

# A TYPE THEORY WITH SCOPED EQUALITY REFLECTION

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Feel free to change the stuff below as our thinking evolves. This is just a starting point. We have the following judgments:

$\vdash \Sigma \text{ sig}$	$\Sigma$ is a valid signature
$\mathcal{H} \vdash_{\Sigma} \Gamma \text{ ctx}$	$\Gamma$ is a valid context
$\Gamma \vdash_{\Sigma} \mathcal{H} \text{ hints}$	$\mathcal{H}$ is a valid hints set
$\Gamma; \mathcal{H} \vdash_{\Sigma} M \Rightarrow A$	$M$ infers type $A$
$\Gamma; \mathcal{H} \vdash_{\Sigma} M \Leftarrow A$	$M$ checks type $A$
$\Gamma; \mathcal{H} \vdash_{\Sigma} M \equiv N : A$	$M$ equals $N$ at type $A$

The rules for the theory are given inductive-recursively below.

(Signatures)	$\overline{\vdash \diamond \text{ sig}}$	$\frac{\begin{array}{c} \vdash \Sigma \text{ sig} \\ \diamond \vdash_{\Sigma}^{\circ} A \Leftarrow \mathbb{U} \end{array} \quad \Sigma \# c}{\vdash \Sigma, c : A \text{ sig}}$
(Contexts)	$\overline{\mathcal{H} \vdash_{\Sigma} \diamond \text{ ctx}}$	$\frac{\begin{array}{c} \mathcal{H} \vdash_{\Sigma} \Gamma \text{ ctx} \\ \Gamma; \mathcal{H} \vdash_{\Sigma} A \Leftarrow \mathbb{U} \end{array} \quad \Gamma \# x}{\mathcal{H} \vdash_{\Sigma} \Gamma, x : A \text{ ctx}}$
(Hints)	$\overline{\Gamma \vdash \diamond \text{ hints}}$	$\frac{\begin{array}{c} \Gamma \vdash_{\Sigma} \mathcal{H} \text{ hints} \\ \Gamma; \mathcal{H} \vdash_{\Sigma} A \Leftarrow \mathbb{U} \\ \Gamma; \mathcal{H} \vdash_{\Sigma} a, b \Leftarrow A \end{array}}{\Gamma \vdash_{\Sigma} \mathcal{H}, a \equiv b : A \text{ hints}}$
(Projection)	$\overline{\Gamma; \mathcal{H} \vdash_{\Sigma} x \Rightarrow A} \quad \overline{\Gamma; \mathcal{H} \vdash_{\Sigma} c \Rightarrow A} \quad \overline{\Gamma; \mathcal{H} \vdash_{\Sigma} a \equiv b : A}$	$\frac{x : A \in \Gamma}{\Gamma; \mathcal{H} \vdash_{\Sigma} x \Rightarrow A} \quad \frac{c : A \in \Sigma}{\Gamma; \mathcal{H} \vdash_{\Sigma} c \Rightarrow A} \quad \frac{a \equiv b : A \in \mathcal{H}}{\Gamma; \mathcal{H} \vdash_{\Sigma} a \equiv b : A}$
(Reflect)	$\overline{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{reflect } p \text{ in } e \Leftarrow C}$	$\frac{\begin{array}{c} \Gamma; \mathcal{H} \vdash_{\Sigma} p \Rightarrow \text{Id}_A(M; N) \\ \Gamma; \mathcal{H}, M \equiv N : A \vdash_{\Sigma} C \Leftarrow \mathbb{U} \\ \Gamma; \mathcal{H}, M \equiv N : A \vdash_{\Sigma} e \Leftarrow C \end{array}}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{reflect } p \text{ in } e \Leftarrow C}$
(Binders)	$\overline{\Gamma; \mathcal{H} \vdash_{\Sigma} Q_A[x]B \Rightarrow \mathbb{U}}$	$\frac{\begin{array}{c} \Gamma; \mathcal{H} \vdash_{\Sigma} A \Leftarrow \mathbb{U} \\ \Gamma, x : A; \mathcal{H} \vdash_{\Sigma} [x]B \Leftarrow \mathbb{U} \end{array} \quad Q \in \{\Sigma, \Pi\}}{\Gamma; \mathcal{H} \vdash_{\Sigma} Q_A[x]B \Rightarrow \mathbb{U}}$

