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ASSET PRICING

Testing the Liquidity Factor for the Cross-Section of Stock Returns

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Abstract

This project investigates the ability of both CAPM as presented by Sharpe (1964) and Lintner (1965) and the liquidity factor as in Pastor and Stambaugh (2003, Journal of Political Economy) to explain the cross-section of portfolio returns. We use the same aggregate liquidity factor as Pastor and Stambaugh (2003), which is defined to be an average measure of individual portfolio measures estimated with monthly data. Over both our initial period (Jan-1968 to Aug-2008) and the subsequent period (sep-2008 to Dec-2018), we also use innovations in liquidity and traded liquidity as additional factors. Over these periods, we find that average return on our portfolios with high sensitivities to liquidity exceeds that for portfolios with lower sensitivities. We can then conclude, just as Pastor and Stambaugh did, that liquidity risk is, in fact, a priced factor.

1 Introduction

In economic and asset pricing theory it seems reasonable that investors require higher returns on investments that have higher risk. [Measured by either volatility, Glosten Jagannathan and Runkle (1993 journal of finance) or sensitivity to risk factors as presented by Chan, Karceski and Lakonishok (1998, Journal of Financial and Quantitative Analysis)]. Hence, we set out to research the extent to which one factor in particular, market-wide liquidity, accounts for systematic variations in portfolio returns and the extent to which it can explain the cross-section of portfolio returns.

We run Fama Macbeth regressions on excess returns and liquidity factors and measure how those returns change for in an out of sample data. Ever since Fama, Macbeth and Black, Jensen and Scholes began forming portfolios of betas with the market, the testing of asset pricing models has followed a simple loop:

1. Find a characteristic or factor that you think is associated with average returns. Sort stocks into portfolios based on the characteristic, and check if there is a difference in average returns between portfolios.
2. Calculate betas for these portfolios and assess whether or not the average return is accounted for by the spread in betas.
3. If not, you have an anomaly. Consider multiple betas.

brief results: This study finds that the expected return on a portfolio in excess of the risk free rate is explained by its sensitivity of its return to these factors ... This model captures the cross-sectional variation of average stock returns

2 Literature Review (*Optional*)

In our data we consider aggregate liquidity which indicates how easily financial transactions can be made in the market or in our case, since we consider stocks, we have a measure of how easily a stock/asset can be traded. Undoubtedly, liquidity is a measure of representation of the state of the market. For instance, during a market crisis we can see that there is a drop-in liquidity in the market since transactions cannot be executed as easily as other times. We can observe this by looking at the following graph from Pastor and Stambaugh data which we include in our project: We can identify some notable spikes in the graph of aggregate liquidity during world renowned market crashes such as:

- Black Monday in 1987 when there was a sudden market crash in October that lasted a day and the market recovered after 2 years.
- The Dot-com bubble burst which happened in March of 2000 and would in fact take another 15 years for the market to regain its dotcom peak. It is necessary here to comment on the rise of

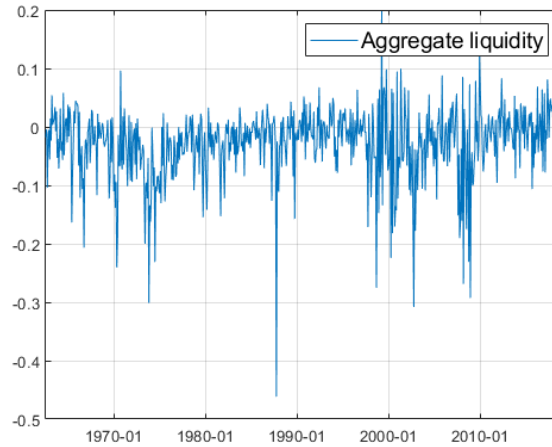


Figure 2.1: Caption

liquidity in the late 90's since investors were affected by the rapid technological development and were investing irresponsibly in companies that were doomed to fail which eventually lead to the bubble burst in 2000.

- The Great Recession which lasted from 2007 to 2009 after the housing market crash and the bankruptcy of Lehman Brothers.

