

MIE 1621 Computational Project Fall 2017

Instructions: You can use MATLAB or Python to do the following assignment. Hand in your report (hardcopy) to Hassan at BA 8145 and send code (soft copy) to Hassan by Dec.7th, 5 PM.

Consider the following financial optimization model:

$$\begin{aligned} & \text{minimize} \sum_{i=1}^n \mu_i x_i - \frac{\delta}{2} \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} x_i x_j \\ & \text{subject to} \\ & \sum_{i=1}^n x_i = 1 \end{aligned}$$

We assume there are n financial assets (e.g. stocks) such that future prices (returns) of each asset is random and so each asset i has associated with it an expected return μ_i and a variance of returns σ_i^2 . Also, asset returns are pairwise correlated as embodied in the covariance quantities σ_{ij} for $i \neq j$. This model seeks to determine a portfolio of asset weights x_i (where x_i is the proportion of investment in asset i) such that the portfolio maximizes risk-adjusted expected return (the second term is a measure of risk of a portfolio x). The quantity $\delta > 0$ is a parameter that controls risk aversion (degree of risk that is tolerable for an investor). The larger δ is the more risk averse an investor is. The constraint of the model ensures that the x_i variables are proportions.

(1) Formulate the model above as an unconstrained problem.

(2) Apply Newton's method, the steepest decent method, and the BFGS quasi-newton method using a step length of one at each iteration for all three methods to solve the unconstrained problem in (1) using the data below. Report on how smoothly the methods ran e.g. how many iterations and how long? was convergence even observed or did the methods just stop. Note that you will have freedom as to how you will design your stopping conditions as well as deciding how to perform the computations required in each iteration. You may want to put a maximum iterations limit on your methods.

(3) Repeat (2) but use backtracking (see class lecture notes) to get an approximate step length at each iteration for each of the three methods. Report on your computational experiments as in part (2).

Data for the Model

Asset	1	2	3
Expected return	$\mu_1 = 10.73\%$	$\mu_2 = 7.37\%$	$\mu_3 = 6.27\%$

σ_{ij}	1	2	3
1	0.02778	0.00387	0.00021
2	0.00387	0.01112	-0.00020
3	0.00021	-0.00020	0.00115

Choose δ to be between 3.5 and 4.5.

(4) **Scaling up:** Collect data for $n > 3$ stocks e.g. Google, Apple, etc... (see Yahoo or Google finance) and get the daily adjusted closing prices and estimate the required parameters (expected returns, standard deviations, and covariances) for the model above using your n assets and run your methods to solve this new model. You choose n . If you can get your methods to work properly for larger the n the better. It is up to you to determine what period of historical data to use to estimate the parameters but the period should be at least 6 months.

Be sure to specify the time period you used and the technique of estimation of the parameters (you do not have to show the computations for these estimations but describe clearly how you estimated them).

Deliverable: You must write up a brief report which details the methods that you developed as well as providing the computational results and analysis of the methods. You must provide the actual code in an appendix of the report. The code you provide in the appendix must be commented and easy to follow. You must also print out in the appendix the actual reported computational results from running your code for the various methods. Finally, the code and data must be submitted to the TA Hassan. You **MUST** make sure that your code is easy to execute so that results can be validated. If it is not easy to get your code running than marks will be lost! Make sure that your report is well written, concise and to the point. We do not want too many pages in a report including the appendix. For example, it suffices to just give the parameter estimations needed for part (4) and do NOT include in the report the historical prices of the n assets you chose.