

# PATTERNS FOR ABTS

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We will work with the very simple type systems of abstract binding trees, where a type is a valence, or it is a vector of valences.

$$\frac{n \in \mathbb{N}}{n \text{ valence}} \quad \frac{|_i m_i \in \mathbb{N} \ (i < n)}{(m_0; \dots; m_n) \text{ arity}}$$

$$\frac{v \text{ valence}}{v \text{ type}} \quad \frac{\vec{v} \text{ arity}}{\vec{v} \text{ type}}$$

The type system is parameterized over a set of operators  $\mathcal{O}$  with a meta-operation  $\mathfrak{A}$  such that  $|_{\vartheta} \mathfrak{A}(\vartheta) \text{ arity}$  ( $\vartheta \in \mathcal{O}$ ).

$$\frac{}{\cdot \text{ ctx}} \quad \frac{\Psi \text{ ctx} \quad x \notin |\Psi|}{\Psi, x : \tau \text{ ctx}}$$

The context concatenation judgment  $\boxed{\Psi \oplus \Psi' \rightsquigarrow \Psi''}$  (presupposing  $\Psi \text{ ctx}$ ,  $\Psi' \text{ ctx}$ ) admits the postsupposition  $\Psi'' \text{ ctx}$ .

$$\frac{}{\Psi \oplus \cdot \rightsquigarrow \Psi} \quad \frac{\Psi \oplus \Psi' \rightsquigarrow \Psi'' \quad x \notin |\Psi'|}{\Psi \oplus \Psi', x : \tau \rightsquigarrow \Psi'', x : \tau}$$

The judgment  $\boxed{\Psi \mid \Lambda \Vdash p : \tau}$  (presupposing  $\Psi \text{ ctx}$ ,  $\tau \text{ type}$ ) expresses the checking of patterns  $p$  at type  $\tau$  with respect to an intuitionistic context  $\Psi$ , synthesizing a linear context  $\Lambda$  (admitting the postsupposition  $\Lambda \text{ ctx}$ ).

$$\frac{}{\Psi \mid \cdot \Vdash (\cdot) : (\cdot)} \text{ nil} \quad \frac{\Psi \mid \Lambda_1 \Vdash (\vec{p}) : (\vec{v}) \quad \Psi \mid \Lambda_2 \Vdash p : v \quad \Lambda_1 \oplus \Lambda_2 \rightsquigarrow \Lambda}{\Psi \mid \Lambda \Vdash (\vec{p}; p) : (\vec{v}; v)} \text{ snoc}$$

$$\frac{}{\Psi \mid \cdot \Vdash \_ : \tau} \text{ wildcard} \quad \frac{\Psi \ni x : \tau}{\Psi \mid \cdot \Vdash x : \tau} \text{ var} \quad \frac{}{\Psi \mid \mathbf{p} : \tau \Vdash \mathbf{p} : \tau} \text{ pat-var}$$

$$\frac{\vartheta \in \mathcal{O} \quad \Psi \mid \Lambda \Vdash \vec{p} : \mathfrak{A}(\vartheta)}{\Psi \mid \Lambda \Vdash \vartheta(\vec{p}) : 0} \text{ op} \quad \frac{\Psi, x : 0 \mid \Lambda \Vdash p : \tau}{\Psi \mid \Lambda \Vdash [x]p : \tau + 1} \text{ abs}$$

The linear context  $\Lambda$  specifies the *pattern variables* which occur in a pattern, and would have to be substituted during unification. On the other hand, the intuitionistic context  $\Psi$  contains variables which arise from the binding structure of the language of patterns, and such variables may occur any number of times; in a simpler system that did not contain abstraction  $[x]p$ , the intuitionistic context would be unnecessary.