Business Cycles in Space

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Motivation

Macro literature usually focuses on country-level outcomes.

- This can mask regional heterogeneity.
- Can miss interaction between regional and aggregate outcomes.

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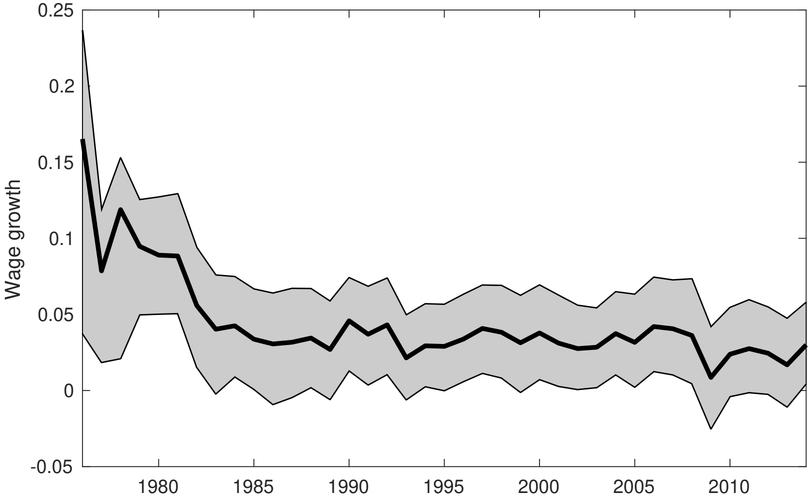

Figure: US county-level wage growth: mean & 10th/90th percentiles (BLS, unweighted)

▸ Histograms ▸ Dispersion ▸ Rural/urban ▸ Maps

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New economic geography (NEG) literature has looked at regional outcomes, but chiefly in static models.

- Does not look at business cycles; usually no shocks in the model.
- NEG models are usually not forward looking.
- Dynamic NEG models usually contain just a few regions.
  - Difficult to track spatial dynamics.
  - Non-atomistic regions make optimal policy very difficult.
- Those in continuous space usually have very restrictive assumptions.

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New Economic Geography Literature

We attempt to bring two literatures together:

- Propose new approach to building macro models with spatial heterogeneity.
- Study spatial effects of business cycles.

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- Firms physically closer to frontier firms catch up quicker (Griffith, Redding & Simpson 2009, Comin, Dmitriev & Rossi-Hansberg 2012, Cardamone 2017).
- Physical distance more important then economic distance in cross-country spillover of TFP growth (Glass, Kenjegalieva & Paez-Farrell 2013).

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Fortunately, spatial correlation enhances tractability of heterogeneous agent models.

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Individual shocks generate aggregate volatility:

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Many interesting questions, for example:

- How do regional asymmetries effect transmission of monetary policy?
- How do local shocks affect internal migration / housing markets / labour markets across physical space?
- Who are the regional winners/losers to policy programmes or other macro/local shocks?

For example... [To be examined fully in another paper]

The shale oil and gas "revolution" in the US provides a natural experiment to look at the effects of regional shocks on broader outcomes.

- Even with assorted controls, "high oil (gas) growth" (ERS-USDA) counties experienced 43.3ppts (25.4ppts) higher wage growth than other counties between 2000 and 2011.
- Over the period, population growth was significantly above average in such counties.
- Looking at the Bakken Shale Play area in North Dakota, up to 2012 we see sharp increases in population and wage growth not just in the shale counties, but also in neighbouring ones.
- After 2012, there is a corresponding decline.

Contributions

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   - Space need not be physical space
   - The geometry of space is flexible

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2. As a benchmark case, we develop a DSGE model of economic geography.
    - Real business cycle model + firm dynamics + typical ingredients from NEG model
    - Currently treated as story-telling device, capturing dynamics of economic geography in the U.S.
        - Agglomeration forces will lead to persistent movements in population and capital
        - Exploring short-medium frequency cycles

An example spatially correlated shock process

Suppose space is 1-dimensional over the interval $[0, 1]$.

