Congratulations! You passed!

Grade received 100%

To pass 80% or higher

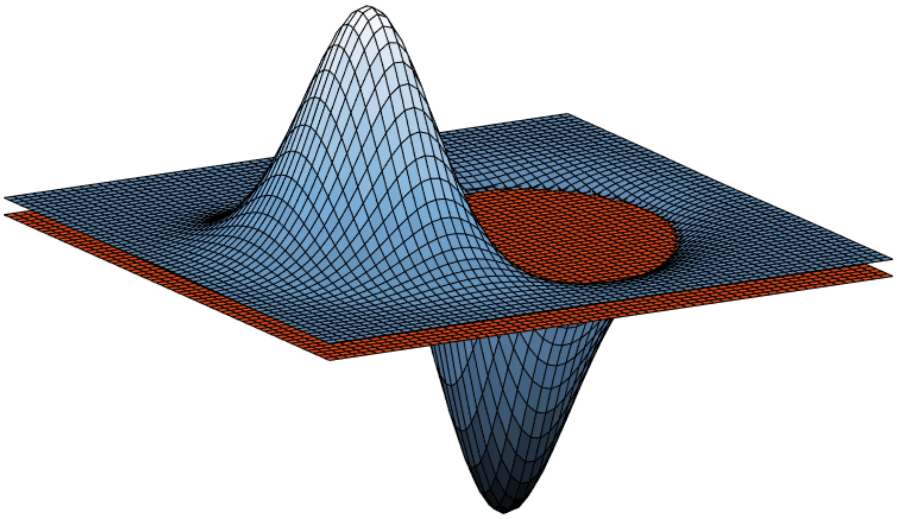
Go to next item

1. Now you have seen how to calculate the multivariate Taylor series, and what zeroth, first and second order approximations look like for a function of 2 variables. In this course we won't be considering anything higher than second order for functions of more than one variable.

1 / 1 point

In the following questions you will practise recognising these approximations by thinking about how they behave with different x and y, then you will calculate some terms in the multivariate Taylor series yourself.

The following plot features a surface and its Taylor series approximation in red, around a point given by a red circle. What order is the Taylor series approximation?

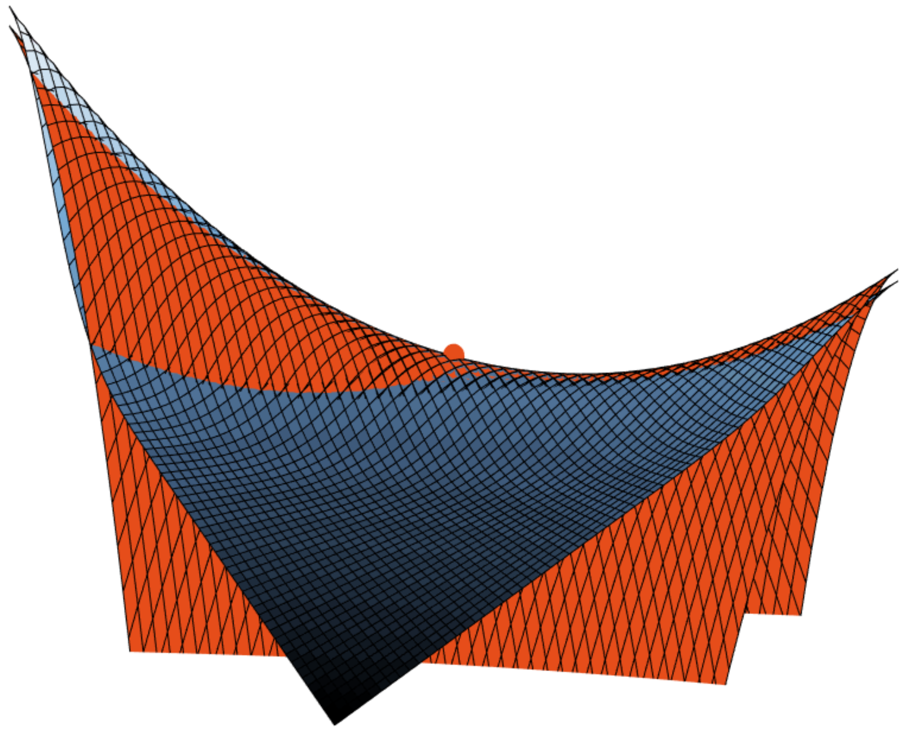


- [x] Zeroth order
- [ ] First order
- [ ] Second order
- [ ] None of the above

Correct
The red surface is constant everywhere and so has no terms in $\Delta\mathbf{x}$ or $\Delta\mathbf{x}^2$

