High-Performance Bidding Agents for the Continuous Double Auction

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ABSTRACT

We develop two bidding algorithms for real-time Continuous Double Auctions (CDAs) using a variety of market rules that offer what we believe to be the strongest known performance of any published bidding strategy. Our algorithms are based on extensions of the "ZIP" (Cliff, 1997) and "GD" (Gjerstad and Dickhaut, 1998) strategies: we have made essential modifications to these strategies which enable trading multiple units in real-time markets. We test these strategies against each other and against the sniping strategy of (Rust et al., 1992) and the baseline "Zero Intelligence" strategy of (Gode and Sunder, 1992), using both a discrete-time simulator and a genuine real-time multi-agent environment called MAGENTA (Das et al., 2001). Under various market rules and limit price distributions, our modified Gjerstad-Dickhaut ("MGD") strategy outperforms the original GD, and generally dominates the other strategies.

1. INTRODUCTION

There is now considerable interest in designing software agents to perform economic tasks on behalf of humans in various electronic marketplaces. Particular emphasis has been placed on auctions, which achieve robust convergence to equilibrium, with high profits or surplus for the participants. The speed and connectivity of the internet will enable future real-time auctions to be held on a much larger scale.

This paper examines agent bidding algorithms for the Continuous Double Auction (CDA) institution. In the CDA, bids and asks may be submitted and traded at any time during the trading period. The CDA is the dominant institution for real-world trading of equities, derivatives, etc.. Experiments with human subjects in simulated CDAs find reliable price convergence close to theoretical equilibria [9, 10].

Several studies examined CDAs with various computerized bidding agents [1, 2, 4, 5, 8], including a major comparative study in the Santa Fe Double Auction Tournament (SFDAT) [7]. The principal conclusion was that Todd Kaplan's simple "sniping" strategy, which waits for the bid/ask

spread to become small and then "steals the deal," clearly outperformed all other strategies. However, there is room for further performance improvements. Kaplan agents do not adapt to market activity and are unable to infer the market's equilibrium, hence they will snipe any profitable deal regardless of how far away it is from equilibrium. A further caveat is that the SFDAT did not test agents against copies of themselves. Finally, the SFDAT was a discrete-time auction with synchronized bidding and non-persistent orders. This does not conform to real CDA markets in which orders are submitted asynchronously in a stochastic sequence, and in which orders may remain open for long times. Hence the results may not apply to real-time CDA markets.

The goal of this paper is to develop high-performance bidding agents for real-time CDAs. We desire that: (1) as in SFDAT, the algorithm should achieve high profit or surplus for the individual agent; (2) performance should be robust with respect to various opponent strategies, potentially including copies of itself. This is important because in real-world electronic CDAs, agents will have little or no information about what type of human and/or agent opponents are present.

We develop agents that can function in real-time environments under a variety of different market rules, and we study their behavior in both homogeneous and heterogeneous agent populations. As a first step, we emulate stochastic, asynchronous market dynamics in a discretet-time simulator, using the standard methodology of randomly enabling activity in a subset of agents at each time step. Each agent i has a probability α_i of being active in each time step. For simplicity α_i is set to a constant value for all agents: this enforces a fairness principle in which no agent has a speed/timing advantage over any other agent. Any activity during a time step is processed by the institution in a random order, and agents are informed of the results at the end of the time step. Orders submitted by agents are persistent, subject to various termination conditions: they can be traded, modified, or expire untraded after some expiration time (typically the end of a period).

The discrete-time simulator is fast enough to run large numbers of trials for each market configuration. We have also ported the strategies to MAGENTA, a real-time multi-agent environment [3]. MAGENTA simulations are event-driven and multi-threaded, and support general agent-agent communication through a conversational protocol. Such simulations are two orders of magnitude slower, so fewer trials can be run, but qualitatively they can provide a proof-of-concept of the real-time bidding agent design.

Our agents are based on two different mechanisms for adapting bid prices to market activity. First, we have implemented a modified version of Cliff's "Zero-Intelligence-Plus" (ZIP) algorithm [2], which essentially tracks the trade price as trades occur and tends to outbid/undercut the competition when trades are not occurring. Second, we have implemented a modified version of the Gjerstad-Dickhaut (GD) algorithm [4]. This is a more sophisticated algorithm that uses recent market activity to estimate the probability that a bid or ask at any given price will be traded. We test these strategies against themselves and against each other, and also against two benchmark opponents: Kaplan agents and so-called "Zero-Intelligence" (ZI) agents [5].

