PATTERNS FOR ABTS

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We will work with the very simple type systems of abstract binding trees, where a type is a valence, or it is a vector of valences.

$$\frac{n \in \mathbb{N}}{n \ \mathit{valence}} \qquad \frac{|_i \ m_i \in \mathbb{N} \ (i < n)}{(m_0; \ldots; m_n) \ \mathit{arity}}$$

$$\frac{v \ \mathit{valence}}{v \ \mathit{type}} \qquad \frac{\vec{v} \ \mathit{arity}}{\vec{v} \ \mathit{type}}$$

The type system is parameterized over a set of operators $\mathcal{O}$ with a meta-operation $\mathfrak{A}$ such that $|_\vartheta \ \mathfrak{A}(\vartheta) \ \mathit{arity}$ $(\vartheta \in \mathcal{O})$.

$$\frac{}{\cdot \ \mathit{ctx}} \qquad \frac{\Psi \ \mathit{ctx} \quad x \notin |\Psi|}{\Psi, x : \tau \ \mathit{ctx}}$$

The context concatenation judgment $\boxed{\Psi \oplus \Psi' \rightsquigarrow \Psi''}$ (presupposing $\Psi \ \mathit{ctx}$, $\Psi' \ \mathit{ctx}$) admits the postsupposition $\Psi' \ \mathit{ctx}$.

$$\frac{}{\Psi \oplus \cdot \rightsquigarrow \Psi} \qquad \frac{\Psi \oplus \Psi' \rightsquigarrow \Psi'' \quad x \notin |\Psi'|}{\Psi \oplus \Psi', x : \tau \rightsquigarrow \Psi'', x : \tau}$$

The judgment $\boxed{\Psi \mid \Lambda \Vdash p : \tau}$ (presupposing $\Psi \ \mathit{ctx}$, $\tau \ \mathit{type}$) expresses the checking of patterns p at type τ with respect to an intuitionistic context Ψ, synthesizing a linear context Λ (admitting the postsupposition $\Lambda \ \mathit{ctx}$).

$$\frac{}{\Psi \mid \cdot \Vdash (\cdot) : (\cdot)} \ \text{nil} \qquad \frac{\Psi \mid \Lambda_1 \Vdash (\vec{p}) : (\vec{v}) \quad \Psi \mid \Lambda_2 \Vdash p : v \quad \Lambda_1 \oplus \Lambda_2 \rightsquigarrow \Lambda}{\Psi \mid \Lambda \Vdash (\vec{p}; p) : (\vec{v}; v)} \ \text{snoc}$$

$$\frac{}{\Psi \mid \cdot \Vdash \_ : \tau} \ \text{wildcard} \qquad \frac{\Psi \ni x : \tau}{\Psi \mid \cdot \Vdash x : \tau} \ \text{var} \qquad \frac{}{\Psi \mid \mathfrak{p} : \tau \Vdash \mathfrak{p} : \tau} \ \text{pat-var}$$

$$\frac{\vartheta \in \mathcal{O} \quad \Psi \mid \Lambda \Vdash \vec{p} : \mathfrak{A}(\vartheta)}{\Psi \mid \Lambda \Vdash \vartheta(\vec{p}) : 0} \ \text{op} \qquad \frac{\Psi, x : 0 \mid \Lambda \Vdash p : \tau}{\Psi \mid \Lambda \Vdash [x]p : \tau + 1} \ \text{abs}$$

The linear context Λ specifies the *pattern variables* which occur in a pattern, and would have to be substituted during unification. On the other hand, the intuitionistic context Ψ contains variables which arise from the binding structure of the language of patterns, and such variables may occur any number of times; in a simpler system that did not contain abstraction $[x]p$, the intuitionistic context would be unnecessary.