

**Mathematics.** — *Demonstration that the concept of spreads of higher order does not come into consideration as a fundamental notion in intuitionistic mathematics.* By Prof. L. E. J. BROUWER\*

(Communicated at the meeting of September 26, 1942.)

In my note, “*Concerning the free development of spreads and functions*”,<sup>1)</sup> the process  $M_\sigma$  was considered, through which the fundamental sequence  $F'$ , which is enumerated in an arbitrary, predetermined way, is associated one-to-one with finite choice sequences of numbers and likewise an arbitrary element  $\sigma$  of the spread<sup>2)</sup>  $M$ . We want to call this process  $M_\sigma$  a spread of second order, and the successive sequences of figures thus associated to the unrestricted choice sequences of numbers [we shall call] *the elements of the second-order spread*  $M_\sigma$ .

The assertion stated in my quoted note, that  $M_\sigma$  acts as a subspecies of a spread  $M_1$  which is derivable from  $M$ , and that the union of all  $M_\sigma$  generated from  $M$  is identical with this  $M_1$ , shall be demonstrated as follows. First, we will deal with the construction of the spread  $M_1$ .

Let  $\alpha_1 \alpha_2 \dots \alpha_m$  be a finite choice sequences of numbers. We will notate the rank thereof in the fundamental sequence  $F'$  with  $\varrho(\alpha_1 \alpha_2 \dots \alpha_m)$  and the maximum of the numbers  $\varrho(\alpha_1)$ ,  $\varrho(\alpha_1 \alpha_2)$ ,  $\dots$   $\varrho(\alpha_1 \alpha_2 \dots \alpha_m)$  with  $\zeta(\alpha_1 \alpha_2 \dots \alpha_m)$ .

We will call the combination of an arbitrary number  $\alpha_1$  with  $\varrho(\alpha_1)$  arbitrary numbers  $\beta_1, \beta_2, \dots \beta_{\varrho(\alpha_1)}$  a  $K$ -combination. We enumerate the  $K$ -combinations through a fundamental sequence  $F$ . We notate any  $K$ -combination which receives the rank  $\nu_1$  in  $F$  with  $K_{\nu_1}$ .

For a given  $\nu_1$ , and thence also a given  $\alpha_1$ , and arbitrary  $\alpha_2$ , we call the number  $\alpha_2$  a  $K_{\nu_1}$ -combination in case  $\zeta(\alpha_1 \alpha_2) = \zeta(\alpha_1)$ , and likewise, in case  $\zeta(\alpha_1 \alpha_2) > \zeta(\alpha_1)$ , we call the combination of  $\alpha_2$  with  $\zeta(\alpha_1 \alpha_2) - \zeta(\alpha_1)$  arbitrary numbers  $\beta_{\zeta(\alpha_1)+1}, \dots \beta_{\zeta(\alpha_1 \alpha_2)}$  a  $K_{\nu_1}$ -combination. For each  $\nu_1$  we enumerate the  $K_{\nu_1}$ -combinations through a fundamental sequence  $F_{\nu_1}$ . We notate each  $K_{\nu_1}$ -combination, which receives rank  $\nu_2$  in  $F_{\nu_1}$ , with  $K_{\nu_1 \nu_2}$ .

For arbitrary  $\nu_1$  and  $\nu_2$ , and thence also given  $\alpha_1$  and  $\alpha_2$ , and arbitrary  $\alpha_3$ , we call the number  $\alpha_3$  a  $K_{\nu_1 \nu_2}$ -combination in case  $\zeta(\alpha_1 \alpha_2 \alpha_3) = \zeta(\alpha_1 \alpha_2)$ , and, in case  $\zeta(\alpha_1 \alpha_2 \alpha_3) > \zeta(\alpha_1 \alpha_2)$ , we likewise call the combination of  $\alpha_3$  with  $\zeta(\alpha_1 \alpha_2 \alpha_3) - \zeta(\alpha_1 \alpha_2)$  arbitrary numbers  $\beta_{\zeta(\alpha_1 \alpha_2)+1}, \dots \beta_{\zeta(\alpha_1 \alpha_2 \alpha_3)}$  a  $K_{\nu_1 \nu_2}$ -combination. For

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\* Translated from the original German by Jon Sterling.

<sup>1)</sup> Proc. Ned. Akad. v. Wetensch. Amsterdam, **45**, 322 (1942).

<sup>2)</sup> For the sake of simplicity, we restrict ourselves in this note to such spreads, in the process of whose creation neither inhibition nor termination occurs. This restriction is inessential.

each pair of numbers  $\nu_1, \nu_2$  we enumerate the  $K_{\nu_1 \nu_2}$ -combinations through a fundamental sequence  $F_{\nu_1 \nu_2}$ . We notate each  $K_{\nu_1 \nu_2}$  combination which receives the rank  $\nu_3$  in  $F_{\nu_1 \nu_2}$  with  $K_{\nu_1 \nu_2 \nu_3}$ .

Proceeding in this way, we define  $K_{\nu_1 \nu_2 \dots \nu_s}$  for each natural number  $s$ . In doing so, we take care from the outset to determine a law through which the fundamental sequences  $F_{\nu_1 \nu_2 \dots \nu_s}$ , each enumerating  $K_{\nu_1 \nu_2 \dots \nu_s}$ , shall be defined once and for all.

The construction of  $M_1$  will now be now be carried out, in which we shall associate each sequence of figures with the finite sequence  $\nu_1 \nu_2 \dots \nu_s$ , which is associated to the finite choice sequence  $\beta_1 \beta_2 \dots \beta_{\varrho(\alpha_1 \alpha_2 \dots \alpha_s)}$  for the respective numbers  $\alpha_1 \alpha_2 \dots \alpha_s$ ,  $\beta_1 \beta_2 \dots \beta_{\varrho(\alpha_1 \alpha_2 \dots \alpha_s)}$  in  $M$ .

Let  $\sigma$  be the element of  $M$  generated by the infinite choice sequence  $\gamma_1 \gamma_2 \gamma_3 \dots$