2. What order Taylor series approximation, expanded around the red circle, is the red surface in the following plot?

1 / 1 point



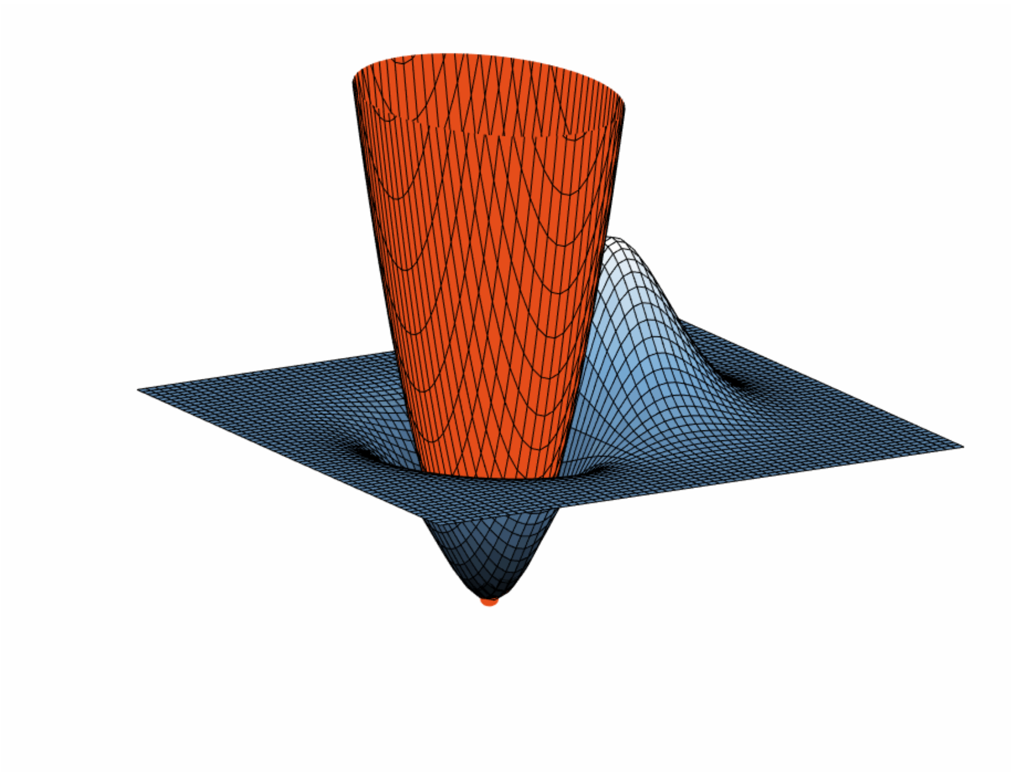
- ○ Zeroth order
- ○ First order
- ◉ Second order
- ○ None of the above

Correct
The gradient of the surface is not constant, so we must have a term of higher order than $\Delta\mathbf{x}$.

