A TYPE THEORY WITH SCOPED EQUALITY REFLECTION

JONATHAN STERLING

Feel free to change the stuff below as our thinking evolves. This is just a starting point. We have the following judgments:

$\vdash \Sigma\ \mathrm{sig}$	Σ is a valid signature
$\mathcal{H} \vdash_\Sigma \Gamma\ \mathrm{ctx}$	Γ is a valid context
$\Gamma \vdash_\Sigma \mathcal{H}\ \mathrm{hints}$	$\mathcal{H}$ is a valid hints set
$\Gamma; \mathcal{H} \vdash_\Sigma M \Rightarrow A$	M infers type A
$\Gamma; \mathcal{H} \vdash_\Sigma M \Leftarrow A$	M checks type A
$\Gamma; \mathcal{H} \vdash_\Sigma M \equiv N : A$	M equals N at type A

The rules for the theory are given inductive-recursively below.

(Signatures)
$$\frac{}{\vdash \diamond\ \mathrm{sig}} \qquad \frac{\begin{array}{l}\vdash \Sigma\ \mathrm{sig} \\ \diamond \vdash^{\diamond}_{\Sigma} A \Leftarrow \mathbb{U}\end{array} \quad \Sigma \mathbin{\#} c}{\vdash \Sigma, c : A\ \mathrm{sig}}$$

(Contexts)
$$\frac{}{\mathcal{H} \vdash_\Sigma \diamond\ \mathrm{ctx}} \qquad \frac{\begin{array}{l}\mathcal{H} \vdash_\Sigma \Gamma\ \mathrm{ctx} \\ \Gamma; \mathcal{H} \vdash_\Sigma A \Leftarrow \mathbb{U}\end{array} \quad \Gamma \mathbin{\#} x}{\mathcal{H} \vdash_\Sigma \Gamma, x : A\ \mathrm{ctx}}$$

(Hints)
$$\frac{}{\Gamma \vdash \diamond\ \mathrm{hints}} \qquad \frac{\begin{array}{l}\Gamma \vdash_\Sigma \mathcal{H}\ \mathrm{hints} \\ \Gamma; \mathcal{H} \vdash_\Sigma A \Leftarrow \mathbb{U} \\ \Gamma; \mathcal{H} \vdash_\Sigma a, b \Leftarrow A\end{array}}{\Gamma \vdash_\Sigma \mathcal{H}, a \equiv b : A\ \mathrm{hints}}$$

(Projection)
$$\frac{x : A \in \Gamma}{\Gamma; \mathcal{H} \vdash_\Sigma x \Rightarrow A} \qquad \frac{c : A \in \Sigma}{\Gamma; \mathcal{H} \vdash_\Sigma c \Rightarrow A} \qquad \frac{a \equiv b : A \in \mathcal{H}}{\Gamma; \mathcal{H} \vdash_\Sigma a \equiv b : A}$$

(Reflect)
$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma p \Rightarrow \mathsf{Id}_A\, a\, b \\ \Gamma; \mathcal{H} \vdash_\Sigma C \Leftarrow \mathbb{U} \\ \Gamma; (\mathcal{H}, a \equiv b : A) \vdash_\Sigma e \Leftarrow C\end{array}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathsf{reflect}\ p\ \mathsf{in}\ e \Leftarrow C}$$

(Binders)
$$\frac{\begin{array}{l}\Gamma; \mathcal{H} \vdash_\Sigma A \Leftarrow \mathbb{U} \\ \Gamma, x : A; \mathcal{H} \vdash_\Sigma [x]B \Leftarrow \mathbb{U}\end{array} \quad \mathrm{Q} \in \{\Sigma, \Pi\}}{\Gamma; \mathcal{H} \vdash_\Sigma \mathrm{Q}_A[x]B \Rightarrow \mathbb{U}}$$

(Sigma)

$$\frac{\Gamma; \mathcal{H} \vdash_\Sigma a \Leftarrow A \qquad \Gamma; \mathcal{H} \vdash_\Sigma b \Leftarrow [a/x]B}{\Gamma; \mathcal{H} \vdash_\Sigma \langle a, b \rangle \Leftarrow \Sigma_A[x]B} \qquad \frac{\Gamma; \mathcal{H} \vdash_\Sigma p \Rightarrow \Sigma_A[x]B \qquad \Gamma, u : \Sigma_A[x]B; \mathcal{H} \vdash_\Sigma [u]C \Leftarrow \mathbb{U} \qquad \Gamma, v : A, w : [v/x]B; \mathcal{H} \vdash_\Sigma [v,w]M \Leftarrow [\langle v, w \rangle/u]C}{\Gamma; \mathcal{H} \vdash_\Sigma \mathsf{split}([u]C; [v,w]M; p) \Rightarrow [p/u]C}$$

(Pi)

$$\frac{\Gamma, x : A; \mathcal{H} \vdash_\Sigma [x]M \Leftarrow [x]B}{\Gamma; \mathcal{H} \vdash_\Sigma \lambda[x]M \Leftarrow \Pi_A[x]B} \qquad \frac{\Gamma; \mathcal{H} \vdash_\Sigma f \Rightarrow \Pi_A[x]B \qquad \Gamma; \mathcal{H} \vdash_\Sigma c \Leftarrow A}{\Gamma; \mathcal{H} \vdash_\Sigma f \mathbin{@} c \Rightarrow [c/x]B}$$