2.1 Expected Conclusions:

Consequently, if we take the above into consideration, we can argue that the returns of our stock portfolios would have also been affected by such market crashes. Intuition supports the fact that after a market crash, we will see a violent change in our returns and liquidity factors. However, we need to contemplate also the role that the Federal Reserve System or central banks in general have in such circumstances. Central banks aim to stabilize the economic growth when a market crisis happens. One of the monetary policies that a central bank will conduct is, they will generally lower interest rates and thus lowering financing costs which will promote borrowing and investing. For example, after the Housing market crash the Fed lowered a key interest rate to essentially zero while also providing the banks with an amount of \$7.7 trillion of emergency loans, a monetary policy also known as quantitative easing (Theweek.com, 2011). These tools helped stabilize the liquidity in the market after the crisis. In conclusion, our expected returns should be lower when we re-estimate the model out-of-sample (including the period from September 2008 to 2019). Although we expect to see a massive drop in our expected returns the following years and to continue decreasing, the Fed's immediate actions and measures contained the catastrophe. The (aggregate)liquidity factors and hence our expected returns should be stabilized through the following years, but they will not reach the same height they had before the market crash since after 2008 banks adapted to a more restrictive policy for lending money.

Also, assuming the investors are rational, i.e. our portfolios obtained should be diversifiable (since you don't get compensated for taking on risk that could be diversified away) we can interpret our results as follows: If we include the dates of the Great Recession then we'd have a rise in systematic risk since now the market is at a low point and everyone is inherent to that risk. As a result, equity risk premium should be higher get compensated more since they take on more systematic risk. A potential reason for this positive change is the fact that after the market crisis of 2008 the equity risk premiums are of higher value than those before as equity investors would want to be compensated more for taking on more systematic risk.

2.2 Something...

2.3 Something else...

3 Methodology

Our data for the U.S. stock portfolio returns is sorted into 25 portfolios according to size and momentum. Therefore there is no need to re-balance the portfolios using a rolling window. We implement the Fama-MacBeth regression as follows:

First, we run time series regression to estimate betas for each portfolio from excess returns regressed on to factor returns. Here we obtain a beta corresponding to each factor within the model for every portfolio. Then we apply cross-sectional regression at each period where we regress excess returns on our estimated betas. From this, we end up with a gamma for each factor for each month within the sample. For each sample, we take the arithmetic average across the period and denote this by $\lambda_i = \frac{1}{T} \sum_{t=1}^T \gamma_{i,t}$ for ease of interpretation and comparison.

The Fama Macbeth regression takes the form of:

$$\mathbb{E}[R_i] - r^f = \lambda_0 + \beta_{1,i}\lambda_1 + \beta_{2,i}\lambda_2 + \beta_{3,i}\lambda_3 + \beta_{4,i}\lambda_4 + \epsilon_{i,t} \quad (1)$$

where $R_{i,t}$ is the return for portfolio i at time t , r_t^f is the risk free rate corresponding to the period. The γ_{factor} is the risk factor premium, with γ_0 corresponding to the intercept. $\beta_{factor,i}$ is the sensitivity of portfolio i to the respective factor, found through conducting the initial time series regression.

The above was implemented in Matlab, and details of this can be found in the appendix. We test the statistical significance of the parameters found by calculating the corresponding t-statistic using the Newey West estimator, which accounts for heteroscedasticity and autocorrelation using the default Bartlett method. In Matlab, we used the `hac.m` function to get an adjusted covariance matrix to calculate t-statistics. As the statistics behind our methodology is beyond the scope of this project, we will not discuss the validity of this approach but acknowledge that this method is necessary in order to have valid t-statistic comparisons.

It is worth noting that we run OLS at each stage in Fama Macbeth, and so we are assuming that the errors are normally distributed with a conditional mean of 0.

In this study we will be fitting the following models:

CAPM: $R_{i,t} - r_t^f = \gamma_{0,t} + \beta_{1,i}\gamma_{1,t} + \epsilon_{i,t}$

where the parameters are as described above and $\gamma_{1,t}$ corresponds to excess market return $\mathbb{E}[r_{M,t} - r_t^f]$

Factor models: $R_{i,t} - r_t^f = \gamma_{0,t} + \beta_{1,i}\gamma_{1,t} + \beta_{2,i}\gamma_2 + \epsilon_{i,t}$

where the parameters are as described above and γ_2 corresponds to one of three liquidity factors

For completeness we also fit a saturated model containing all additional factors.

