1.1. Example OBCs in Macroeconomics

Occasionally Binding Constraints in DSGE Models

Jonathan Swarbrick[1]
Bank of Canada

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Occasionally binding constraints

Nearly every model you write down will have an inequality constraint somewhere, for example:

- Household borrowing constraints
- Irreversible investment
- Zero-lower bound on nominal interest rates
- Firm or bank financing constraints
- Cash in advance constraints

Household Borrowing Constraints

In the representative agent model with borrowers and savers, the borrowing constraint is often assumed to always assumed to bind (see e.g. Iacoviello & Neri 2010)

Guerrieri & Iacoviello (2017) relax the assumption to show how the inequality constraint can drive macro asymmetries

Intuition

- During the housing boom, borrowing constraints slackened as the value of household collateral increased.
- House prices collapsed $\longrightarrow$ rapid depreciation in the value of household collateral $\longrightarrow$ borrowing constraints to tighten.
- This led to a deeper/sharper fall during the downturn than the boom.

Patient Households

Proportion $\lambda \in (0,1)$ patient households that solves

$$\max_{c_t^p, h_t^p, b_t^p} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{p\,t} u\left(c_t^p, h_t^p\right) \tag{1}$$

subject to

$$c_t^p + q_t h_t^p + b_t^p = y_t^p + (1-\delta) q_t h_{t-1}^p + r_{t-1} b_{t-1}^p \tag{2}$$

- Note: h_t^p is housing not hours – depreciates at δ.

The household chooses h_t^p, c_t^p and b_t^p subject to exogenous y_t and q_t, and the market determined r_t, leading to

$$u'\left(h_t^p\right) = q_t u'\left(c_t^p\right) - \beta^p (1-\delta) \mathbb{E}_t \left[q_{t+1} u'\left(c_{t+1}^p\right)\right] \tag{3}$$

$$1 = \beta^p \mathbb{E}_t \left[\frac{u'\left(c_{t+1}^p\right)}{u'\left(c_t^p\right) p}\right] r_t \tag{4}$$

Impatient Households

The $1-\lambda$ impatient households solve an analogous problem but also subject to a borrowing constraint

$$b_t^i \leq \mathrm{m} q_t h_t^i. \tag{5}$$

Leading to:

$$u'\left(h_t^i\right) - q_t u'\left(c_t^i\right) + \beta^i (1-\delta)\mathbb{E}_t\left[q_{t+1} u'\left(c_{t+1}^i\right)\right] + \varrho_t q_t m = 0 \tag{6}$$

$$\varrho_t = u'\left(c_t^i\right) - \beta^i \mathbb{E}_t\left[u'\left(c_{t+1}^i\right)\right] r_t \tag{7}$$

where ϱ_t is the Lagrange multiplier on the borrowing constraint where:

$$\varrho_t \geq 0 \tag{8}$$

$$\varrho_t \left(m q_t h_t^i - b_t^i\right) = 0. \tag{9}$$

These are the Kuhn-Tucker conditions (equivalent to Lagrange conditions), (9) is the complementary slackness condition.

Irreversible Investment

Investment is the act of giving up something today for future gain.

- In virtually all situations this is, at the least, partially irreversible.
- I.e., investment is a sunk cost that cannot be recovered if economic conditions change significantly.

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- In virtually all situations this is, at the least, partially irreversible.
- I.e., investment is a sunk cost that cannot be recovered if economic conditions change significantly.
- $\implies$ there is an additional opportunity cost of investment not present in standard model that introduces an **uncertainty effect**.

Intuition

- When a firm invests, it observes distribution of future states.
  - In good states the firm will want to invest more
  - In bad states the firm will want to dis-invest.
- We define an increase in uncertainty as an increase in variance of this distribution.
  - Higher uncertainty $\implies$ higher probability of dis-investment.
- With irreversible investment, this will:.
  1. cause firms to insure against this by waiting to invest
  2. Increase the threshold value of positive investment, i.e. firms will need to see higher expected returns from a project in order to invest.