We follow the standard practice of giving each agent a fixed role of either Buyer (submits only bids) or Seller (submits only asks), and a fixed schedule of limit prices (seller costs or buyer valuations) for each unit of commodity bought or sold. A uniform random distribution is used to generate the limit prices (results are qualitatively similar using other distributions). We test agent performance in homogeneous populations and in two different heterogeneous settings: (1) a "one-in-many" test in which a single agent of one type competes against an otherwise homogeneous population of a different type; (2) a "balanced-group" test in which buyers and sellers are evenly split between two types, and every agent of one type has a counterpart of the other type with identical limit prices. The first test measures incentive for strategy deviation in a homogeneous population, while we believe the latter test to be the fairest way to test two different algorithms against each other.

2. CDA TRADING STRATEGIES

2.1 Zero-Intelligence (ZI) Traders

ZI agents [5] use an extremely simple strategy: they generate random order prices, ignoring the state of the market. For a given unit, prices are drawn from a uniform distribution between the unit's limit price and either a maximum allowable price for sellers, or a minimum allowable price for buyers. One wouldn't expect this algorithm to perform well, but it does provide a useful baseline level of performance.

2.2 Kaplan's Sniping Traders

Kaplan agents [7] tend to let other agents do all the negotiating. Kaplan buyers will bid at the best ask only when one of the following three conditions are met: (1) Juicy offer: the best ask is less than the minimum trade price in the previous period. (2) Small spread: the best ask is less than the maximum trade price in the previous period, and the ratio of the bid-ask spread and the best ask is less than a spread factor F_s, while the expected profit is more than a minimum profit factor F_p. (3) Time running out: the fraction of time remaining in the period is less than a time factor F_t. (Kaplan sellers makes symmetric choices.)

For the SFDAT, Kaplan used $F_s = 0.1$, $F_p = 0.02$ and $F_t = 0.1$. However, in this work we set $F_s = 0.025$, as this was found to give significantly better performance in our one-in-many tests. We also add $\pm 50\%$ random noise to each parameter to avoid synchronized behavior.

2.3 Zero-Intelligence Plus (ZIP) Traders

In the ZIP strategy [2], each agent maintains a scalar variable m denoting its desired profit margin, and it combines

this with a unit's limit price to compute a bid or ask price p. At the start of trading, p is initialized to a random positive-surplus value (as in ZI), and it is adjusted by small random increments after every successful or failed trade. For successful trades, p is adjusted towards the trade price. Changes in p that increase margins are always made, whereas margin decreases are only made if the agent has not yet traded all its units. For failed bids, buyers adjust p in the direction of beating the failed bid, again only if they still have tradeable units. Sellers behave similarly for failed asks. Finally, p is constrained to correspond to non-negative surplus.

Our simulations utilize the following modifications to ZIP. First, we use an array of profit margins m_j of the size of the number of units: since the units have different limit prices, they require different profit margins to trade at equilibrium. Second, we use larger initial margins than in [2], and the m_j values are not statistically independent: the limit prices of the less valuable units influence the initial margins of the more valuable units. We find it is important to start with a low bid or high ask price to negotiate up or down from, otherwise a significant amount of surplus may be lost. Finally, the margin decrease for failed trades is modified as follows: if a sufficient amount of time passes without a trade occurring (one time step in our discrete-time simulator, and 1.0 seconds in MAGENTA), the agent adjusts margins and prices of units that have not yet traded to beat the best current open bid or ask. This allows the algorithm to be used in a real-time environment. Our modifications for real-time trading and persistent orders are related to those independently proposed in [6].

Our ZIP experiments find efficiencies and rates of convergence somewhat higher than in prior studies, possibly because we have done a better job of optimizing the initialization and dynamic behavior.

2.4 Gjerstad-Dickhaut (GD) Traders

GD agents [4] use the history H_M of recent market activity (the bids and asks leading to the last M trades) to calculate a "belief" function $f(p)$ estimating the probability for a bid or ask at price p to be accepted. For example, for a seller,

$$f(p) = \frac{AAG(p) + BG(p)}{AAG(p) + BG(p) + UAL(p)} \qquad (1)$$

where $AAG(p)$ is the number of accepted asks in H_M with price $\geq p$, $BG(p)$ is the number of bids in $H_M \geq p$, and $UAL(p)$ is the number of unaccepted asks in H_M with price $\leq p$. Interpolation is used for prices at which no orders or trades are registered in H_M. The GD agent then chooses a price that maximizes its expected surplus, defined as the product of $f(p)$ and the gain from trade at that price (equal to $p - l$ for sellers and $l - p$ for buyers, where l is the limit price).

The original GD algorithm can suffer from excessive volatility in certain pathological situations, such as when there are no rejected orders in H_M. We have fixed this problem by enabling the GD agents to remember the highest trade price p_h and lowest trade price p_l from the previous period. After interpolating the belief function $f(p)$, the seller resets $f(p) = 0$ for all prices above p_h, and $f(p) = 1$ for all prices below p_l. Buyers makes symmetric adjustments to their belief function. This change greatly reduces volatility in homogeneous GD populations.