One would anticipate for the CAPM model to find that the term corresponding to the intercept $\lambda_0 = 0$ if the CAPM model holds true.

Also we would also expect to observe a positive return-risk trade off within the market.

These are all confirmed by conducting the following hypothesis test at a 5% significance level.

$$H_0 : \lambda_i = 0 \qquad H_1 : \lambda_i \neq 0$$

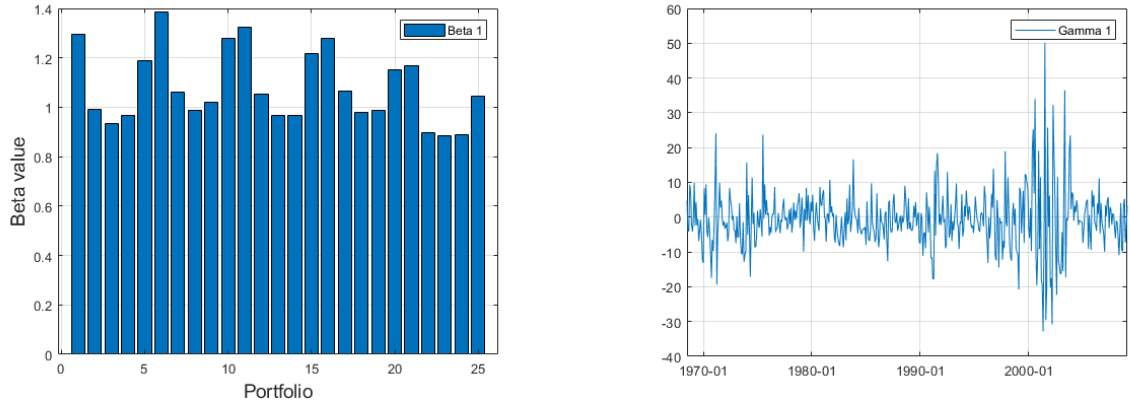
4 Data and Empirical Results

4.1 Data Description

For this study, we observe monthly returns of US stock portfolios sorted by size and momentum and obtain data for excess market return and risk free rate from Kenneth R. French's website. We compare the effects (???) of the liquidity factors of Pastor and Stambaugh using data from 1962 to 2018. The dates in the raw data were misaligned so the start and end dates of our periods were synchronised before running the regressions. After adjusting for null values, we consider only the periods between January 1968 and December 2018.

4.2 Testing CAPM

In order to test the validity of CAPM, we follow the methodology described above.



(a) Measure of exposure to excess market return risk for portfolios sorted by size and momentum.

(b) Excess market return premia as a function of time.

Figure 4.1: GRAPH BETA_1 and GAMMA_1 IN SAMPLE

Table 1: In sample t-test at 5% significance level

Model	λ_0	λ_1	λ_2	λ_3	λ_4
CAPM	1.3541 (2.33)	-0.761 (-1.3295)			
Aggregate liquidity	0.8662 (1.3630)	-0.3977 (-0.6283)	0.0616 (2.5152)		
Innovation liquidity	1.4658 (2.2684)	-0.7817 (-1.3279)	-0.0261 (-1.0216)		
Traded liquidity	0.8800 (1.8419)	-0.3549 (-0.8617)	-0.0748 (-6.2689)		
Saturated	0.2181 (0.6054)	0.3069 (0.9631)	0.0720 (4.6196)	0.0256 (2.4805)	-0.0549 (-6.0122)

Using our in-sample data from 1968 to 2008 we obtain an average of γ_0 (averaged over the whole period) $\lambda_0 = 1.3541$. From table 1, we find this to be significant, implying that there is sufficient evidence to reject our null hypothesis that $\lambda_0 = 0$ and so we can conclude that the CAPM model does not hold within the market we are studying. For completeness, we also obtain a factor value for excess market return (equity risk premium) of -0.761 , which tests insignificant at a 95% confidence level. Therefore as the t-test is insignificant, nothing can be said about the null hypothesis and so it is inconclusive as to whether the factor has a positive risk premium, as one should expect.

