A TYPE THEORY WITH SCOPED EQUALITY REFLECTION

JONATHAN STERLING

Feel free to change the stuff below as our thinking evolves. This is just a starting point. We have the following judgments:

$\vdash \Sigma$ sig	Σ is a valid signature
$\mathcal{H} \vdash_\Sigma \Gamma$ ctx	Γ is a valid context
$\Gamma \vdash_\Sigma \mathcal{H}$ hints	$\mathcal{H}$ is a valid hints set
$\Gamma; \mathcal{H} \vdash_\Sigma M \Rightarrow A$	M infers type A
$\Gamma; \mathcal{H} \vdash_\Sigma M \Leftarrow A$	M checks type A
$\Gamma; \mathcal{H} \vdash_\Sigma M \equiv N : A$	M equals N at type A

The rules for the theory are given inductive-recursively below.

(Signatures)
$$\frac{}{\vdash \diamond \text{ sig}} \qquad \frac{\begin{array}{l}\vdash \Sigma \text{ sig} \\ \diamond \vdash^{\diamond}_{\Sigma} A \Leftarrow \mathbb{U}\end{array} \quad \Sigma \# c}{\vdash \Sigma, c : A \text{ sig}}$$

(Contexts)
$$\frac{}{\mathcal{H} \vdash_\Sigma \diamond \text{ ctx}} \qquad \frac{\begin{array}{l}\mathcal{H} \vdash_\Sigma \Gamma \text{ ctx} \\ \Gamma; \mathcal{H} \vdash_\Sigma A \Leftarrow \mathbb{U}\end{array} \quad \Gamma \# x}{\mathcal{H} \vdash_\Sigma \Gamma, x : A \text{ ctx}}$$

(Hints)
$$\frac{}{\Gamma \vdash \diamond \text{ hints}} \qquad \frac{\begin{array}{l}\Gamma \vdash_\Sigma \mathcal{H} \text{ hints} \\ \Gamma; \mathcal{H} \vdash_\Sigma A \Leftarrow \mathbb{U} \\ \Gamma; \mathcal{H} \vdash_\Sigma a, b \Leftarrow A\end{array}}{\Gamma \vdash_\Sigma \mathcal{H}, a \equiv b : A \text{ hints}}$$

$$\frac{x : A \in \Gamma}{\Gamma; \mathcal{H} \vdash_\Sigma x \Rightarrow A} \qquad \frac{c : A \in \Sigma}{\Gamma; \mathcal{H} \vdash_\Sigma c \Rightarrow A} \qquad \frac{a \equiv b : A \in \mathcal{H}}{\Gamma; \mathcal{H} \vdash_\Sigma a \equiv b : A}$$

$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma p \Rightarrow \mathsf{Id}_A(M; N) \\ \Gamma; \mathcal{H}, M \equiv N : A \vdash_\Sigma C \Leftarrow \mathbb{U} \\ \Gamma; \mathcal{H}, M \equiv N : A \vdash_\Sigma e \Leftarrow C\end{array}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathsf{reflect}\ p\ \mathsf{in}\ e \Leftarrow C} \text{(Reflect)}$$

$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma A \Leftarrow \mathbb{U} \\ \Gamma; \mathcal{H} \vdash_\Sigma M \Leftarrow A \\ \Gamma; \mathcal{H} \vdash_\Sigma N \Leftarrow A\end{array}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathsf{Id}_A(M; N) \Rightarrow \mathbb{U}} \ (\mathsf{Id}\text{-form}) \qquad \frac{\Gamma; \mathcal{H} \vdash_\Sigma M \Leftarrow A}{\Gamma; \mathcal{H} \vdash_\Sigma \mathsf{refl} \Leftarrow \mathsf{Id}_A(M; M)} \ (\mathsf{Id}\text{-intro})$$

$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma p \Rightarrow \mathsf{Id}_A(M; N) \\ \Gamma; \mathcal{H} \vdash_\Sigma q \Leftarrow \mathsf{Id}_A(M; N)\end{array}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathsf{uip}(p; q) \Rightarrow \mathsf{Id}_{\mathsf{Id}_A(M;N)}(p; q)} \ (\mathsf{Id}\text{-eq})$$

$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma A \Leftarrow \mathbb{U} \\ \Gamma, x : A; \mathcal{H} \vdash_\Sigma [x]B \Leftarrow \mathbb{U}\end{array} \qquad \mathrm{Q} \in \{\Sigma, \Pi\}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathrm{Q}_A[x]B \Rightarrow \mathbb{U}} \ (\text{Binder-form})$$