One shock process over this space is the Orstein-Uhlenbeck process:

- Continuous time (space) analogue to Gaussian AR(1) process.
- Characterised by covariance structure:

$$\text{cov}\left(\varepsilon_x, \varepsilon_{\tilde{x}}\right) = \exp\left(-\zeta|x - \tilde{x}|\right)$$

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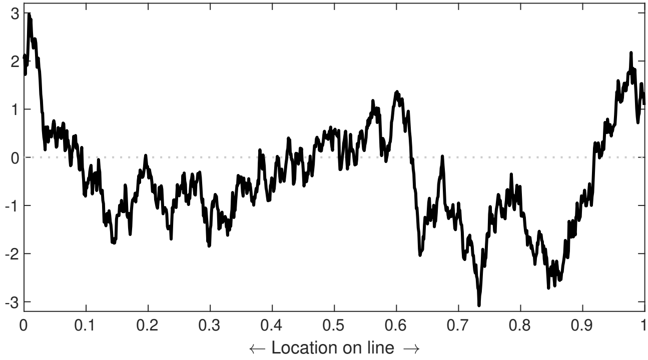

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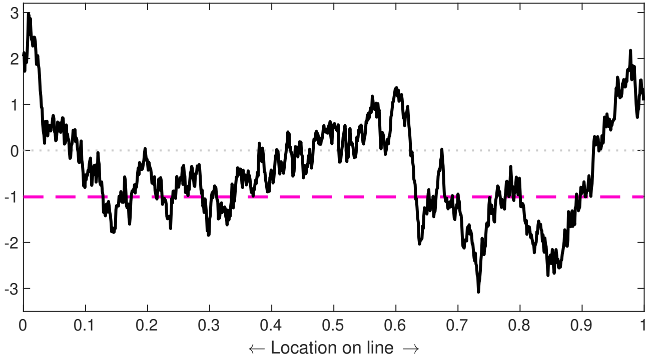

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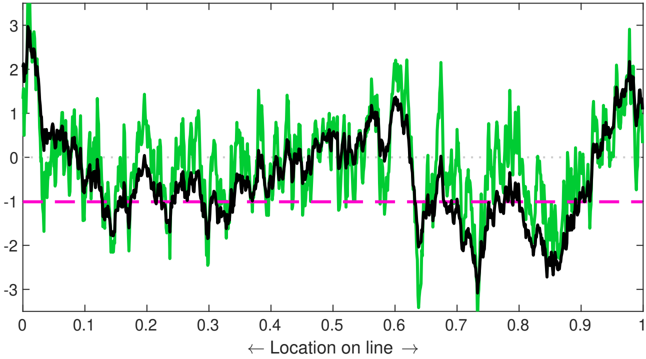
E.g. $\zeta = 14$, $\zeta = 0$ and $\zeta = 100$:



General modelling approach I

1. Define the geometry of the relevant space: plane, circle, torus, network, etc.
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2. Define the model: objectives, markets, frictions and spatial shock.
   - There will be conditions at each location x giving decisions and state evolution.
   - There will be some aggregate / market conditions.
   - Example spatial stochastic process:

$$a_{x,t} = \rho a_{x,t-1} + \sigma \varepsilon_{x,t}$$
$$\operatorname{cov}\left(\varepsilon_{x,t}, \varepsilon_{\tilde{x},t}\right) = s\left(\zeta, d\left(x, \tilde{x}\right)\right)$$

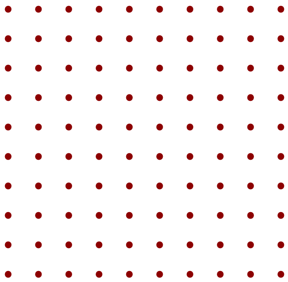
   - ζ will control spatial correlation.
   - s and d must fulfil certain conditions to produce a valid process. See paper.
   - Often $s(\zeta, d) = \exp\left(-\zeta d\right)$.
   - Note, with $a_t \equiv \int_0^1 a_{x,t}\, \mathrm{d}x$ and $\varepsilon_t \equiv \int_0^1 \varepsilon_{x,t}\, \mathrm{d}x$, we have:

$$a_t = \rho a_{t-1} + \sigma \varepsilon_t$$

General modelling approach II

3. Choose grid geometry, e.g. $N = 100$ evenly spaced points.
   - Note on accuracy:
     - Bounded variation of shock implies error from using the trapezium rule decays at $1/N^2$ at the slowest.
     - For sufficiently smooth functions the rate is k^{-N} for some $k > 1$ (thanks to periodicity).
     - Compare with $1/\sqrt{N}$, with Monte Carlo used in e.g. Krusell-Smith.

- Approximate outcomes at locations between nodes via linear interpolation.
- Approximate integrals over space via the trapezium rule.
- E.g. market clearing conditions of the form: $0 = \int_0^1 B_{x,t}\,\mathrm{d}x$ become $0 = \sum_{n=1}^{N} B_{x_n,t}$.