In terms of the validity of the CAPM model, one can observe from the residual plot **big dot plot**

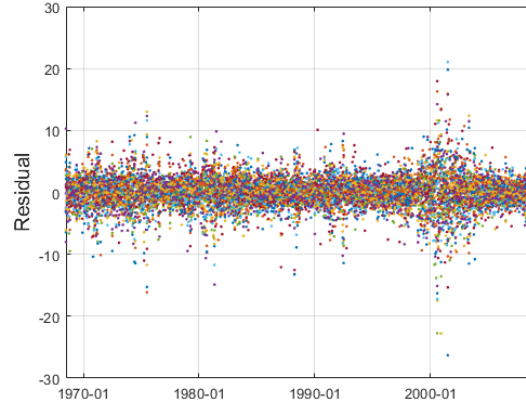


Figure 4.2: Caption

that the residual values are predominantly centred around zero. We witness large deviations mostly at notable dates such as 2000 and 2008, where it can be assumed that this is driven by global economic shock/event. Therefore it can be said that this model performs well in explaining returns despite not being successful at explaining the cross section of returns.

Overall we find that the CAPM model with only one factor fails to explain the cross-section of stock returns that are sorted by size and momentum and so we proceed to introduce additional factors corresponding to liquidity and compare their performance.

4.3 Testing CAPM - MultiFactor Models 1968 - 2008

We now conduct the same procedure for liquidity factors as an additional factor on the CAPM model using in sample data.

Aggregate liquidity

Using our in-sample data from 1968 to 2008 we obtain an average of γ_0 , $\lambda_0 = 0.8662$. From table 1, we find this to be significant, implying that there is sufficient evidence to reject our null hypothesis that $\lambda_0 = 0$. Also, we also obtain a factor value for excess market return of $\lambda_1 = -0.3977$, which also tests insignificant at a 95% confidence level. Therefore as the t-test is insignificant, nothing can be said about the null hypothesis and so it is inconclusive as to whether the factor has a positive risk premium, as one should expect. However the risk premium for aggregate liquidity with a value of $\lambda_2 = 0.061$ tests significant. Therefore as the value is positive, then investors are compensated for additional risk and so the liquidity risk is priced.