We have also modified the belief function calculation so as

Strategy	Efficiency	Trade Ratio	Avg. Price (std. dev)
ZI	0.983	0.965	151.69 ($\pm$14.1)
Kaplan	0.964	1.153	147.90 ($\pm$25.0)
ZIP	0.997	1.060	148.51 ($\pm$2.9)
GD	0.995	1.058	149.57 ($\pm$14.2)
MGD	0.997	1.046	148.13 ($\pm$3.6)

Table 1: Homogeneous population results. Columns show the average efficiency $\hat{\mu}$, average trade ratio $\hat{\tau}$ and average final period trade price $\hat{p}_f$ (std. deviation in parentheses). The theoretical equilibrium is 148.5.

Strategy	Many ZI	Many Kaplan	Many ZIP	Many GD	Many MGD
1-ZI	-	-0.238 (-0.52)	-0.124 (-0.19)	-0.005 (-0.20)	-0.105 (-0.18)
1-Kaplan	0.011 (0.04)	-	0.011 (-0.07)	0.068 (-0.09)	0.018 (-0.06)
1-ZIP	0.091 (0.32)	0.117 (0.08)	-	0.028 (0.02)	-0.022 (0.02)
1-GD	0.130 (0.27)	-0.039 (-0.17)	-0.290 (0.01)	-	-
1-MGD	0.158 (0.24)	0.235 (-0.21)	-0.162 (0.01)	-	-

Table 2: One-in-Many Tests: Average improvement in single-agent efficiency when the agent changes strategy in an otherwise homogeneous population. Figures in parentheses show differential single-agent trade ratio.

to enable stingier bidding leading to potentially higher surplus. We set the boundary conditions in the belief function to 0 (or 1) after interpolation rather than before as advocated in [4]. Additionally, if the agent has several tradeable units, we allow the agent to submit a bid corresponding to the least valuable unit. Our modified GD algorithm is referred to as MGD to distinguish it from the original GD specification in [4]. Experiments with both GD and MGD use a memory parameter $M = 8$; this is somewhat larger than in [4] because our agents have a significantly larger number of tradeable units. As with ZIP, there is excellent convergence to equilibrium in a few periods.

3. CDA EXPERIMENTAL SETUP

Our experiments are conducted in a discrete-time simulator that emulates stochastic asynchronous activity through random activation of a subset of agents at each time step. Each agent has a constant activation probability per time step $\alpha_i = 0.25$. This qualitatively emulates behavior seen in the MAGENTA simulations. The experiments use 10 buyer and 10 seller agents. Each agent is given a list of ten limit prices, ordered from lowest to highest seller cost, or highest to lowest buyer value. The limit prices are fixed values drawn from a uniform random distribution between 100 and 200. About half of the units are tradeable for positive surplus at equilibrium. Allowable prices in the auction range from 0 to 400.

An experiment consists of a sequence of five consecutive trading periods, each lasting 300 time steps. Buyers and sellers receive a fresh supply of cash/commodity at the start of each period.

When an agent is active at a given time step, it may submit an order. As in previous studies, orders are for a single unit only, and agents are allowed to have at most one open order at any time. If an agent currently has an open order, it generally is allowed to modify the order, although we have experimented with market rules that prohibit order modification. Active agents calculate an order price using their bidding algorithm, and then submit either a new or a replacement order, subject to any market constraints. For example, we commonly use the "NYSE" spread-improvement rule, which states that any new bid or ask must improve upon the current best bid or ask in the market.

Orders placed at each time step are processed by the exchange in a random sequence. The exchange first attempts to trade each order with an existing open order. If it can do so, the trade will be executed, using the earlier order to set the trade price. Otherwise, the order is appended to a queue of current open orders. In some experiments we employ a "No Queue" market rule, which terminates any existing orders that are improved upon by a new order. All orders are assumed to be limit-price orders, although one can obviously obtain the equivalent of a market order by a sufficiently high bid or low ask. After processing the orders at a given time step, the results are made available to the agents, and the simulation moves to the next time step. Any open orders after the end of a period expire untraded.

4. RESULTS

The results in this section are averages over 100 experiments of 5 periods each. Market rules include NYSE, an open order queue, and allowance of bid modification.