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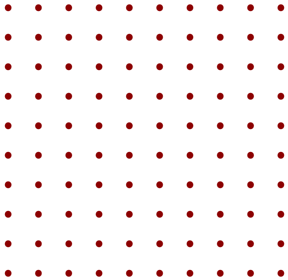


4. Solve via perturbation. e.g., with Dynare.
   - We provide a toolkit to help define spatially correlated shocks: https://github.com/tholden/DynareTransformationEngine

Model Overview

- RBC model + standard new economic geography features (following Krugman 1991).
- Key features:
  - Population movement.
  - Competing land usage: farming and residential.
  - Non-tradeable raw goods (production services).
  - Two types of final good: agricultural products and manufactured products.
  - Differentiated intermediate manufactured goods subject to iceberg trade costs.
  - Firm entry à la Bilbiie, Ghironi & Melitz (2012).

Agglomeration mechanism

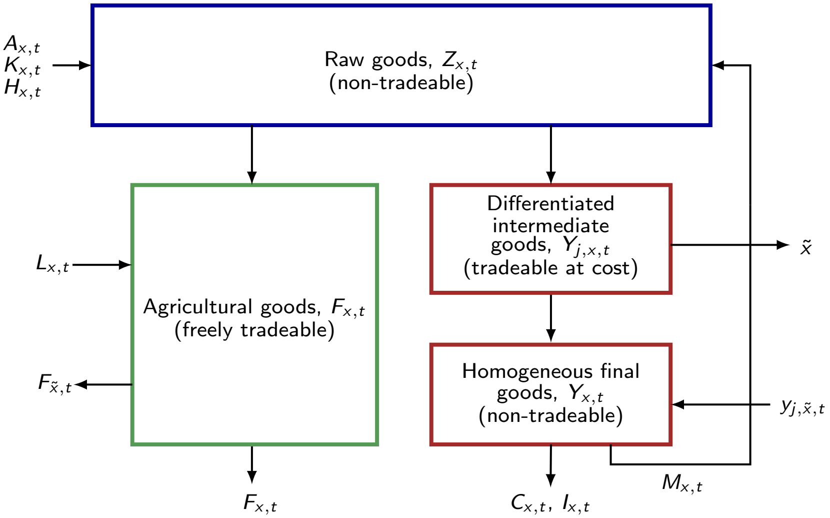
The key mechanism works as follows:

- Productivity shock at x increases wages there.
- People move to x for higher wages.
- The increased demand leads to firm entry.
- More products on sale implies increased productivity due to the taste for variety.
- This feeds back to higher wages, more migration, more firm entry etc.
- Nearby locations also benefit as iceberg transport costs mean the increased demand from x is concentrated in its neighbourhood.

Households and firms over space

- Set of points in space $x \in X$, normalised so $\int_X \mathrm{d}x = 1$.
- Distance between any $x, \tilde{x} \in X$ given by $d(x, \tilde{x})$.
- Firms, capital and population have a density over space.
- Land is uniform over space.
- Representative household at each location, each household is part of a representative family.
- Representative family maximises a utilitarian social welfare function.
  - Equivalent to complete markets.

Productive sectors at $x \in X$



Production at $x \in X$

Skip details

- Raw good: $Z_{x,t} = \left[K_{x,t-1}^{\alpha} \left(A_{x,t} H_{x,t} \right)^{1-\alpha} \right]^{1-\kappa} M_{x,t}^{\kappa}$
  - Non-tradeable.
  - Sold at $\mathcal{P}_{x,t}$.

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- Manufactured good: $Y_{j,x,t} = Z_{j,x,t}$.
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Capital holding companies at $x \in X$

- Capital law of motion:

$$K_{x,t} = (1 - \delta_K) K_{x,t-1} + \left[1 - \Phi\left(\frac{I_{x,t}}{I_{x,t-1}} \right) I_{x,t} \right]$$

- Capital rented out at $\mathcal{R}_{K,x,t}$ per unit to firms at x.
- Standard CEE assumptions about investment adjustment costs, $\Phi(\cdot)$.
- Location specific capital stocks and adjustment costs make it particularly hard to move capital between locations.

Households and the representative family

Family head maximizes:

▸ Utility

$$\mathbb{E}_t \sum_{s=0}^{\infty} \left[\prod_{k=1}^{s} \beta_{t+k-1} \right] \int_X N_{x,t+s-1} \frac{U_{x,t+s}^{1-\varsigma}}{1-\varsigma} \, \mathrm{d}x,$$

s.t.

