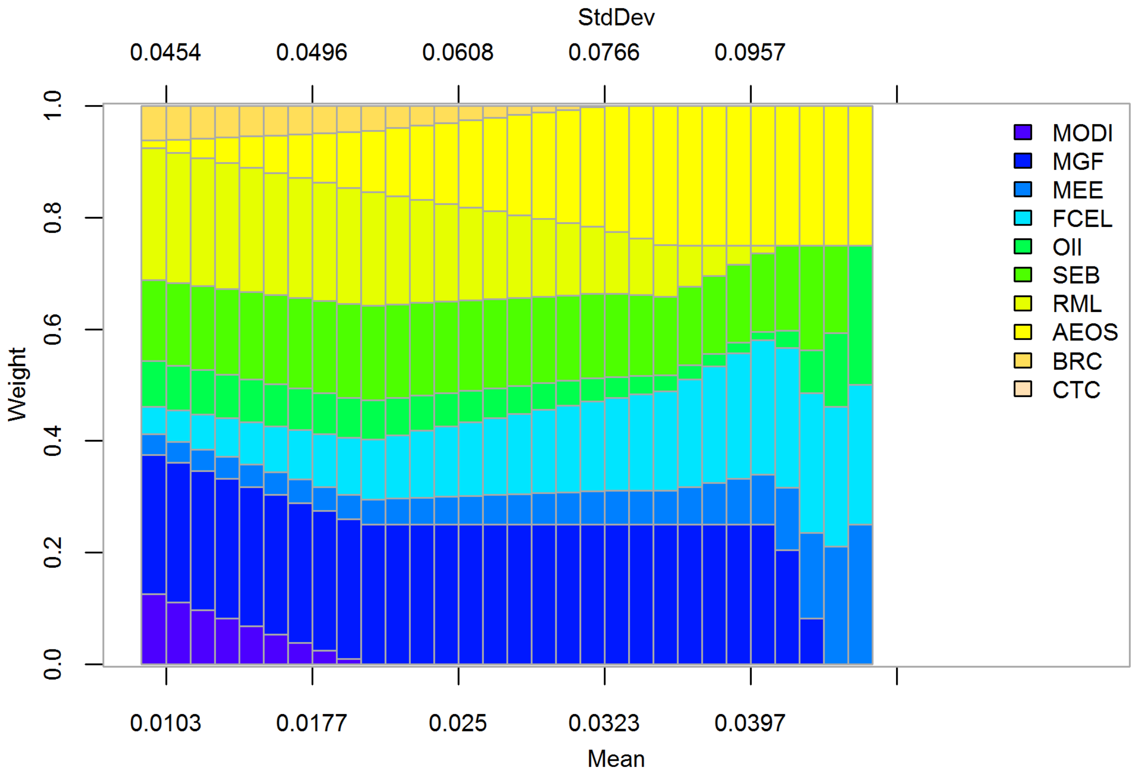
mu = 0.03382412

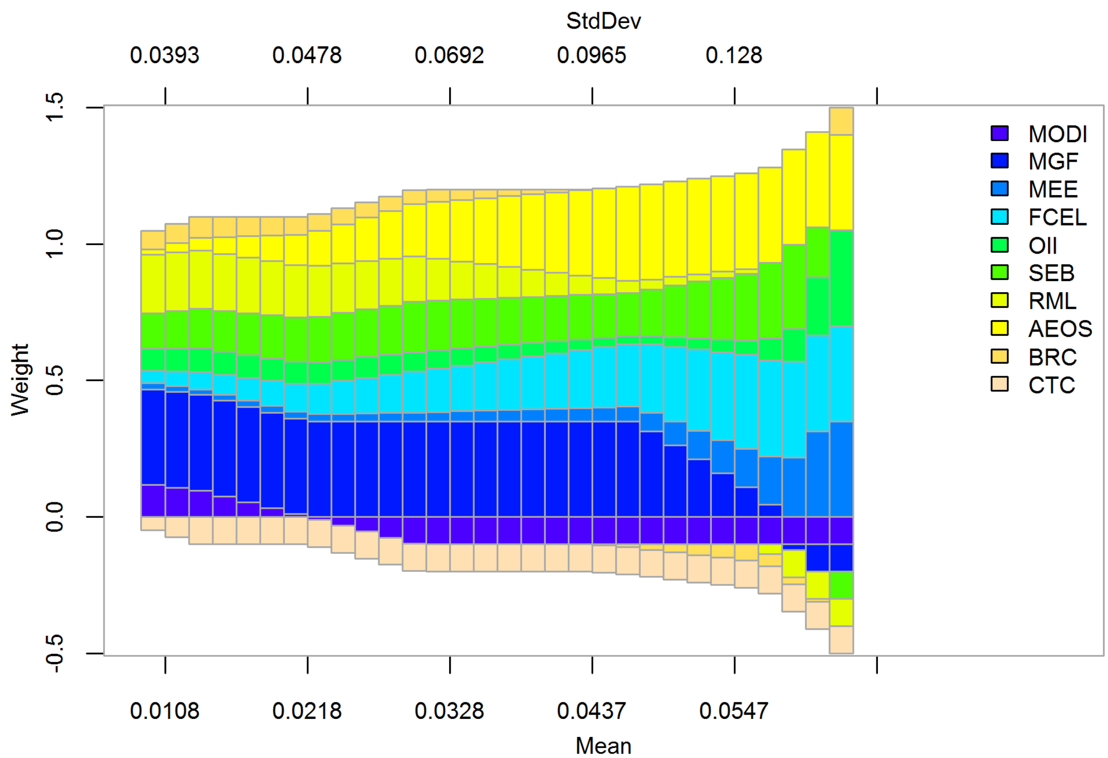
Problem 6



In the next two plots, one can see that CTC is the first stock to be shorted due to its low mean return and relatively low risk. The short position for this stock increases with increasing sigma of the portfolio. Analogous to this, the next good stock to short is MODI. The only stocks that are never shorted are FCEL and MEE.

The first plot shows portfolio weights along the efficient frontier subject to box constraint. The second plot shows portfolio weights along the efficient frontier subject to box with shorting constraint.


StdDev
0.0454
0.0496
0.0608
0.0766
0.0957
Weight
1.0
0.8
0.6
0.4
0.2
0.0
0.0103
0.0177
0.025
0.0323
0.0397
Mean
MODI
MGF
MEE
FCEL
OII
SEB
RML
AEOS
BRC
CTC



StdDev
0.0393
0.0478
0.0692
0.0965
0.128
Weight
1.5
1.0
0.5
0.0
-0.5
MODI
MGF
MEE
FCEL
OII
SEB
RML
AEOS
BRC
CTC
0.0108
0.0218
0.0328
0.0437
0.0547
Mean