Table 1 summarizes the behavior of homogeneous populations in terms of three different measures: (i) average efficiency $\hat{\mu}$ (the ratio of actual to theoretical population surplus); (ii) average trade ratio $\hat{\tau}$ (the ratio to actual to theoretical number of trades); (iii) average trade price $\hat{p}_f$ (and its standard deviation) in the final trading period. Due to NYSE and the order queue, all strategies achieve high efficiency, with ZIP and GD strategies obtaining $\hat{\mu}$ extremely close to 1.0, the maximum possible value. Note that trade ratios over 1.0 can occur due to out-of-equilibrium units being traded. Kaplan agents obtain the lowest efficiency and the highest variance in trade prices. The original GD algorithm has high efficiency, but can exhibit high fluctuations in trade prices. MGD significantly reduces such fluctuations, while retaining high efficiency.

Table 2 shows the differential performance matrix for the various strategies in the one-in-many tests. Positive (negative) values indicates that, on average, a single agent can obtain more (less) surplus using a different strategy than when following the population strategy. Several observations are noteworthy. Ther is no incentive to deviate to ZI in any population, and it is better to defect to any other

Groups	Wins	Surplus Difference	Efficiency	Trade Ratio	Price (std dev)
Kaplan vs ZI	0-100	-1080.0	0.964	1.055	150.6 ($\pm$17.3)
ZIP vs. ZI	100-0	+1037.0	0.955	1.003	151.7 ($\pm$8.3)
ZIP vs. Kaplan	99-1	+347.9	0.982	1.128	148.8 ($\pm$4.6)
GD vs. ZI	99-1	+575.1	0.978	1.049	150.7 ($\pm$10.0)
GD vs. Kaplan	93-7	+323.0	0.978	1.141	149.3 ($\pm$10.1)
GD vs. ZIP	36-63	-54.4	0.996	1.062	149.2 ($\pm$6.3)
MGD vs. ZI	99-1	+787.4	0.960	0.972	150.9 ($\pm$10.1)
MGD vs. Kaplan	98-2	+210.3	0.993	1.086	148.2 ($\pm$4.8)
MGD vs. ZIP	71-29	+44.9	0.997	1.054	148.6 ($\pm$2.9)

Table 3: Balanced group test results. Columns show: the number of wins for each group in 100 trials, the average difference between first group and second group surplus, population average efficiency, trade ratio, and final period trade price $\pm$ (standard deviation). The magnitude of surplus differences can be compared to the total population surplus of 2612.0 theoretically available in each experiment.

strategy in a ZI population. Defections to Kaplan are always beneficial, although the incentive to deviate in ZIP or MGD populations is tiny. On the flip side, defections from all-Kaplan to either ZIP or MGD are significantly beneficial. A single ZIP agent also shows strong performance and can profitably invade all strategies except MGD. ZIP populations are also less vulnerable to defection to other strategies. Finally, MGD is superior to GD in being both more resistant to defections to other strategies, and more likely to invade into populations of other strategies.

Table 3 shows the results of balanced group testing. Each experiment tallies the total surplus obtained by each group, and the group with greater surplus is declared the winner. The margin of victory is the magnitude of surplus difference. Total wins and average surplus difference are reported in the first two columns. The final three columns report population efficiency, trade ratio, and average final period trade price ($\pm$ std. dev.). Note that both Kaplan and ZI are crushed by either ZIP or by GD and MGD in these tests. Kaplan also suffers the further embarrassment of losing 100-0 to the random ZI agents. Comparing ZIP with GD, we note that while the original GD loses to ZIP, the MGD agents win over 70% of the experiments against ZIP.

5. CONCLUSIONS

Results of the homogeneous and two different heterogeneous tests of five different strategies provide a largely consistent understanding of the merits of each strategy. Both GD and ZIP consistently perform well, and our implementation of MGD consistently performs better than the original formulation of GD. By any measure, both ZI and Kaplan are clearly dominated either by ZIP or by MGD, the one exception being that in a homogeneous ZIP or a homogeneous MGD population, there may be a slight incentive to deviate to the Kaplan strategy. However, such deviation is dangerous because if multiple agents deviate to Kaplan, the performance of all the Kaplan agents degrades considerably. Our finding in this regard is consistent with the analysis of Kaplan given in [7]. Comparing ZIP with GD, we see that ZIP beats the original GD by a number of measures, but it in turn is outperformed by the improved MGD version. These qualitative comparisons hold regardless of the presence or absence of NYSE, an open order queue, and allowance of order modification. The body of evidence suggests that under a variety of different market rules, MGD outperforms any previously published algorithm for this class of CDAs.

While beyond the scope of this paper, our experiments have also measured other individual agent statistics, such as average trade time, trade price and trade ratio. Such measures provide insights into whether a strategy is trading too early or too late, too infrequently, or accepting sub-optimal deals. These insights, together with the above competitive analysis, suggest means by which the strategies might be improved further, depsite their already high efficiencies.

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