Simple Model with Irreversible Investment 1/3

Firm produces output using

$$Y_t = A_t f\left(K_{t-1}\right), \tag{10}$$

where A_t is exogenous TFP, and K_t the capital stock which evolves according to

$$K_t = I_t + (1-\delta) K_{t-1} \tag{11}$$

Every period, the firm choose to either invest or pay dividends in order to maximise the sum of expected discounted future utility of dividend payments. The problem can be written

$$\max_{I_t, D_t} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u\left(D_{t+s}\right) \tag{12}$$

$$\text{s.t.} \quad D_t + I_t = A_t f\left(K_{t-1}\right) \tag{13}$$

$$K_t = I_t + (1-\delta) K_{t-1} \tag{14}$$

$$I_t \geq 0 \tag{15}$$

Simple Model with Irreversible Investment 2/3

Problem is simplified to

$$\max_{K_t} \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \begin{array}{c} \beta^s u\left(A_{t+s} f\left(K_{t+s-1}\right) - K_{t+s} + (1-\delta) K_{t+s-1}\right) \\ +\lambda_{t+s}\left(K_{t+s} - (1-\delta) K_{t+s-1}\right) \end{array} \right\} \tag{16}$$

where $\lambda \geq 0$ is the Lagrange multiplier on the positivity constraint on investment.
This leads to

$$\mathbb{E}_t \left[\beta u'\left(D_{t+1}\right)\left(A_{t+1} f'\left(K_t\right) + 1 - \delta\right)\right] + \lambda_t - \mathbb{E}_t\left[\lambda_{t+1}\right](1-\delta) = u'\left(D_t\right) \tag{17}$$

Notice that with $\lambda_t 0 \forall t$, this would become

$$\beta \mathbb{E}_t \left[\frac{u'\left(D_{t+1}\right)}{u'\left(D_t\right)}\left(A_{t+1} f'\left(K_t\right) + 1 - \delta\right)\right] = 1 \tag{18}$$

The pricing equation of capital (Euler equation) from a standard RBC model.

Simple Model with Irreversible Investment 3/3

Lagrange multiplier λ_t can be defined as the **shadow value** of relaxing the constraint, i.e. how much a firm would pay to reduce investment by \$1.

- If I_t is positive, then this is strictly zero.
- If $I_t = 0$, then $\lambda_t \geq 0$.
- If there are any states of the world for which investment could be constrained at zero then $\mathbb{E}_t \left[\lambda_{t+1}\right] > 0$ is strictly positive.

Suppose $I_t > 0$, the FOC becomes

$$\beta \mathbb{E}_t \left[u'\left(D_{t+1}\right) \left(A_{t+1} f'\left(K_t\right) + 1 - \delta\right) \right] - \mathbb{E}_t \left[\lambda_{t+1}\right] (1 - \delta) = u'\left(D_t\right) \tag{19}$$

If make standard assumptions that $f'\left(K_t\right) > 0$, $f''\left(K_t\right) < 0$, then:

- A non-zero $\mathbb{E}_t \left[\lambda_{t+1}\right]$ implies a higher marginal product $f'\left(K_t\right)$ relative to that with unconstrained investment, which implies relatively lower investment.

Empirical Support and Further Reading

Empirical evidence:

- Caballero & Pindyck (1996) – an increase in the marginal profitability of capital increases the required return on investment (U.S. manufacturing data).
- Bloom, Bond & Van Reenen (2007) – firms with higher volatility in returns are more cautious about investment (U.K. manufacturing firms).
- Gilchrist, Sim & Zakrajšek (2014) – find evidence for a positive relationship between uncertainty and credit spreads, consistent with the option value (U.S. stock data).

For further study, see Bernanke (1983), Caballero & Pindyck (1996), Bloom, Bond & Van Reenen (2007), Gilchrist, Sim & Zakrajšek (2014)

Effective-lower Bound on Nominal Interest Rates

The interest rate rule in the standard NK model becomes, e.g.,:

$$r_t = \begin{cases} r + \phi_\pi (\pi_t - \pi^*) + \phi_y (y_t - \bar{y}) & \text{if } > 0 \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

Absent the ELB, there is a single determinate equilibrium if the response to output/inflation are within a required range – Taylor principle was interest rate should increase/fall more than change in inflation.

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What happens when interest rates are stuck at zero?

- There is some disagreement – issues include:
    - Multiple equilibria and indeterminacy
    - Size of multiplier – is it unusually large?
        - Paradox of thrift
        - Paradox of toil

Indeterminacy and multiple equilibria 1/2

While the uniqueness of equilibria can be guaranteed in the standard model by choice of parameters, this is not obviously true when interest rates are constrained at zero.

Consider standard 3-equation NK model:

- implies two long-run equilibria that depend on which agents expect:
  1. Normal/inflationary: $r = \bar{r}$, $\pi = \pi^*$;
  2. Deflationary: $r = 0$, $\pi = -r$

Non-fundamental (i.e. not to do with technology, output etc) shifts in expectations can cause economy to jump between equilibria (sunspots).

