3. Occasionally Binding Constraints in DSGE Models

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Bank of Canada – CMFE-Carleton Virtual Series
Advanced Topics in Macroeconomic Modelling

January 2021

[1]The views expressed are those of the authors and should not be interpreted as reflecting the views of the Bank of Canada.

Previously...

In the first couple of classes we looked at:

1. Using global approximation methods to macro models with OBCs
   - Value function iteration (numerically solve household problem)
   - Projection methods (approximate policy function)

2. Using expectational approximation and perturbation based methods:
   - Perfect foresight
   - Extended path
   - Penalty function
   - Piecewise linear

Recap: general problem

- A **D**ynamic **S**tochastic **G**eneral **E**quilibrium model has the general form:

$$\mathbb{E}_t f\left(x_{t+1}, x_t, x_{t-1}, u_t\right) = 0 \tag{1}$$

  where $f(\cdot)$ is a *known* function.

- Usually recursive representation, i.e., same state variables $\Longrightarrow$ same decisions, implies policy:

$$x_t = g\left(x_{t-1}, u_t\right) \tag{2}$$

  in general, $g(\cdot)$ is an *unknown* function.

- Because $g(\cdot)$ is unknown, it must be solved (approximated) numerically

Week 2 recap: perfect foresight

We can solve non-linearly more readily by abstracting from uncertainty.

If we assume that agents know the full sequence of shocks z, we can stack the model over T periods:

$$F(X) = \left\{ \begin{array}{c} f(x_0, x_1, x_2, z_1) \\ f(x_1, x_2, x_3, z_2) \\ \vdots \\ f(x_{T-1}, x_T, x_{T+1}, z_T) \end{array} \right\} = 0 \tag{3}$$

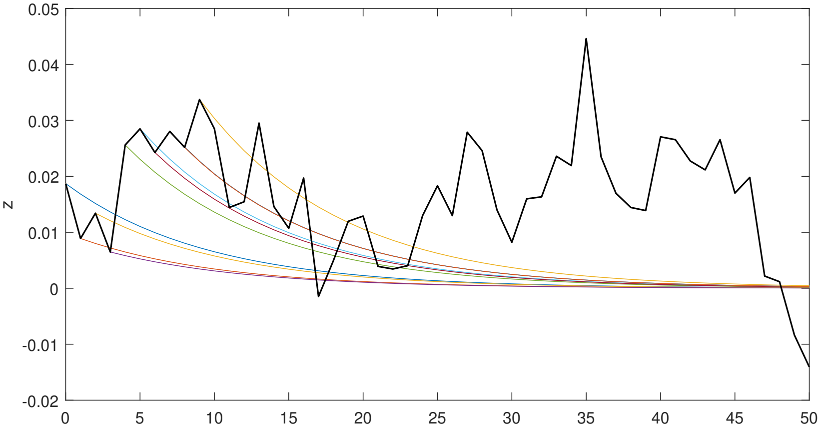
Decide on initial and end conditions ($x_0 = x_{T+1} = \bar{x}$)

- For an N equation model over T periods, there will be $N \times T$ unknowns and equations
- Can use a root-finding algorithm to find x to satisfy (3).
- By default dynare uses a Newton-type algorithm.

Week 2 recap: extended path

Here we assume firms and households are always surprised by shocks (see Fair & Taylor 1983)

- Every period they observe current and past shocks, and expect future shocks to equal zero
- Every period the perfect foresight simulation is solved to compute current decisions
- Actual series for z (black), with expected path at different points in time.



Week 2 recap: penalty function

We replace the inequality constraint with a penalty function

- We could update the optimization problem to

$$\max_{c_t, b_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, h_t\right) - P\left(b_t\right) \tag{4}$$

  where

$$P_t = \begin{cases} 0 & \text{if } b_t \geq \underline{b} \\ \infty & \text{if } b_t < \underline{b} \end{cases} \tag{5}$$

- An approximation could be

$$\tilde{P}\left(b_t\right) = \eta_0 + \eta_1 b_t + \eta_2 b_t^2 + \eta_3 b_t^3 + \eta_4 b_t^4 \tag{6}$$

- Solve using standard local methods.

Week 2 recap: regime switching

An OBC represents two regimes:

1. In one regime the constraint is slack
2. In the other, the constraint binds

In each regime, the model is approximated via perturbation.

There is an endogenous 'switching' between regimes.

The transition probabilities may or may not be endogenous.

Can generalize to a higher number of constraints (and regimes)

Course overview

1. First session:
   - Introduction – OBCs in macro models
   - Model approximations and problem of OBCs
   - Global approximation
   - Intro to perturbation

2. Second session:
   - Deterministic (Newton Method)
   - Intro to local-based methods
   - Penalty function approximation
   - Regime-switching

3. Third session:
   - Regime-switching
   - News shocks
   - Precautionary behaviour
   - Multiple equilibria

References I

Fair, R. C. & Taylor, J. B. (1983), 'Solution and Maximum Likelihood Estimation of Dynamic Nonlinear Rational Expectations Models', *Econometrica* **51**(4), 1169–1185.