$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma \mathrm{Q}_A[x]B \Leftarrow \mathbb{U} \\ \Gamma; \mathcal{H} \vdash_\Sigma \mathrm{Q}'_A[x]B' \Leftarrow \mathbb{U} \\ \Gamma; \mathcal{H} \vdash_\Sigma M \Leftarrow \mathsf{Id}_\mathbb{U}(A; A') \\ \Gamma, u : A; \mathcal{H}, A \equiv A' : \mathbb{U} \vdash_\Sigma N \Leftarrow \mathsf{Id}_\mathbb{U}([u/x]B; [u/x]B')\end{array} \qquad \mathrm{Q} \in \{\Pi, \Sigma\}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathsf{binderEq}(Q_A[x]B; Q_{A'}[x]B'; M; [y]N) \Rightarrow \mathsf{Id}_\mathbb{U}(\mathrm{Q}_A[x]B; \mathrm{Q}_{A'}[x]B')} \ (\text{Binder-eq})$$

$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma a \Leftarrow A \\ \Gamma; \mathcal{H} \vdash_\Sigma b \Leftarrow [a/x]B\end{array}}{\Gamma; \mathcal{H} \vdash_\Sigma \langle a, b\rangle \Leftarrow \Sigma_A[x]B} \ (\Sigma\text{-intro}) \qquad \frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma p \Rightarrow \Sigma_A[x]B \\ \Gamma, u : \Sigma_A[x]B; \mathcal{H} \vdash_\Sigma [u]C \Leftarrow \mathbb{U} \\ \Gamma, v : A, w : [v/x]B; \mathcal{H} \vdash_\Sigma [v, w]M \Leftarrow [\langle v, w\rangle/u]C\end{array}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathsf{spread}([u]C; [v, w]M; p) \Rightarrow [p/u]C} \ (\Sigma\text{-elim}^+)$$

$$\frac{\Gamma; \mathcal{H} \vdash_\Sigma M \Rightarrow \Sigma_A[x]B}{\Gamma; \mathcal{H} \vdash_\Sigma \pi_1 M \Rightarrow A} \ (\Sigma\text{-elim}_1^-) \qquad \frac{\Gamma; \mathcal{H} \vdash_\Sigma M \Rightarrow \Sigma_A[x]B}{\Gamma; \mathcal{H} \vdash_\Sigma \pi_2 M \Rightarrow [\pi_1 M/x]B} \ (\Sigma\text{-elim}_2^-)$$

$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma M \Rightarrow \Sigma_A[x]B \\ \Gamma; \mathcal{H} \vdash_\Sigma N \Leftarrow \Sigma_A[x]B \\ \Gamma; \mathcal{H} \vdash_\Sigma p \Leftarrow \mathsf{Id}_A(\pi_1 M; \pi_1 N) \\ \Gamma; (\mathcal{H}, \pi_1 M \equiv \pi_1 N : A) \vdash_\Sigma q \Leftarrow \mathsf{Id}_{[\pi_1 M/x]B}(\pi_2 M; \pi_2 N)\end{array}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathsf{pairEq}(M; N; p; q) \Rightarrow \mathsf{Id}_{\Sigma_A[x]B}(M; N)} \ (\Sigma\text{-eq})$$

$$\frac{\Gamma, x : A; \mathcal{H} \vdash_\Sigma [x]M \Leftarrow [x]B}{\Gamma; \mathcal{H} \vdash_\Sigma \lambda[x]M \Leftarrow \Pi_A[x]B} \ (\Pi\text{-intro}) \qquad \frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma f \Rightarrow \Pi_A[x]B \\ \Gamma; \mathcal{H} \vdash_\Sigma c \Leftarrow A\end{array}}{\Gamma; \mathcal{H} \vdash_\Sigma f \mathbin{@} c \Rightarrow [c/x]B} \ (\Pi\text{-elim})$$

$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma f \Rightarrow \Pi_A[x]B \\ \Gamma; \mathcal{H} \vdash_\Sigma g \Leftarrow \Pi_A[x]B \\ \Gamma, u : A; \mathcal{H} \vdash_\Sigma M \Leftarrow \mathsf{Id}_{[u/x]B}(f \mathbin{@} u; g \mathbin{@} u)\end{array}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathsf{funext}(f; g; [u]M) \Rightarrow \mathsf{Id}_{\Pi_A[x]B}(f; g)} \ (\Pi\text{-eq})$$