2.3. Penalty Functions

Occasionally Binding Constraints in DSGE Models

Jonathan Swarbrick[1]
Bank of Canada

Bank of Canada – CMFE-Carleton Virtual Series
Advanced Topics in Macroeconomic Modelling

January 2021

[1] The views expressed are those of the authors and should not be interpreted as reflecting the views of the Bank of Canada.

The idea

Idea: replace the complimentary slackness condition with an approximation.

E.g. by using a penalty function

The idea

Idea: replace the complimentary slackness condition with an approximation.

E.g. by using a penalty function

- We could update the optimization problem to

$$\max_{c_t, b_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, h_t\right) - P\left(b_t\right) \tag{1}$$

  where

$$P_t = \begin{cases} 0 & \text{if } b_t \geq \underline{b} \\ \infty & \text{if } b_t < \underline{b} \end{cases} \tag{2}$$

The idea

Idea: replace the complimentary slackness condition with an approximation.

E.g. by using a penalty function

- We could update the optimization problem to

$$\max_{c_t, b_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, h_t\right) - P\left(b_t\right) \tag{1}$$

  where

$$P_t = \begin{cases} 0 & \text{if } b_t \geq \underline{b} \\ \infty & \text{if } b_t < \underline{b} \end{cases} \tag{2}$$

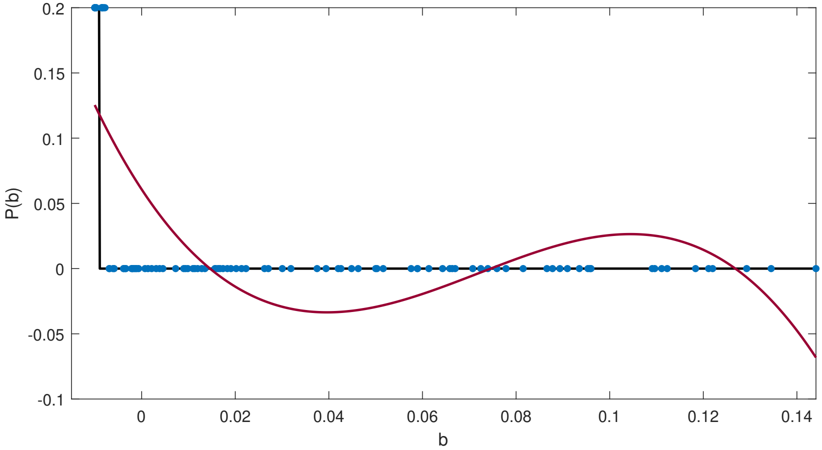
- an approximation could be

$$\tilde{P}\left(b_t\right) = \eta_0 + \eta_1 b_t + \eta_2 b_t^2 + \eta_3 b_t^3 + \eta_4 b_t^4 \tag{3}$$

Penalty function

Sample points for b from projection simulation, fit a 4th order polynomial

- $P = 0.07 - 5.38b + 83.2b^2 - 333b^3 + 77.5b^4$



- What are the drawbacks of this approach?

Borrowing constraints model

See code: **/penalty-function/soe_obc.mod** The household problem is written

$$\max_{c_t, h_t, b_t} \mathbb{E}_0 \sum_t^{\infty} \beta^t \left(\log\left(c_t\right) + \chi \log\left(1 - h_t\right) - \delta b_t^2 \right) - P(b_t) \tag{4}$$

$$\text{s.t.} \quad c_t + b_t = \exp\left(z_t\right) h_t + r b_{t-1} \tag{5}$$

$$P(b_t) = \eta_0 + \eta_1 b_t + \eta_2 b_t^2 + \eta_3 b_t^3 + \eta_4 b_t^4 \tag{6}$$

Leads to:

$$\frac{1}{c_t} = r\beta\mathbb{E}_t\left[\frac{1}{c_{t+1}}\right] - 2\delta b_t - \eta_1 - 2\eta_2 b_t - 3\eta_3 b_t^2 - 4\eta_4 b_t^3 \tag{7}$$

Notice we could write:

$$\frac{1}{c_t} = r\beta\mathbb{E}_t\left[\frac{1}{c_{t+1}}\right] - 2\delta b_t - \mu_t \tag{8}$$

$$\mu_t = -\left(\eta_1 + 2\eta_2 b_t + 3\eta_3 b_t^2 + 4\eta_4 b_t^3\right) \tag{9}$$

So $-P'\left(b_t\right)$ approximates μ_t.

Simulated Moments

Projection results:

	Mean	Standard deviation	Skewness	
	Relative to no constraint	Relative to no constraint	Baseline	No constraint
Consumption	+0.03%	+15%	-0.22	0.09
Hours	-0.01%	-43%	-0.09	-0.04
Bonds / $\bar{c}$	0.3% $\longrightarrow$ 5%	-59%	1.18	0.007

All methods:

		Mean	Standard deviation	Skewness (projection)
		Relative to projection	Relative to projection	
VFI	Consumption	≈ 0	+0.02%	-0.23 (-0.22)
	Hours	≈ 0	+0.6%	-0.09 (-0.09)
	Bonds / $\bar{c}$	5.0% $\longrightarrow$ 4.9%	$<$1%	1.18 (1.18)
Extended-path	Consumption	-0.03%	+0.5%	-0.29 (-0.22)
	Hours	+0.003%	+1%	-0.1 (-0.09)
	Bonds / $\bar{c}$	5.0% $\longrightarrow$ 4.0%	-1.4%	1.3 (1.18)
Penalty function	Consumption	-0.02%	+25%	0.01 (-0.22)
	Hours	-0.002%	-99%	-0.1 (-0.09)
	Bonds / $\bar{c}$	5.0% $\longrightarrow$ 4.4%	$\approx$-100%	-0.19 (1.18)