THE EQUIVARIANT UNIFORM KAN FIBRATION MODEL OF CUBICAL HOMOTOPY TYPE THEORY (E. RIEHL)

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(0.1) Joint work with Awodey, Caallo, Coquand, Sattler

(0.2) We want a $\mathbf{P}(\square)$-based model of HoTT which is a Quillen model category equivalent to spaces (Kan complexes in simplicial sets). We may further ask that this equivalence be a *nice* functor (for instance, a triangulation), and that $\square$ supports inductive constructions (in the sense of being an Eilenberg-Zilber category).

Team	Cubes	Equivalent to spaces?
BCH	symmetric monoidal cubes	No: consider $\mathbb{I}^2/\mathsf{swap}$ (Buchholtz)
ABCFHL, A	Cartesian cubes	No: analogous argument by Sattler
CCHM	De Morgan cubes	No: consider $\mathbb{I}/\mathsf{rev}$ (Buchholtz)
CCHM	Dedekind	open problem

(0.3) We will work with Cartesian cubes, but change the notion of fibration to rule out the counterexample. We need $* \longrightarrow \mathbb{I}^2/\mathsf{swap}$ to be a trivial cofibration. The idea is to add an equivariance condition $j(x,y)\sigma = j(x,y\sigma)$ relative to symmetries σ:

$$\begin{tikzcd}
* \arrow[r] \arrow[d] & * \arrow[r] \arrow[d] & * \arrow[r, "x"] \arrow[d] & X \arrow[d] \\
\mathbb{I}^2 \arrow[r, "\sigma"'] \arrow[urrr, bend left, "j(x,y\sigma)"] & \mathbb{I}^2 \arrow[r, two heads, "e"'] \arrow[urr, bend left, "j(x,y)"] & \mathbb{I}^2/\mathsf{swap} \arrow[r, "y"'] & Y
\end{tikzcd}$$

We get the desired lift using the universal property of the quotient map e.