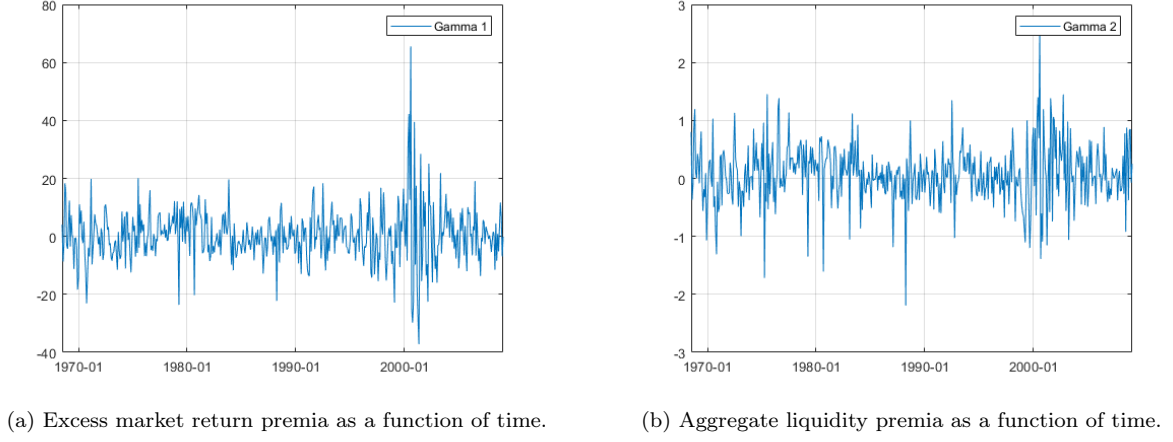


Figure 4.3: GRAPH GAMMA 1 and GAMMA 2 IN SAMPLE

Innovation in liquidity

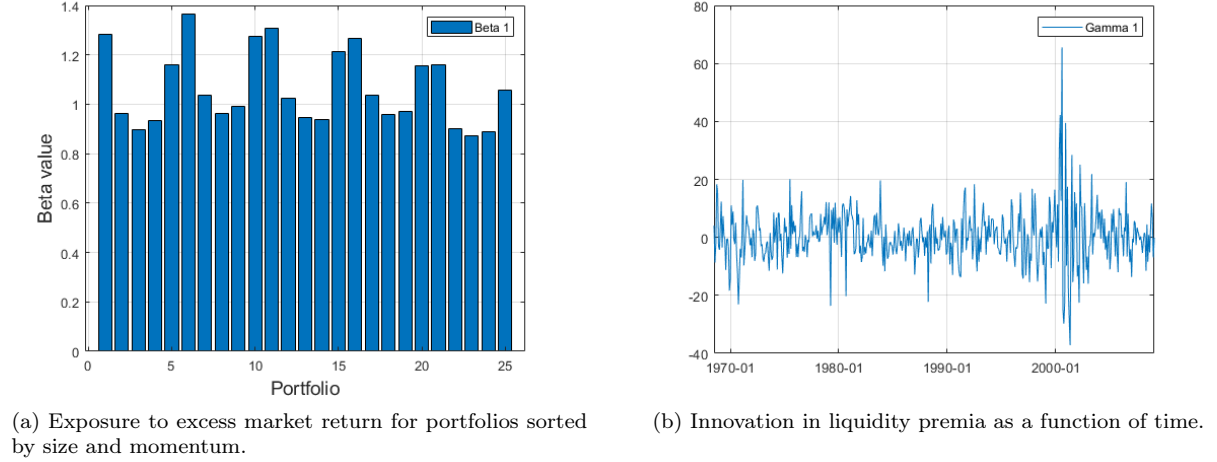


Figure 4.4: BETA 1 GAMMA 1 IN SAMPLE

For an additional factor of innovation in liquidity observe that $\lambda_0 = 1.4658$. From table 1, we find this to be significant, implying that there is sufficient evidence to reject our null hypothesis that $\lambda_0 = 0$. For completeness, we also obtain a factor value for excess market return (equity risk premium) of -0.7817 , not too dissimilar from the CAPM model built above, which also tests insignificant at a 95% confidence level. Therefore as the t-test is insignificant, nothing can be said about the null hypothesis and so it is inconclusive as to whether the factor has a positive risk premium, as one should expect. Similarly the risk premium for innovations in liquidity with a value of -0.0261 also tests insignificant. Overall we find that the CAPM model with an additional factor of innovations in liquidity fails to explain the cross-section of stock returns that are sorted by size and momentum.

Traded liquidity

Using in-sample data from 1968 to 2008 we find that $\lambda_0 = 0.8800$. From table 1, we find this to be insignificant, and so no conclusions can be made about the value of λ_0 . We also obtain a factor value for excess market return of -0.3549 , not too dissimilar from the model containing aggregate liquidity built above, which also tests insignificant at a 95% confidence level. Therefore as the t-test is insignificant, nothing can be said about the null hypothesis and so it is inconclusive as to whether the factor has a positive risk premium, as one should expect. However We find that the risk premium for traded liquidity with a value of -0.0748 tests significant and so we have evidence to suggest that this value is non-zero. As this value is negative, then trading liquidity is not priced.

