

A TYPE THEORY WITH SCOPED EQUALITY REFLECTION

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Feel free to change the stuff below as our thinking evolves. This is just a starting point. We have the following judgments:

$\vdash \Sigma \text{ sig}$	Σ is a valid signature
$\mathcal{H} \vdash_{\Sigma} \Gamma \text{ ctx}$	Γ is a valid context
$\Gamma \vdash_{\Sigma} \mathcal{H} \text{ hints}$	\mathcal{H} is a valid hints set
$\Gamma; \mathcal{H} \vdash_{\Sigma} M \Rightarrow A$	M infers type A
$\Gamma; \mathcal{H} \vdash_{\Sigma} M \Leftarrow A$	M checks type A
$\Gamma; \mathcal{H} \vdash_{\Sigma} M \equiv N : A$	M equals N at type A

The rules for the theory are given inductive-recursively below.

(Signatures)	$\frac{}{\vdash \diamond \text{ sig}} \quad \frac{\vdash \Sigma \text{ sig} \quad \Sigma \# c \quad \diamond \vdash_{\Sigma}^{\circ} A \Leftarrow \mathbb{U}}{\vdash \Sigma, c : A \text{ sig}}$
(Contexts)	$\frac{}{\mathcal{H} \vdash_{\Sigma} \diamond \text{ ctx}} \quad \frac{\mathcal{H} \vdash_{\Sigma} \Gamma \text{ ctx} \quad \Gamma \# x \quad \Gamma; \mathcal{H} \vdash_{\Sigma} A \Leftarrow \mathbb{U}}{\mathcal{H} \vdash_{\Sigma} \Gamma, x : A \text{ ctx}}$
(Hints)	$\frac{}{\Gamma \vdash \diamond \text{ hints}} \quad \frac{\Gamma \vdash_{\Sigma} \mathcal{H} \text{ hints} \quad \Gamma; \mathcal{H} \vdash_{\Sigma} A \Leftarrow \mathbb{U} \quad \Gamma; \mathcal{H} \vdash_{\Sigma} a, b \Leftarrow A}{\Gamma \vdash_{\Sigma} \mathcal{H}, a \equiv b : A \text{ hints}}$
(Projection)	$\frac{x : A \in \Gamma}{\Gamma; \mathcal{H} \vdash_{\Sigma} x \Rightarrow A} \quad \frac{c : A \in \Sigma}{\Gamma; \mathcal{H} \vdash_{\Sigma} c \Rightarrow A} \quad \frac{a \equiv b : A \in \mathcal{H}}{\Gamma; \mathcal{H} \vdash_{\Sigma} a \equiv b : A}$
(Reflect)	$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} p \Rightarrow \text{ld}_A(M; N) \quad \Gamma; \mathcal{H}, M \equiv N : A \vdash_{\Sigma} C \Leftarrow \mathbb{U} \quad \Gamma; \mathcal{H}, M \equiv N : A \vdash_{\Sigma} e \Leftarrow C}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{reflect } p \text{ in } e \Leftarrow C}$
(Binders)	$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} A \Leftarrow \mathbb{U} \quad \Gamma, x : A; \mathcal{H} \vdash_{\Sigma} [x]B \Leftarrow \mathbb{U} \quad Q \in \{\Sigma, \Pi\}}{\Gamma; \mathcal{H} \vdash_{\Sigma} Q_A[x]B \Rightarrow \mathbb{U}}$

(Sigma)

$$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} a \Leftarrow A \quad \Gamma; \mathcal{H} \vdash_{\Sigma} b \Leftarrow [a/x]B}{\Gamma; \mathcal{H} \vdash_{\Sigma} \langle a, b \rangle \Leftarrow \Sigma_A[x]B} \quad \frac{\Gamma; \mathcal{H} \vdash_{\Sigma} p \Rightarrow \Sigma_A[x]B \quad \Gamma, u : \Sigma_A[x]B; \mathcal{H} \vdash_{\Sigma} [u]C \Leftarrow \mathbb{U} \quad \Gamma, v : A, w : [v/x]B; \mathcal{H} \vdash_{\Sigma} [v, w]M \Leftarrow [\langle v, w \rangle / u]C}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{split}([u]C; [v, w]M; p) \Rightarrow [p/u]C}$$

(Pi)

$$\frac{\Gamma, x : A; \mathcal{H} \vdash_{\Sigma} [x]M \Leftarrow [x]B}{\Gamma; \mathcal{H} \vdash_{\Sigma} \lambda[x]M \Leftarrow \Pi_A[x]B} \quad \frac{\Gamma; \mathcal{H} \vdash_{\Sigma} f \Rightarrow \Pi_A[x]B \quad \Gamma; \mathcal{H} \vdash_{\Sigma} c \Leftarrow A}{\Gamma; \mathcal{H} \vdash_{\Sigma} f @ c \Rightarrow [c/x]B}$$

(Id)

$$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} A \Leftarrow \mathbb{U} \quad \Gamma; \mathcal{H} \vdash_{\Sigma} M \Leftarrow A \quad \Gamma; \mathcal{H} \vdash_{\Sigma} N \Leftarrow A}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{ld}_A(M; N) \Rightarrow \mathbb{U}} \quad \frac{\Gamma; \mathcal{H} \vdash_{\Sigma} M \Leftarrow A}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{refl} \Leftarrow \text{ld}_A(M; M)}$$

(BinderEq)

$$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} Q_A[x]B \Leftarrow \mathbb{U} \quad \Gamma; \mathcal{H} \vdash_{\Sigma} Q'_A[x]B' \Leftarrow \mathbb{U} \quad \Gamma; \mathcal{H} \vdash_{\Sigma} M \Leftarrow \text{ld}_{\mathbb{U}}(A; A') \quad \Gamma, u : A; \mathcal{H}, A \equiv A' : \mathbb{U} \vdash_{\Sigma} N \Leftarrow \text{ld}_{\mathbb{U}}([u/x]B; [u/x]B')}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{binderEq}(M; [y]N) \Leftarrow \text{ld}_{\mathbb{U}}(Q_A[x]B; Q_{A'}[x]B')} \quad Q \in \{\Pi, \Sigma\}$$

(Funext)

$$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} f \Leftarrow \Pi_A[x]B \quad \Gamma; \mathcal{H} \vdash_{\Sigma} g \Leftarrow \Pi_A[x]B \quad \Gamma, u : A; \mathcal{H} \vdash_{\Sigma} M \Leftarrow \text{ld}_{[u/x]B}(f @ u; g @ u)}{\Gamma; \mathcal{H} \vdash_{\Sigma} \text{funext}([u]M) \Leftarrow \text{ld}_{\Pi_A[x]B}(f; g)}$$