Indeterminacy and multiple equilibria 2/2

Being in the deflationary equilibrium implies that agents believe that the economy is converging to the **bad** steady-state

- Many papers rule this out explicitly both on theoretical and empirical grounds.

It turns out under many specifications, the NK model under inflation targeting is still indeterminate when ELB binds, even if the **bad** steady state is ruled out

- Holden (2017) provides necessary and sufficient conditions for determinacy.
- There is a unique equilibrium under a price-level targeting regime.

Fiscal multipliers

A number of papers claim that the fiscal multiplier is large when the ELB binds.

- See e.g. Woodford (2011), Eggertsson (2010*b*), Christiano, Eichenbaum & Rebelo (2012), Miyamoto, Nguyen & Sergeyev (2018), Di Serio, Fragetta & Gasteiger (2020)

Other papers claim that the fiscal multiplier is only about 1 – i.e. no multiplier!

- See e.g. Braun & Körber (2011), Braun, Boneva & Waki (2016), Braun, Korber & Waki (2013), Mertens & Ravn (2014)

Others find whether multiplier is large depends on:

- persistence of government spending Ngo (Forthcoming)
- monetary policy inertia Hills & Nakata (2018)

Fiscal multiplier – paradox of thrift

One explanation for the high multipliers is the **paradox of thrift**.

- Originally discussed in Keynes (1936)

Intuition:

- Propensity to save increases $\longrightarrow$ fall in AD $\longrightarrow$ fall in output/incomes $\longrightarrow$ agents want to save more, but aggregate savings fall because everyone poorer!
- At ELB this implies a deflationary spiral.
  - Lower expected inflation $\Longrightarrow$ higher real rates $\longrightarrow$ higher saving etc
- Consider an increase in government spending:
  - Increase in AD and expected inflation.
  - Higher inflation $\Longrightarrow$ lower real rates $\longrightarrow$ lower saving so boost to private demand.
- This implies a £1 increase in government spending increases AD by more than £1.

Fiscal multiplier – paradox of toil

Another explanation is the (related) **paradox of toil**.

- Discussed in Eggertsson (2010*a*)

Intuition:

- An adverse shock lowers output and causing deflation and ELB to bind.
- People feel poorer and increase hours worked.
  - I.e., leisure is a normal good, so households reduce both consumption and leisure following negative shock.
- Labour supply goes up $\longrightarrow$ wages go down $\longrightarrow$ prices go down
- Expected deflation $\longrightarrow$ higher real savings rates $\longrightarrow$ lower private demand.
- Ultimately less work in aggregate.
- Increased government spending has same effect as previous slide.

Small multiplier?

Braun, Boneva & Waki (2016) and Braun, Korber & Waki (2013) simulate an NK model and find fiscal multipliers close to one.

What is different?

- Rotemberg vs. Calvo
- Non-linear vs. log-linear

The pricing mechanism near ELB exhibits different properties than around zero inflation SS. Paradox of toil disappears.

- Braun et al. (2016) also argue that models with predictions consistent with empirical time-series exhibit low multipliers.

Can still exhibit large multipliers with Calvo prices (see e.g. Fernández-Villaverde, Gordon, Guerrón-Quintana & Rubio-Ramírez 2015)

Eggertsson & Singh (2019) discuss log-linear vs. non-linear, and Calvo vs. Rotemberg.

Further reading and other issues

- Deflationary sunspot equilibria: Aruoba, Cuba-Borda & Schorfheide (2018) and Mertens & Ravn (2014)
- Determinacy: Holden (2017)
- ELB and unconventional policy: Gilchrist, López-Salido & Zakrajšek (2015)
- ELB and non-linearities: Fernández-Villaverde, Gordon, Guerrón-Quintana & Rubio-Ramírez (2015)
- Optimal monetary policy and ELB: Eggertsson & Woodford (2003), Jung, Teranishi & Watanabe (2005), Evans, Fisher, Gourio & Krane (2015)
- Estimation Gust, Herbst, López-Salido & Smith (2017)

Financial Constraints

Another set of OBCs in the literature is on either firm financing or financial intermediation.
For further reading see:

- Adverse selection in SME lending: Swarbrick (2019).
- Bank borrowing constraints: Holden, Levine & Swarbrick (2020), Gertler, Kiyotaki & Prestipino (2019)
- Equity constraints: Brunnermeier & Sannikov (2014), He & Krishnamurthy (2013).
- Cash in advance constraints: Dixon & Pourpourides (2016).

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