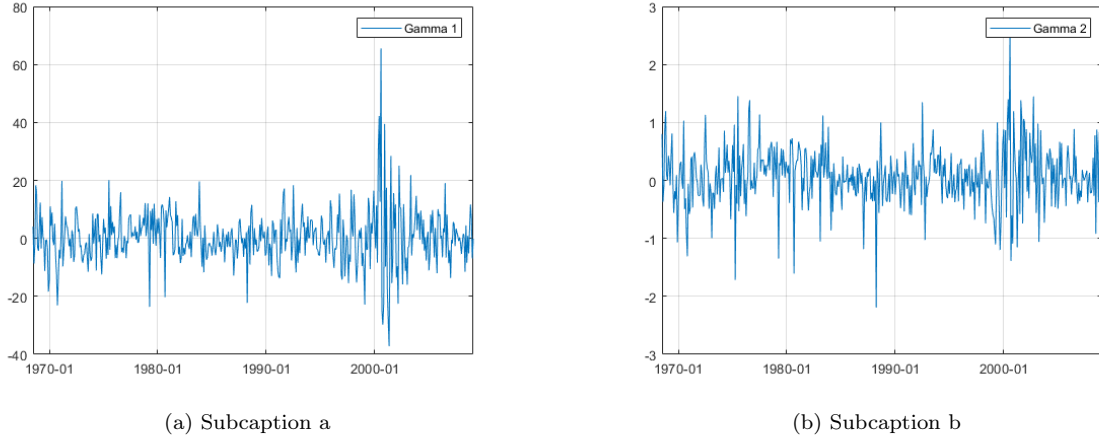


Figure 4.5: GRAPH GAMMA 1 AND GAMMA 2 INSAMpLE

Saturated model

When testing the model containing the excess market return and all three liquidity factors, we find that λ_0 and λ_1 are not significant and so nothing can be said. However, we find that the risk premium of the aggregate liquidity factor and innovation in liquidity are both positive and statistically significant. Therefore, portfolios with higher exposure to the liquidity factor are expected to have higher returns. This means that investors are compensated for additional risk. In other words, liquidity risk is priced. However, the risk premium for traded liquidity is negative, which contrasts the results obtained for the risk premiums for the first two liquidity factors. A negative liquidity risk premium suggests that the liquidity risk is not priced.

In the CAPM model, the liquidity factors are not included. Hence, it is assumed in the CAPM that the only determinant of excess returns is the exposure to systematic risk. However, from an economic point of view, other factors like liquidity may predict the changes in excess returns. For example, a portfolio with high exposure to liquidity risk can be undesirable by investors. That is because investors prefer more liquid assets to those that are less liquid. Hence, the demand for portfolios with high liquidity risk may be low. Therefore, the market will offer the investor a higher return to compensate them for the added risk.

Therefore, the extended factor model makes a more realistic assumption about the determinants of the excess returns. By omitting the liquidity factor from the CAPM model, the model may suffer from the omitted variable problem. Also, the fact that liquidity is correlated with the excess market returns when the market is performing poorly, the liquidity available will decline significantly, leading to the CAPM to perform worse than the extended factor model. (Hameed, Kang and Viswanathan, 2010).

4.4 Testing CAPM - MultiFactor Models 2008 - 2019

Now we refit the models using out of sample data.

CAPM

With out-of-sample data from 2008 to 2019 we obtain a $\lambda_0 = 0.9266$. From table 1, we find this to be significant, implying that there is sufficient evidence to reject our null hypothesis that $\lambda_0 = 0$. Also, we also obtain a factor value for excess market return of $\lambda_1 = -0.0043$, which tests insignificant at a 95% confidence level. Therefore as the t-test is insignificant, nothing can be said about the null hypothesis and so it is inconclusive as to whether the factor has a positive risk premium, as one should expect. Compared to our in-sample estimates, we find the intercept to be much lower post-crisis.

Table 2: Out of sample t-test at 5% significance level

Model	γ_0	γ_1	γ_2	γ_3	γ_4
CAPM	0.9266 (4.3343)	-0.0043 (-0.0230)			
Aggregate liquidity	1.2887 (6.1620)	-0.3456 (-1.9401)	-0.0146 (-2.8181)		
Innovation liquidity	1.1337 (5.8912)	-0.2408 (-1.4178)	-0.0240 (-2.8202)		
Traded liquidity	0.9527 (4.7396)	-0.0130 (-0.0705)	-0.0023 (-0.6057)		
Saturated	1.2754 (3.8828)	-0.3863 (-1.2295)	-0.222 (-2.2258)	-0.0255 (-2.7887)	0 (0.0004)