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$$\mathbb{E}_t \sum_{s=0}^{\infty} \left[\prod_{k=1}^{s} \beta_{t+k-1} \right] \int_X N_{x,t+s-1} \frac{U_{x,t+s}^{1-\varsigma}}{1-\varsigma} \, \mathrm{d}x,$$

s.t.

$$\int \left(P_{x,t} C_{x,t} + E_{x,t} \right) \mathrm{d}x + B_t = \int \left(\mathcal{R}_{L,x,t} L_{x,t} + W_{x,t} H_{x,t} \right) \mathrm{d}x + R_{t-1} B_{t-1} + T_t$$

where:

$$U_{x,t} = u \left(\frac{C_{x,t}}{N_{x,t-1}}, \frac{E_{x,t}}{N_{x,t-1}}, \frac{1-L_{x,t}}{N_{x,t-1}}, \frac{H_{x,t}}{N_{x,t-1}}, \frac{N_{x,t-1}}{N_{t-1}}, g \left(N_{x,t-1}, \left\{ \mathcal{N}_{x,\tilde{x},t}, N_{\tilde{x},t-1} \right\}_{\tilde{x} \in X} \right) \right)$$

$$N_{t-1} \equiv \int_X N_{\tilde{x},t-1} \, \mathrm{d}\tilde{x},$$

$$N_{x,t} = G_{N,t} N_{x,t-1} - \int_X \mathcal{N}_{x,\tilde{x},t} \, \mathrm{d}\tilde{x} + \int_X \mathcal{N}_{\tilde{x},x,t} \, \mathrm{d}\tilde{x}.$$

First-order conditions

FOCs in full

- Single saving decision:

$$1 = \beta_t \mathbb{E}_t \left[\frac{\mu_{E,x,t+1}}{\mu_{E,x,t}} \right] R_t \tag{1}$$

- where

$$\mu_{E,x,t} = \theta_F \frac{N_{x,t-1} U_{x,t}^{1-\varsigma}}{E_{x,t}} = \theta_F \frac{N_{\tilde{x},t-1} U_{\tilde{x},t}^{1-\varsigma}}{E_{\tilde{x},t}} \forall \tilde{x} \tag{2}$$

First-order conditions

FOCs in full

- Single saving decision:

$$1 = \beta_t \mathbb{E}_t \left[\frac{\mu_{E,x,t+1}}{\mu_{E,x,t}} \right] R_t \tag{1}$$

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- Consumption, land use and labour supply:

$$\frac{E_{x,t}}{N_{x,t-1}} \propto \frac{P_{x,t} C_{x,t}}{N_{x,t-1}} \tag{3}$$

First-order conditions

FOCs in full

- Single saving decision:

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First-order conditions

FOCs in full

- Single saving decision:

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First-order conditions

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- Migration ($x \longrightarrow \tilde{x}$):

$$\mathcal{N}_{x,\tilde{x},t} = \mathbb{E}_t n \left(\left\{ \frac{H_{x,s+1}}{N_{x,s}}, U_{x,s+1}, \mathcal{N}_{x,\tilde{x},s}, N_{x,s} \right\}_{s \in (t,\infty), x, \tilde{x} \in X} \right) \tag{4}$$

First-order conditions

FOCs in full

- Single saving decision:

$$1 = \beta_t \mathbb{E}_t \left[\frac{\mu_{E,x,t+1}}{\mu_{E,x,t}} \right] R_t \tag{1}$$

- where

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First-order conditions

▸ FOCs in full

- Single saving decision:

$$1 = \beta_t \mathbb{E}_t \left[\frac{\mu_{E,x,t+1}}{\mu_{E,x,t}} \right] R_t \tag{1}$$

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Market clearing

$$B_t = 0$$

$$Y_{x,t} = C_{x,t} + I_{x,t} + M_{x,t}$$

$$Z_{x,t} = Z_{F,x,t} + \phi_t \left[J_{x,t} - (1-\delta_J) J_{x,t-1} \right] + \int_0^{J_{x,t}} Z_{j,x,t} \, \mathrm{d}j$$

$$\int E_{x,t} \, \mathrm{d}x = \int F_{x,t} \, \mathrm{d}x$$

Stochastic processes

Technology:

$$A_{x,t} = A_t^P A_{x,t}^T$$
$$A_t^P = G_{A,t} A_{t-1}^P$$
$$\log G_{A,t} = \left(1 - \rho_{G_A}\right) \log G_A + \rho_{G_A} \log G_{A,t-1} + \sigma_{G_A} \varepsilon_{G_A,t}$$
$$\log A_{x,t}^T = \rho_A \log A_{x,t-1}^T + \sigma_A \varepsilon_{A,x,t}$$

Similar AR(1) processes for $G_{N,t}$, τ_t, ϕ_t and β_t.

Stochastic processes

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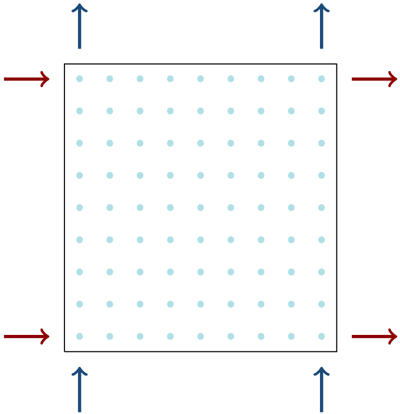
Similar AR(1) processes for $G_{N,t}$, τ_t, ϕ_t and β_t.

