Analysis of an Adaption of the Adaptive Aggressive Algorithm: Asking are AA Algorithms Actually All that Accurate in Applicable Auctions Anyway?

Rupert Bedford
rb9281@bristol.ac.uk

Max Robinson
mr9388@bristol.ac.uk

Jonathan Simmonds
js9721@bristol.ac.uk

ABSTRACT

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1. INTRODUCTION

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An automated trader is an algorithm that automatically places trading orders on a stock market. The trader receives buy or sell orders from customers with a price and quantity. The trader makes money by buying for less than the customers price or selling for more than the customers price. Therefore the goal of a trading strategy is maximise these margins and the trading volume to maximise the amount of profit.

IBM published a paper in 2001 that showed that their MGD trading strategy and Dave Cliff's ZIP were able to outperform human traders. Algorithmic strategies are able to react more quickly to changing market conditions and combine information from a multiple sources.

We have implemented the Adaptive Aggressiveness trading algorithm and compare its performance with other adaptive algorithms as well as simpler traders.

- What is automated trading (history)
- What our goal was

2. ENVIRONMENT

2.1 BSE

For this assignment we use the Bristol Stock Exchange (BSE) as a virtual trading environment or, more accurately, a minimal simulation of a limit order book financial exchange. As it is minimalistic it has the advantage of being both easy to understand and quick to run.

BSE acts like a dark pool continuous double auction as both buyers and sellers simultaneously bid towards their limit price and can see all other offers but do not know the identity of the trader. This has a number of implications, the main one being the difficulty of implementing the MGD trader in this environment as it requires non-anonymised data to calculate which trades have been updated each time step. This is covered in more detail in Section 2.2.2. Other traders (e.g. ZIP and AA) do not have this limitation.

2.2 Traders

2.2.1 ZIP

Zero-Intelligence-Plus (ZIP) traders

- What was significant — key points

- Advantages/disadvantages

2.2.2 GD-Variants

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- Shavers / Sniper / XKCD, etc.

- MGD anonymised data (see BSE paragraph 2)

- We implemented it but couldn't do a full implementation

2.3 Adaptive Aggressive Traders

The adaptive aggressive algorithm relies on the principal that you can be in two different states. One where you trade more frequently closer to your limit price, higher chance of trading but lower profit per trade. The other where you trade further from your limit price, which decreases the likelihood of making a trade but increases the amount of profit you will make when you do trade.

The Adaptive-Aggressive algorithm maintains an aggressiveness value (r), which determines how close to your limit price you will be trading. A completely aggressive buyer ($r = 1$) will buy at their limit price, and a completely passive trader ($r = -1$) will buy at the minimum market price ($1 in our case)

2.4 Price Equilibrium Estimator

One of the key features that most trading algorithms rely on is the ability to estimate the equilibrium price of the market. For this we used the following equation which estimates the market equilibrium price based on the N most recent transactions, known as a moving average.

$$\hat{p}^* = \frac{\sum_{i=T-N+1}^{T} w_i p_i}{\sum_{i=T-N+1}^{T} w_i} \text{where } w_T = 1 \text{ and } w_{i-1} = \lambda w_i \quad (1)$$

The w_i represents the weight of the transaction so that we weight more recent transactions more heavily with the λ affect how severe this weighting is. We set the value of λ to 0.9 which was the value recommended in the original paper [1, p. 100].

2.4.1 Price estimation

Short term learning reflects how the value of our aggressive is calculated and then subsequently updated. However, first we must define two concepts. An intra-marginal trader is a trader that bids above the equilibrium if they are a buyer (or asks below the equilibrium if they are a seller). Whereas an extra-marginal trader will never trade above the equilibrium if they are a buyer (or below the equilibrium if they are a seller).

Whether our trader is acting in an extra-marginal or intra-marginal capacity will depend upon the limit price for our order. If we have a limit price above the equilibrium (below the equilibrium for a seller) then we are considered an intra-marginal trader otherwise we are considered extra-marginal.

The price that we then submit will depend upon which capacity we are acting under based on the follow equations.

Intra-marginal buyer

$$\tau = \begin{cases} \hat{p}^*(1 - \frac{e^{-r\theta}-1}{e^{\theta}-1}), & \text{if r } \in (-1, 0) \\ \hat{p}^* + (l_i - \hat{p}^*)(\frac{e^{r\theta}-1}{e^{\theta}-1}), & \text{if r } \in (0, 1) \end{cases} \quad (2)$$

where θ (2.4.3) measures the volatility of the market, l_i is the limit price for the buyer, $\hat{p}^*$ is the estimator of the equilibrium price.