Aggregate liquidity

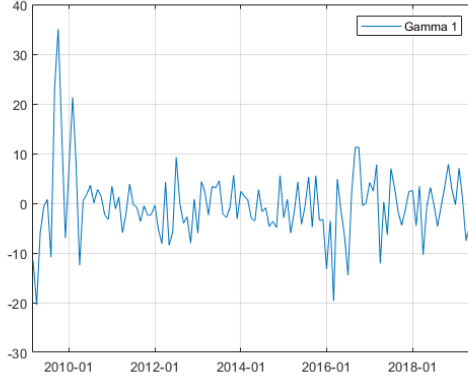
Using our out of sample data from 2008 to 2019 we obtain an average of γ_0 , $\lambda_0 = 0.8662$. From table 1, we find this to be significant, implying that there is sufficient evidence to reject our null hypothesis that $\lambda_0 = 0$. Also, we also obtain a factor value for excess market return of $\lambda_1 = -0.3977$, which also tests insignificant at a 95% confidence level. Therefore as the t-test is insignificant, nothing can be said about the null hypothesis and so it is inconclusive as to whether the factor has a positive risk premium, as one should expect. However the risk premium for aggregate liquidity with a value of $\lambda_2 = 0.061$ tests significant. Therefore as the value is positive, then investors are compensated for additional risk and so the liquidity risk is priced.

Innovation in liquidity

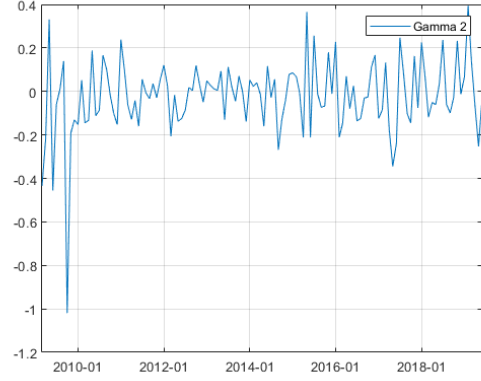
We now conduct the same procedure for the innovation in liquidity factor as an additional factor on the CAPM model for the out of sample data.

Using our out-of-sample data from 2008 to 2018 we obtain an average of γ_0 (averaged over the whole period) $\lambda_0 = 1.1337$. From table 1, we find this to be significant, implying that there is sufficient evidence to reject our null hypothesis that $\lambda_0 = 0$. For completeness, we also obtain a factor value for excess market return (equity risk premium) of -0.2408 , not too dissimilar from the value obtained from the in-sample data, which also tests insignificant at a 95% confidence level. Therefore as the t-test is insignificant, nothing can be said about the null hypothesis and so it is inconclusive as to whether the factor has a positive risk premium, as one should expect. Surprisingly the risk premium for innovations in liquidity with a value of -0.0240 also tests significant, and so there is evidence to suggest this value is non-zero, which is contradictory/opposite to what was found in the in-sample case.

Overall we find that the CAPM model with an additional factor of innovations in liquidity appears to explain the cross-section of stock returns that are sorted by size and momentum.



(a) Subcaption a

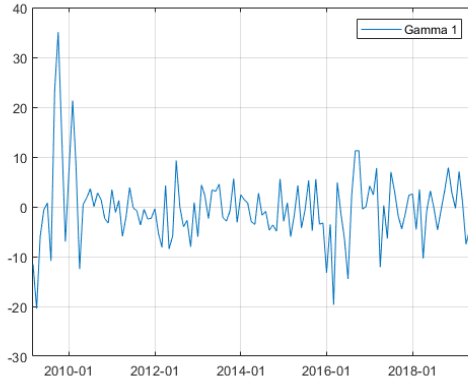


(b) Subcaption b

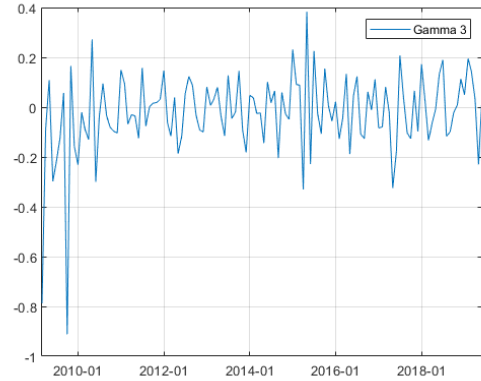
Figure 4.6: GAMMA 1 GAMMA 2 OUT SAMPLE

Traded liquidity

We now conduct the same procedure for the traded liquidity factor as an additional factor on the CAPM model using out-of-sample data.



