2. Occasionally Binding Constraints in DSGE Models

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[1] The views expressed are those of the authors and should not be interpreted as reflecting the views of the Bank of Canada.

Last Week

Last week we mentioned two reasons solving models with DSGE models with OBCs is difficult:

1. Model is forward-looking and stochastic
2. Local approximation cannot be used because OBCs are non-differentiable

We can use Value Function Iteration (VFI) or projection for small models

Recap: general problem

- A **D**ynamic **S**tochastic **G**eneral **E**quilibrium model has the general form:

$$\mathbb{E}_t f\left(x_{t+1}, x_t, x_{t-1}, u_t\right) = 0 \tag{1}$$

where $f(\cdot)$ is a *known* function.

- Usually recursive representation, i.e., same state variables $\Longrightarrow$ same decisions, implies policy:

$$x_t = g\left(x_{t-1}, u_t\right) \tag{2}$$

in general, $g(\cdot)$ is an *unknown* function.

- Because $g(\cdot)$ is unknown, it must be solved (approximated) numerically

Recap: VFI 1/2

Can use dynamic programming:

- Suppose household problem is

$$V(b,z) = \max_{b'} U(b,b',z) + \beta \int_{z'} p(z'|z) V\left(b',z'\right) \tag{3}$$

$$\text{s.t.} \qquad b' \geq \underline{b} \tag{4}$$

  where $U(\cdot)$ is known, but $V(\cdot)$ unknown.

- Conditional on b, z and an nth iteration $V^n(\cdot)$, we select b' to maxmimise the RHS
  - Everything else on RHS is known
  - This gives $V^{n+1}(\cdot)$
- Check if $V^{n+1}(\cdot) = V^n(\cdot)$, if not, we use $V^{n+1}(\cdot)$ for next iteration
- Solution gives optimal b' conditional on b, z, i.e., the policy function $b' = \tilde{g}(b,z)$

Recap: VFI 2/2

- Requires:
  1. Functional approximation: use basis functions to approximate $V^n(\cdot) \approx V(\cdot)$
  2. Numerical integration to approximate integral
  3. Optimization to solve b' each iteration

- The OBC may affect the choice of each

Recap: projection 1/2

VFI solves household problem directly.

- Alternatively, we can first derive the household first-order conditions
- Note we can write the model conditions:

$$\mathcal{H}\left(b'', b', b, z\right) = 0 \tag{5}$$

- $\mathcal{H}(\cdot)$ is a *functional equation* as it depends on unknown function $b' = g\left(b, z\right)$
- Idea of projection is to 'project' $\mathcal{H}\left(\cdot\right)$ against an approximating function $b' = \tilde{g}\left(b, z; \eta\right)$:

$$e\left(b, z; \eta\right) = \mathcal{H}\left(\tilde{g}\left(\tilde{g}\left(b, z; \eta\right), z'; \eta\right), \tilde{g}\left(b, z; \eta\right), b, z\right) \tag{6}$$

  where we want to minimize error $e\left(b, z; \eta\right)$.

Recap: projection

$$e\left(b, z; \eta_k\right) = \mathcal{H}\left(\tilde{g}\left(\tilde{g}\left(b, z; \eta\right), z'; \eta\right), \tilde{g}\left(b, z; \eta\right), b, z\right) \tag{7}$$

- The error depends on basis function parameters η and known b and z
- Objective then is to choose η to minimize e
- Either minimization routine/solver or iteration
- The OBC may affect the choice of basis functions and grid

Course overview

1. First session:
   - Introduction – OBCs in macro models
   - Model approximations and problem of OBCs
   - Global approximation
   - Intro to perturbation

2. Second session:
   - Deterministic (Newton Method)
   - Intro to local-based methods
   - Penalty function approximation
   - Regime-switching
   - News shocks

3. Third session:
   - News shocks
   - Precautionary behaviour
   - Multiple equilibria