Choice of space

- We choose a torus since this has a uniform steady state:



- Distance metric and continuous stochastic process:

ACF

$$d\left([x_1, x_2], [\tilde{x}_1, \tilde{x}_2]\right) = \sqrt{\left(\min\left\{|x_1 - \tilde{x}_1|, 1 - |x_1 - \tilde{x}_1|\right\}\right)^2 + \left(\min\left\{|x_2 - \tilde{x}_2|, 1 - |x_2 - \tilde{x}_2|\right\}\right)^2}$$

$$\operatorname{cov}\left(\varepsilon_{A,x,t}, \varepsilon_{A,\tilde{x},t}\right) = s\left(\zeta, d\left(x, \tilde{x}\right)\right)$$

Estimating the spatial shock

We estimate the spatial process using state-level average wage data via a feasible generalized least squares approach.

- Writing the model:

$$\log w_{i,t} = \alpha_t + \beta_i + \gamma_i t + \sigma_i \varepsilon_{i,t} \tag{5}$$

  where

$$\operatorname{cov}\left(\varepsilon_{i,t}, \varepsilon_{j,s}\right) = \rho^{|t-s|} \exp\left[-\zeta d_{i,j}\right] \tag{6}$$

  we estimate ζ and ρ by maximum likelihood.
- On each iteration, α_t, β_i, γ_i and σ_i are estimated by feasible GLS.

▸ Further detail

This gives $\zeta = 14.2$, $\rho_A = 0.76$.

- 95% confidence interval for ζ: [11.8, 17.1]

Spatial correlation

We can compute residuals, $r_{i,t} = \log w_{i,t} - \alpha_t - \beta_i - \gamma_i t$, and remove correlation across time via Cholesky transformation

- the resulting residuals only have the spatial correlation so are our spatial shocks
- We can compute the correlation between the shocks of each state-wise pair, and plot against the distance.

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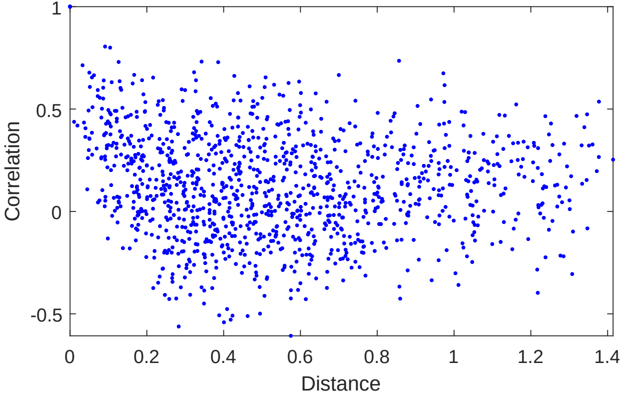
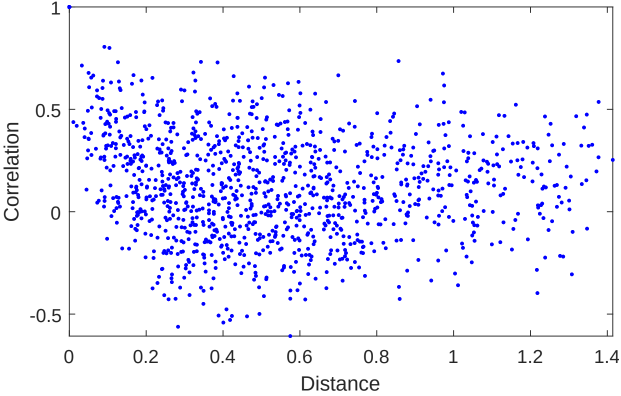
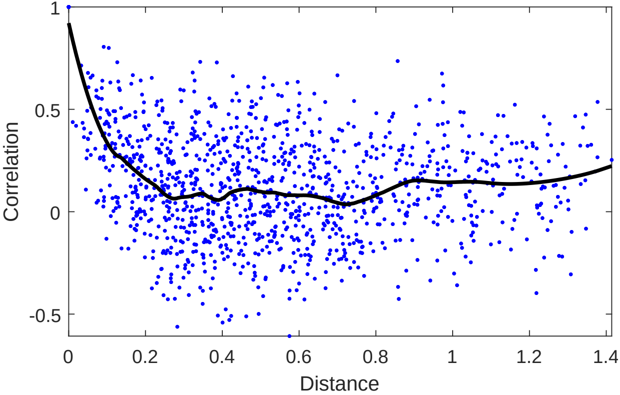