(a) Subcaption a



(b) Subcaption b

Figure 4.7: GAMMA 1 GAMMA 2 OUT OF SAMPLE

Using our out-of-sample data from 2008 to 2018 we obtain an average of γ_0 (averaged over the whole period) $\lambda_0 = 0.9527$. From table 1, we find this to be significant, and so we can conclude that λ_0 is non-zero, which is opposite to in-sample case. For completeness, we also obtain a factor value for excess market return (equity risk premium) of -0.0705 , not to dissimilar from the model containing aggregate liquidity built above, which also tests insignificant at a 95% confidence level. Therefore as the t-test is insignificant, nothing can be said about the null hypothesis and so it is inconclusive as to whether the factor has a positive risk premium, as one should expect. We find that the risk premium for traded liquidity with a value of -0.0023 also tests insignificant.

In the out-of-sample data, all of the three liquidity factors seem to have a negative lambda value. The out-of-sample estimation shows that both the aggregate liquidity and innovation in aggregate liquidity

do predict changes in excess returns. However, it does not provide evidence for where the traded liquidity predicts the changes in excess returns or not.

Similar to the in-sample estimation, the out-of-sample provides negative lambda value for the innovation in liquidity and the traded liquidity. Unlike in the out-of-sample, the in-sample three-factor model provides positive lambda value for the aggregate liquidity factor. The out-of-sample extended factor model implies that the liquidity risk is not price. Since all of the liquidity factors gammas are negative, this implies that higher exposure to liquidity risk leads to lower returns. However, in the in-sample estimation of the risk premium of the aggregate liquidity was positive. The downward spike may explain the change in the gamma relating to aggregate liquidity in the out-of-sample data in 2009. The downward spike coincides with the credit crunch caused by the Financial crisis. In the Financial crisis, portfolios with higher sensitivity to liquidity suffered lower returns.

5 Conclusion

In conclusion, in this project we estimated the CAPM and the extended factor model with liquidity factor for the cross-section of stock returns. Both models provided positive intercept. Which suggests that there exists a factor, not included in our model, that can explain some of the variation in excess returns.

When looking at the liquidity factors, the evidence shows that they do explain partially the movement in the excess returns of the portfolios in both the out-of-sample and in-sample. However, the out-of-sample results suggest that the liquidity factors are not priced. Compared to the out-of-sample, where the in-sample estimates provide mixed results.

The lambda 1 in all the estimations is not significant. The theory of asset pricing tells us that the lambda1 must be positive in expectation only. The fact that lambda1 take both positive and negative values is driven by the volatility of the portfolios whose betas are greater than 1. Portfolios whose betas are greater than 1 exhibit higher fluctuations in their returns. These fluctuations make it difficult to pin down the relationship between the betas and excess return of the portfolios.

Further research might investigate the reason why liquidity risk is not priced. Also, whether it is possible to create an arbitrage strategy as to take advantage of such miss pricing. Other interesting areas to explore is whether such miss pricing is found in other financial markets.

One of the main limitations of our study is that size and momentum were not controlled for. This is something that Pastor and Stambaugh (2003) did and found liquidity to be a priced factor. Another limitation is that, since liquidity is not a traded factor, the way the liquidity measure is constructed may affect our estimation.

6 Bibliography

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A MATLAB Code

The code was written with re-usability in mind and should work for comparable files for historical factors and returns other than the ones used in this study.

The raw data was processed into datatables using `data_processing.m` and `download_data.m`, and the matching dates in the tables were found with the function `index_tables.m`. All of the regressions discussed above were performed in `main.m`, where `run_regression`, `Fama_Macbeth.m` and several plotting functions were called. For further information, refer to the code.

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