A TYPE THEORY WITH SCOPED EQUALITY REFLECTION

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Feel free to change the stuff below as our thinking evolves. This is just a starting point. We have the following judgments:

 $\begin{array}{lll} \vdash \Sigma \ \mathrm{sig} & \Sigma \ \mathrm{is \ a \ valid \ signature} \\ \mathcal{H} \vdash_{\Sigma} \Gamma \ \mathrm{ctx} & \Gamma \ \mathrm{is \ a \ valid \ context} \\ \Gamma \vdash_{\Sigma} \mathcal{H} \ \mathrm{hints} & \mathcal{H} \ \mathrm{is \ a \ valid \ hints \ set} \\ \Gamma; \mathcal{H} \vdash_{\Sigma} M : A & M \ \mathrm{has \ type} \ A \end{array}$

 $\Gamma; \mathcal{H} \vdash_{\Sigma} M \equiv N : A \quad M \text{ equals } N \text{ at type } A$

The rules for the theory are given inductive-recursively below.

$$(\text{Signatures}) \qquad \qquad \frac{\vdash \Sigma \text{ sig} \quad \diamond \vdash_{\Sigma}^{\diamond} A : \mathbb{U} \quad \Sigma \# c}{\vdash \Sigma, c : A \text{ sig}}$$

$$(Contexts) \qquad \frac{\mathcal{H} \vdash_{\Sigma} \Gamma \operatorname{ctx} \quad \Gamma; \mathcal{H} \vdash_{\Sigma} A : \mathbb{U} \quad \Gamma \# x}{\mathcal{H} \vdash_{\Sigma} \Gamma, x : A \operatorname{ctx}}$$

$$(\text{Hints}) \qquad \frac{\Gamma \vdash_{\Sigma} \mathcal{H} \text{ hints} \quad \Gamma; \mathcal{H} \vdash_{\Sigma} A : \mathbb{U} \quad \Gamma; \mathcal{H} \vdash_{\Sigma} a, b : A}{\Gamma \vdash_{\Sigma} \mathcal{H}, a \equiv b : A \text{ hints}}$$

$$\begin{array}{ccc} \underline{x:A\in\Gamma} & \underline{c:A\in\Sigma} & \underline{a\equiv b:A\in\mathcal{H}} \\ \text{(Projection)} & \overline{\Gamma;\mathcal{H}\vdash_{\Sigma}x:A} & \overline{\Gamma;\mathcal{H}\vdash_{\Sigma}c:A} & \overline{\Gamma;\mathcal{H}\vdash_{\Sigma}a\equiv b:A} \end{array}$$

$$\frac{\Gamma; \mathcal{H} \vdash_{\Sigma} p : \mathsf{Id}_{A} \ a \ b \quad \Gamma; (\mathcal{H}, a \equiv b : A) \vdash_{\Sigma} e : C}{\Gamma; \mathcal{H} \vdash_{\Sigma} \mathsf{reflect} \ p \ \mathsf{in} \ e : C}$$