(Sigma)

$$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} a \Leftarrow A \quad \Gamma; \mathcal{H} \vdash_{\Sigma} b \Leftarrow [a/x]B}{\Gamma; \mathcal{H} \vdash_{\Sigma} \langle a, b \rangle \Leftarrow \Sigma_A[x]B} \quad \frac{\Gamma; \mathcal{H} \vdash_{\Sigma} p \Rightarrow \Sigma_A[x]B \quad \Gamma, u : \Sigma_A[x]B; \mathcal{H} \vdash_{\Sigma} [u]C \Leftarrow \mathbb{U} \quad \Gamma, v : A, w : [v/x]B; \mathcal{H} \vdash_{\Sigma} [v, w]M \Leftarrow [\langle v, w \rangle / u]C}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{spread}([u]C; [v, w]M; p) \Rightarrow [p/u]C}$$

(Pi)

$$\frac{\Gamma, x : A; \mathcal{H} \vdash_{\Sigma} [x]M \Leftarrow [x]B}{\Gamma; \mathcal{H} \vdash_{\Sigma} \lambda[x]M \Leftarrow \Pi_A[x]B} \quad \frac{\Gamma; \mathcal{H} \vdash_{\Sigma} f \Rightarrow \Pi_A[x]B \quad \Gamma; \mathcal{H} \vdash_{\Sigma} c \Leftarrow A}{\Gamma; \mathcal{H} \vdash_{\Sigma} f @ c \Rightarrow [c/x]B}$$

(Id)

$$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} A \Leftarrow \mathbb{U} \quad \Gamma; \mathcal{H} \vdash_{\Sigma} M \Leftarrow A \quad \Gamma; \mathcal{H} \vdash_{\Sigma} N \Leftarrow A}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{Id}_A(M; N) \Rightarrow \mathbb{U}} \quad \frac{\Gamma; \mathcal{H} \vdash_{\Sigma} M \Leftarrow A}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{refl} \Leftarrow \text{Id}_A(M; M)}$$

(BinderEq)

$$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} Q_A[x]B \Leftarrow \mathbb{U} \quad \Gamma; \mathcal{H} \vdash_{\Sigma} Q'_{A'}[x]B' \Leftarrow \mathbb{U} \quad \Gamma; \mathcal{H} \vdash_{\Sigma} M \Leftarrow \text{Id}_{\mathbb{U}}(A; A') \quad \Gamma, u : A; \mathcal{H}, A \equiv A' : \mathbb{U} \vdash_{\Sigma} N \Leftarrow \text{Id}_{\mathbb{U}}([u/x]B; [u/x]B')}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{binderEq}(Q_A[x]B; Q_{A'}[x]B'; M; [y]N) \Rightarrow \text{Id}_{\mathbb{U}}(Q_A[x]B; Q_{A'}[x]B')}$$

(Funext)

$$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} f \Rightarrow \Pi_A[x]B \quad \Gamma; \mathcal{H} \vdash_{\Sigma} g \Leftarrow \Pi_A[x]B \quad \Gamma, u : A; \mathcal{H} \vdash_{\Sigma} M \Leftarrow \text{Id}_{[u/x]B}(f @ u; g @ u)}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{funext}(f; g; [u]M) \Rightarrow \text{Id}_{\Pi_A[x]B}(f; g)}$$

(UIP)

$$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} p \Rightarrow \text{Id}_A(M; N) \quad \Gamma; \mathcal{H} \vdash_{\Sigma} q \Leftarrow \text{Id}_A(M; N)}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{uip}(p; q) \Rightarrow \text{Id}_{\text{Id}_A(M; N)}(p; q)}$$