# A TYPE THEORY WITH SCOPED EQUALITY REFLECTION

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Feel free to change the stuff below as our thinking evolves. This is just a starting point. We have the following judgments:

| | |
|---|---|
| $\vdash \Sigma\ \text{sig}$ | $\Sigma$ is a valid signature |
| $\mathcal{H} \vdash_\Sigma \Gamma\ \text{ctx}$ | $\Gamma$ is a valid context |
| $\Gamma \vdash_\Sigma \mathcal{H}\ \text{hints}$ | $\mathcal{H}$ is a valid hints set |
| $\Gamma; \mathcal{H} \vdash_\Sigma M \Rightarrow A$ | $M$ infers type $A$ |
| $\Gamma; \mathcal{H} \vdash_\Sigma M \Leftarrow A$ | $M$ checks type $A$ |
| $\Gamma; \mathcal{H} \vdash_\Sigma M \equiv N : A$ | $M$ equals $N$ at type $A$ |

The rules for the theory are given inductive-recursively below.

(Signatures)
$$\frac{}{\vdash \diamond\ \text{sig}} \qquad \frac{\begin{array}{l}\vdash \Sigma\ \text{sig} \\ \diamond \vdash^{\diamond}_{\Sigma} A \Leftarrow \mathbb{U}\end{array} \quad \Sigma \# c}{\vdash \Sigma, c : A\ \text{sig}}$$

(Contexts)
$$\frac{}{\mathcal{H} \vdash_\Sigma \diamond\ \text{ctx}} \qquad \frac{\begin{array}{l}\mathcal{H} \vdash_\Sigma \Gamma\ \text{ctx} \\ \Gamma; \mathcal{H} \vdash_\Sigma A \Leftarrow \mathbb{U}\end{array} \quad \Gamma \# x}{\mathcal{H} \vdash_\Sigma \Gamma, x : A\ \text{ctx}}$$

(Hints)
$$\frac{}{\Gamma \vdash \diamond\ \text{hints}} \qquad \frac{\begin{array}{l}\Gamma \vdash_\Sigma \mathcal{H}\ \text{hints} \\ \Gamma; \mathcal{H} \vdash_\Sigma A \Leftarrow \mathbb{U} \\ \Gamma; \mathcal{H} \vdash_\Sigma a, b \Leftarrow A\end{array}}{\Gamma \vdash_\Sigma \mathcal{H}, a \equiv b : A\ \text{hints}}$$

(Projection)
$$\frac{x : A \in \Gamma}{\Gamma; \mathcal{H} \vdash_\Sigma x \Rightarrow A} \qquad \frac{c : A \in \Sigma}{\Gamma; \mathcal{H} \vdash_\Sigma c \Rightarrow A} \qquad \frac{a \equiv b : A \in \mathcal{H}}{\Gamma; \mathcal{H} \vdash_\Sigma a \equiv b : A}$$

(Reflect)
$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma p \Rightarrow \mathsf{Id}_A(M; N) \\ \Gamma; \mathcal{H}, M \equiv N : A \vdash_\Sigma C \Leftarrow \mathbb{U} \\ \Gamma; \mathcal{H}, M \equiv N : A \vdash_\Sigma e \Leftarrow C\end{array}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathsf{reflect}\ p\ \mathsf{in}\ e \Leftarrow C}$$

(Binders)
$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma A \Leftarrow \mathbb{U} \\ \Gamma, x : A; \mathcal{H} \vdash_\Sigma [x]B \Leftarrow \mathbb{U}\end{array} \quad \mathrm{Q} \in \{\Sigma, \Pi\}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathrm{Q}_A[x]B \Rightarrow \mathbb{U}}$$

(Sigma)

$$\frac{\Gamma;\mathcal{H}\vdash_\Sigma a \Leftarrow A \qquad \Gamma;\mathcal{H}\vdash_\Sigma b \Leftarrow [a/x]B}{\Gamma;\mathcal{H}\vdash_\Sigma \langle a,b\rangle \Leftarrow \Sigma_A[x]B}
\qquad
\frac{\Gamma;\mathcal{H}\vdash_\Sigma p \Rightarrow \Sigma_A[x]B \qquad \Gamma,u:\Sigma_A[x]B;\mathcal{H}\vdash_\Sigma [u]C \Leftarrow \mathbb{U} \qquad \Gamma,v:A,w:[v/x]B;\mathcal{H}\vdash_\Sigma [v,w]M \Leftarrow [\langle v,w\rangle/u]C}{\Gamma;\mathcal{H}\vdash_\Sigma \mathsf{split}([u]C;[v,w]M;p) \Rightarrow [p/u]C}$$

(Pi)

$$\frac{\Gamma,x:A;\mathcal{H}\vdash_\Sigma [x]M \Leftarrow [x]B}{\Gamma;\mathcal{H}\vdash_\Sigma \lambda[x]M \Leftarrow \Pi_A[x]B}
\qquad
\frac{\Gamma;\mathcal{H}\vdash_\Sigma f \Rightarrow \Pi_A[x]B \qquad \Gamma;\mathcal{H}\vdash_\Sigma c \Leftarrow A}{\Gamma;\mathcal{H}\vdash_\Sigma f\mathbin{@}c \Rightarrow [c/x]B}$$

(Id)

$$\frac{\Gamma;\mathcal{H}\vdash_\Sigma A \Leftarrow \mathbb{U} \qquad \Gamma;\mathcal{H}\vdash_\Sigma M \Leftarrow A \qquad \Gamma;\mathcal{H}\vdash_\Sigma N \Leftarrow A}{\Gamma;\mathcal{H}\vdash_\Sigma \mathsf{Id}_A(M;N) \Rightarrow \mathbb{U}}
\qquad
\frac{\Gamma;\mathcal{H}\vdash_\Sigma M \Leftarrow A}{\Gamma;\mathcal{H}\vdash_\Sigma \mathsf{refl} \Leftarrow \mathsf{Id}_A(M;M)}$$

(BinderEq)

$$\frac{\Gamma;\mathcal{H}\vdash_\Sigma \mathrm{Q}_A[x]B \Leftarrow \mathbb{U} \qquad \Gamma;\mathcal{H}\vdash_\Sigma \mathrm{Q}'_A[x]B' \Leftarrow \mathbb{U} \qquad \Gamma;\mathcal{H}\vdash_\Sigma M \Leftarrow \mathsf{Id}_{\mathbb{U}}(A;A') \qquad \Gamma,y:A;\mathcal{H},A\equiv A':\mathbb{U}\vdash_\Sigma N \Leftarrow \mathsf{Id}_{\mathbb{U}}([y/x]B;[y/x]B')}{\Gamma;\mathcal{H}\vdash_\Sigma \mathsf{binderEq}(M;[y]N) \Leftarrow \mathsf{Id}_{\mathbb{U}}(\mathrm{Q}_A[x]B;\mathrm{Q}_{A'}[x]B')}
\quad \mathrm{Q}\in\{\Pi,\Sigma\}$$