Spatial correlation



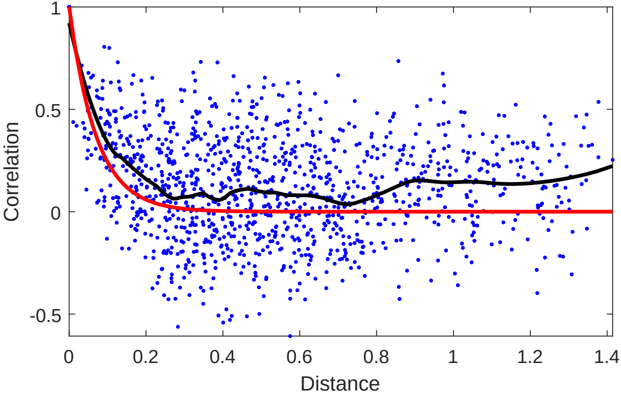
- Blue dots are the correlation between the shocks of each state-wise pair against distance

Spatial correlation



- Blue dots are the correlation between the shocks of each state-wise pair against distance
- Black line is a local quadratic (LOESS) trend

Spatial correlation



- Blue dots are the correlation between the shocks of each state-wise pair against distance
- Black line is a local quadratic (LOESS) trend
- Red line is $\exp(-\zeta d)$

Selected calibration overview

We calibrate parameters in the utility function to target:

- Land use: agriculture vs. residential
- Consumption bundle: ratio food vs non-food consumption
- Average hours worked and labour supply elasticity

Utility calibration targets related to migration and population:

- Distribution of population density
- Proportion of households moving each quarter $\mathcal{N}_x / N_x$
- Typical distance moved
- Population growth volatility

Standard parameters target familiar steady state ratios.

Specifics

Numerical Simulations

- Use a 9×9 square grid.
- IRF simulations:
  - 1% spatial productivity shock.
  - Focus on shock centred on the point $\left(\frac{1}{2}, \frac{1}{2}\right)$.
- 10,000 year stochastic simulation.

IRF to spatial productivity shock – in space I

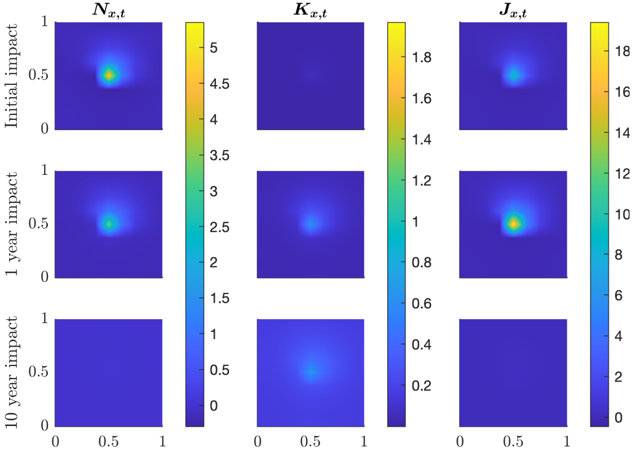


Figure: Impulse responses of key state variables to a 1% spatial productivity shock centred on $\left(\frac{1}{2}, \frac{1}{2}\right)$. Entire space, snapshots in time. Bright colours are high values.

IRF to spatial productivity shock – in time I

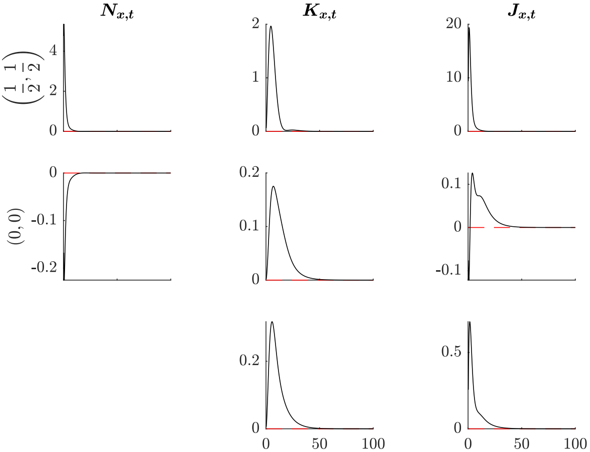


Figure: Impulse responses of key state variables to a 1% spatial productivity shock centred on $\left(\frac{1}{2},\frac{1}{2}\right)$. Percent deviation from steady state. x-axis measured in years.