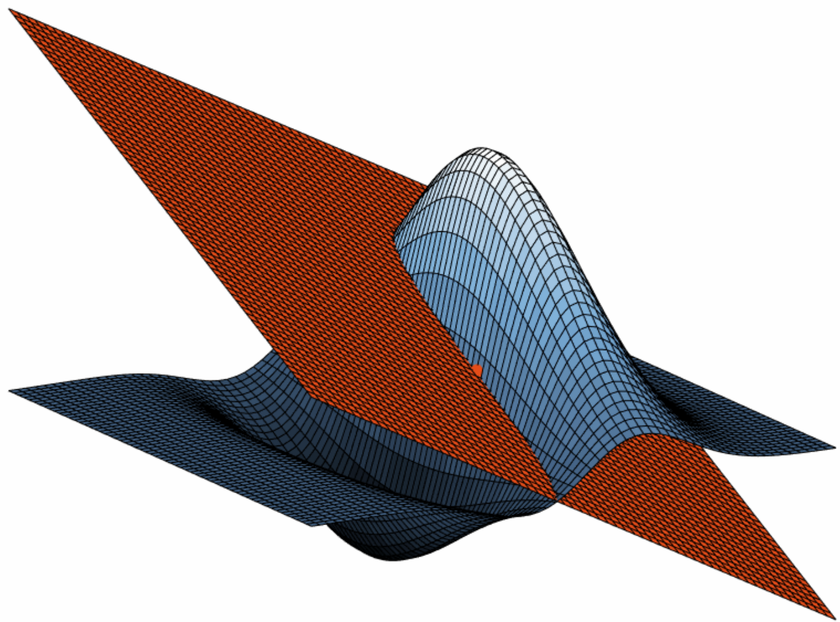
3. Which red surface in the following images is a first order Taylor series approximation of the blue surface? The original functions are given, but you don't need to do any calculations.