Extra-marginal buyer

$$\tau = \begin{cases} l_i(1 - \frac{e^{-r\theta}-1}{e^{\theta}-1} & \text{if r } \in (-1,0) \\ l_i & \text{if r } \in (0,1) \end{cases} \tag{3}$$

Intra-marginal seller

$$\tau = \begin{cases} \hat{p}^* + (MAX - \hat{p}^*)(\frac{e^{-r\theta}-1}{e^{\theta}-1}), & \text{if r } \in (-1,0) \\ c_j + (\hat{p}^* - c_j)(1 - \frac{e^{r\theta}-1}{e^{\theta}-1}), & \text{if r } \in (0,1) \end{cases} \tag{4}$$

where MAX is the maximum value that can be submitted to the market and c_j is the limit price for the seller.

Extra-marginal seller

$$\tau = \begin{cases} c_j + (MAX - c_j)(1 - \frac{e^{-r\theta}-1}{e^{\theta}-1} & \text{if r } \in (-1,0) \\ c_j & \text{if r } \in (0,1) \end{cases} \tag{5}$$

In the paper, there was a different value used instead of θ, designed to ensure that the curve provided by the equations was continuous. However, we found that just using θ was still sufficiently accurate and greatly simplified the above equations.

One of the equations is then used to determine the ideal price that we want to be trading at. We then use the following rules to determine the actual shout that we will make when asked to submit a new order. The o_{bid} and o_{ask} represent the best bid (highest) and best ask (lowest) on the market that have not yet been accepted. The value of η is set as a parameter and reflects how quickly we converge on our estimated price (higher η converge slower).

Bidding rules for Buyer if ($l_i \leq o_bid$) - Submit no bid (market transacting above our limit)
else submit bid given by Equation 6

$$bid_i = \frac{o_{bid} + (\tau - o_{bid}}{\eta} \tag{6}$$

Bidding rules for Seller if ($c_j \geq o_{ask}$) - Submit no ask (market transacting below our limit)
else submit bid given by Equation 7

$$ask_i = o_{ask} - \frac{o_{ask} - \tau}{\eta} \tag{7}$$

Since these all require an existing bid on the market, there were difficult rules that were used to determine what shout to make if we were the first trader selected to make a bid. These can be found in the Adaptive Aggressive paper [2, p. 32] but are not defined here.

Initially when we ran our algorithm we found that it was occasionally making a loss upon receiving a new order from the scheduler with a different limit price. The reason for this was that our previous bids had been submitted using the limit price from the previous order. Since on the Bristol Stock Exchange it is not possible to remove a bid, we added additional logic to notify us of the new order. If it was found that the new order contained a limit that was outside of our trading range, we cleared our previous bid using a stub.

- Graph of all trades with projected price equilibrium
- **MEGA GRAPH**

2.4.2 Short term learning

Once we decided what capacity we are acting in we then use the following equations to calculate our aggressiveness

$r(t+1) = r(t) + \beta_1(\delta(t) - r(t))$
$\delta(t) = (1 \pm \lambda_r)r_{shout} \pm \lambda_a$

- Graphs — `r` vs price equilibrium

2.4.3 Long term learning

Max

- θ

3. CALIBRATION

Group hug

- β_1, β_2, γ, η
- potential to compare statistically?

4. RESULTS

Group hug

- Graph: Average balance over time
- Statistical **anal**ysis — why you used a certain test
  Ed's report: "According to the conducted Wilcoxon-Mann-Whitney two-tailed rank-sum tests, the difference in the observed efficiencies is significant ($U = 2, N_1 = N_2 = 10, p < 0.0003$)."
- Experiment with changing scheduler
- Other graphs

5. CONCLUSION

Group hug

- Thank you and good night
- Hold for applause

References

[1] Perukrishnen Vytelingum. *The Structure and Behaviour of the Continuous Double Auction.* PhD thesis, University of Southampton, December 2006.

[2] P. Vytelingum, D. Cliff, and N. R. Jennings. Strategic bidding in continuous double auctions. *Artif. Intell.*, 172(14):1700–1729, September 2008.