IRF to spatial productivity shock – in space II

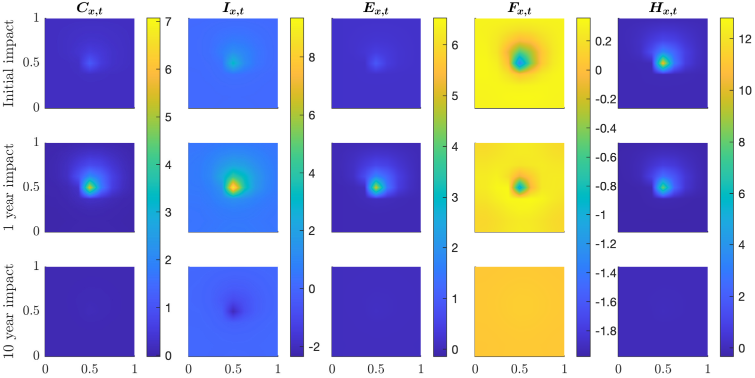


Figure: Impulse responses of key state variables to a 1% spatial productivity shock centred on $\left(\frac{1}{2}, \frac{1}{2}\right)$. Entire space, snapshots in time. Bright colours are high values.

IRF to spatial productivity shock – in time II

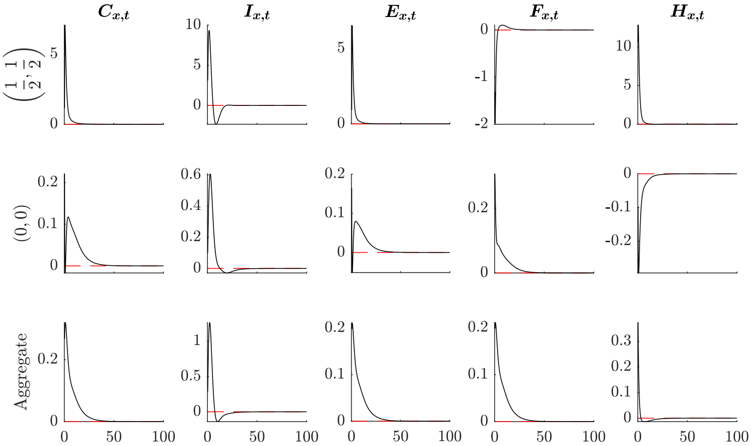


Figure: Impulse responses of key state variables to a 1% spatial productivity shock centred on $\left(\frac{1}{2}, \frac{1}{2}\right)$. Percent deviation from steady state. x-axis measured in years.

Next steps 1/2

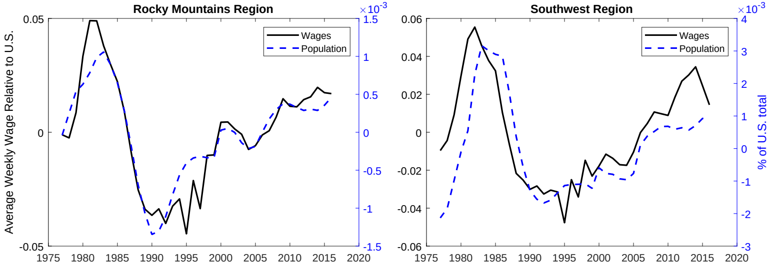
1. Currently doing some work to estimate causal linkages between wages–population
   - VAR with U.S. county-level wages, population and instrument for population (total population divided by distance to coast – justification: immigration traditionally gone to coasts.)

More info

2. Use our model to identify why the wage-population correlation breaks down.

Next steps 2/2

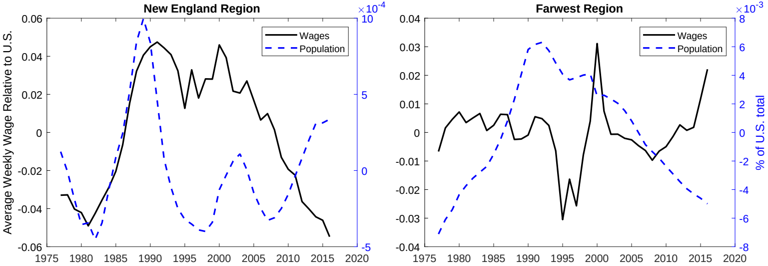
- E.g., consider some U.S. regions:



Note: plots show deviation of relative wage and population around linear trends

Next steps 2/2

- E.g., consider some U.S. regions:
  ... and some other regions:



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Next steps 2/2

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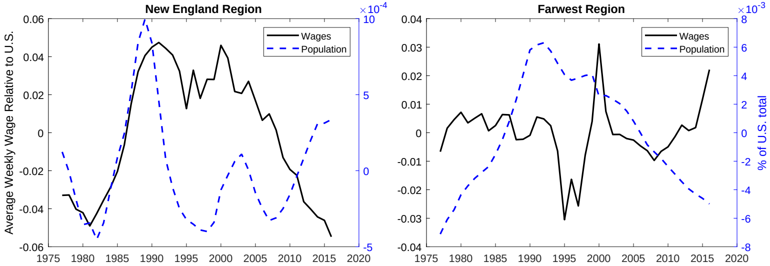


Note: plots show deviation of relative wage and population around linear trends

- We plan to use the model to identify the source of the difference.
  - Land use regulation? Technology changes?
  - Any policy implications?