1 / 1 point

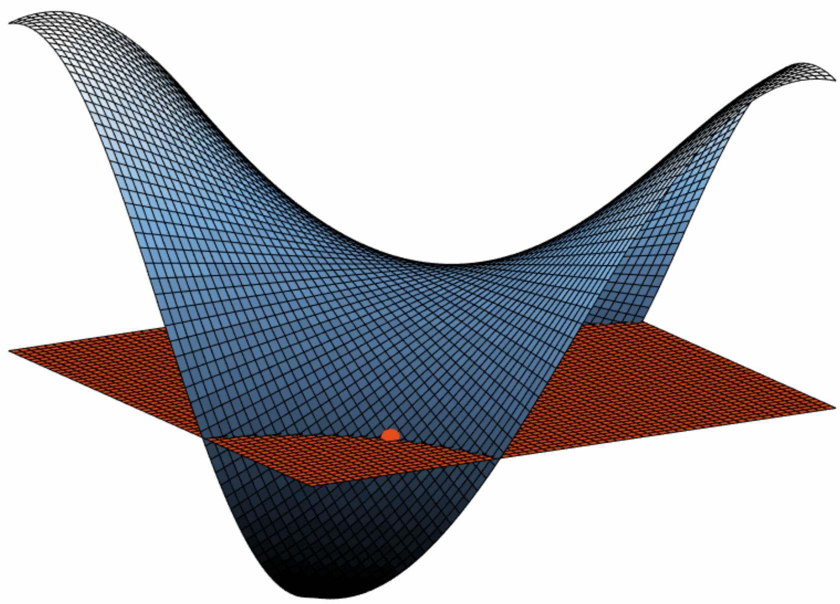
○ $f(x, y) = xe^{-x^2-y^2}$



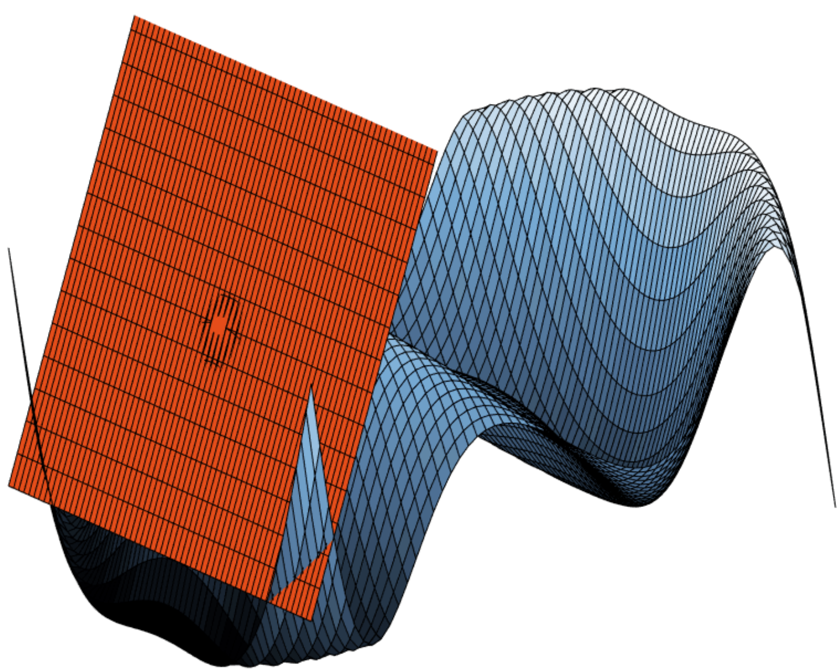
○ $f(x, y) = (x^2 + 2x)e^{-x^2-y^2/5}$



○ $f(x, y) = \sin(xy/5)$



◉ $f(x, y) = x\sin(x^2/2 + y^2/4)$



Correct

The gradient of the red surface is non-zero and constant, so the $\boldsymbol{\Delta}\mathbf{x}$ terms are the highest order.

4. Recall that up to second order the multivariate Taylor series is given by $f(\mathbf{x} + \boldsymbol{\Delta}\mathbf{x}) = f(\mathbf{x}) + J_f\boldsymbol{\Delta}\mathbf{x} + \frac{1}{2}\boldsymbol{\Delta}\mathbf{x}^T H_f \boldsymbol{\Delta}\mathbf{x} + \ldots$

1 / 1 point

Consider the function of 2 variables, $f(x, y) = xy^2 e^{-x^4 - y^2/2}$. Which of the following is the first order Taylor series expansion of f around the point $(-1, 2)$?

- ( ) $f_1(-1 + \Delta x, 2 + \Delta y) = -4e^{-3} + 16e^{-3}\Delta x - 8e^{-3}\Delta y$
- ( ) $f_1(-1 + \Delta x, 2 + \Delta y) = -4e^{-3} - 4e^{-3}\Delta x + 4e^{-3}\Delta y$
- (•) $f_1(-1 + \Delta x, 2 + \Delta y) = -4e^{-3} - 12e^{-3}\Delta x + 4e^{-3}\Delta y$
- ( ) $f_1(-1 + \Delta x, 2 + \Delta y) = 2e^{-33/2} - 63e^{-33/2}\Delta x - 2e^{-33/2}\Delta y$

Correct

5. Now consider the function $f(x, y) = \sin(\pi x - x^2 y)$. What is the Hessian matrix H_f that is associated with the second order term in the Taylor expansion of f around $(1, \pi)$?

1 / 1 point

- ( ) $$\begin{pmatrix} -\pi^2 & \pi \\ \pi & -1 \end{pmatrix}$$
  $H_f = \begin{pmatrix} -\pi^2 & \pi \\ \pi & -1 \end{pmatrix}$

- (•) $$\begin{pmatrix} -2\pi & -2 \\ -2 & 0 \end{pmatrix}$$
  $H_f = \begin{pmatrix} -2\pi & -2 \\ -2 & 0 \end{pmatrix}$

○

$$\begin{pmatrix} -2 & -2\pi \\ 2\pi & 1 \end{pmatrix}$$

$H_f = \begin{pmatrix} -2 & -2\pi \\ 2\pi & 1 \end{pmatrix}$

○

$$\begin{pmatrix} -2\pi & -2 \\ -2 & 1 \end{pmatrix}$$

$H_f = \begin{pmatrix} -2\pi & -2 \\ -2 & 1 \end{pmatrix}$

Correct

Good, you can check your second order derivatives here:

$$\partial_{xx}f(x,y) = -2y\cos(\pi x - x^2 y) - (\pi - 2xy)^2 \sin(\pi x - x^2 y)$$

$$\partial_{xy}f(x,y) = -2x\cos(\pi x - x^2 y) - x^2(\pi - 2xy)\sin(\pi x - x^2 y)$$

$$\partial_{yx}f(x,y) = -2x\cos(\pi x - x^2 y) - x^2(\pi - 2xy)\sin(\pi x - x^2 y)$$

$$\partial_{yy}f(x,y) = -x^4 \sin(\pi x - x^2 y)$$