Conclusions

We've seen the two main contributions of the current version of the paper:

1. A new approach to building macro models with continuous-in-space heterogeneity.
   - Wide range of possible applications (not just physical space).
2. An application – a DSGE model of economic geography.
   - Including firm entry and strong agglomeration forces.
   - Model generates persistent movements in population and capital.
   - Can model the rise and decline of regions

I presented some future plans, in particular:

- Identifying causal linkages channels between wages and population
- Estimating the model to identify causes of observed wage and population dynamics

Comments appreciated!

References I

Acemoglu, D., Carvalho, V. M., Ozdaglar, A. & Tahbaz-salehi, A. (2012), 'The Network Origins of Aggregate Fluctuations', *Econometrica* **80**(5), 1977–2016.

Bilbiie, F. O., Ghironi, F. & Melitz, M. J. (2012), 'Endogenous Entry, Product Variety, and Business Cycles', *Journal of Political Economy* **120**(2), 304–345.

Boucekkine, R., Camacho, C. & Zou, B. (2009), 'Bridging The Gap Between Growth Theory And The New Economic Geography: The Spatial Ramsey Model', *Macroeconomic Dynamics* **13**(01), 20–45.

Brito, P. (2004), The Dynamics of Growth and Distribution in a Spatially Heterogeneous World, Working Papers Department of Economics 2004/14, ISEG - Lisbon School of Economics and Management, Department of Economics, Universidade de Lisboa.

Cardamone, P. (2017), 'A Spatial Analysis of the R&D-Productivity Nexus at Firm Level', *Growth and Change* **48**(3), 313–335.

Caselli, F. & Coleman II, W. J. (2001), 'The u.s. structural transformation and regional convergence: A reinterpretation', *Journal of Political Economy* **109**(3), 584–616.

References II

Comin, D., Dmitriev, M. & Rossi-Hansberg, E. (2012), The Spatial Diffusion of Technology, No. 18534, NBER Working Paper Series, Cambridge, MA.

Desmet, K., Nagy, D. & Rossi-Hansberg, E. (2015), 'The geography of development: Evaluating migration restrictions and coastal flooding'.

Desmet, K. & Rossi-Hansberg, E. (2014), 'Spatial development', **104**(4), 1211–43.

Duranton, G. (2007), 'Urban Evolutions: The Fast, the Slow, and the Still', *American Economic Review* **97**(1), 197–221.

Eckert, F. & Peters, M. (2017), 'Spatial structural change and agricultural productivity'.

Gabaix, X. (2011), 'The Granular Origins of Aggregate Fluctuations', *Econometrica* **79**(3), 733–772.

Glass, A., Kenjegalieva, K. & Paez-Farrell, J. (2013), 'Productivity growth decomposition using a spatial autoregressive frontier model', *Economics Letters* **119**, 291–295.

Griffith, R., Redding, S. & Simpson, H. (2009), 'Technological catch-up and geographic proximity', *Journal of Regional Science* **49**(4), 689–720.

References III

Krugman, P. (1991), 'Increasing Returns and Economic Geography', *Journal of Political Economy* **99**(3), 483–499.

Krugman, P. (1998), 'What's new about the new economic geography?', *Oxford Review of Economic Policy* **14**(2), 7–17.

Michaels, G., Rauch, F. & Redding, S. J. (2012), 'Urbanization and Structural Transformation', *The Quarterly Journal of Economics* **127**(2), 535–586.

Nagy, D. (2016), 'City location and economic development'.

Quah, D. (2002), 'Spatial Agglomeration Dynamics', *American Economic Review* **92**(2), 247–252.

Redding, S. J. (2013), Economic Geography: A Review of the Theoretical and Empirical Literature, *in* R. E. Falvey, D. Greenaway, U. Kreickemeier & D. Bernhofen, eds, 'Palgrave handbook of international trade', 1 edn, Palgrave Macmillan UK, chapter 16, pp. 497–531.

Rossi-Hansberg, E. & Wright, M. L. J. (2007), 'Urban Structure and Growth', *Review of Economic Studies* **74**(2), 597–624.

Appendices

Wage growth distribution

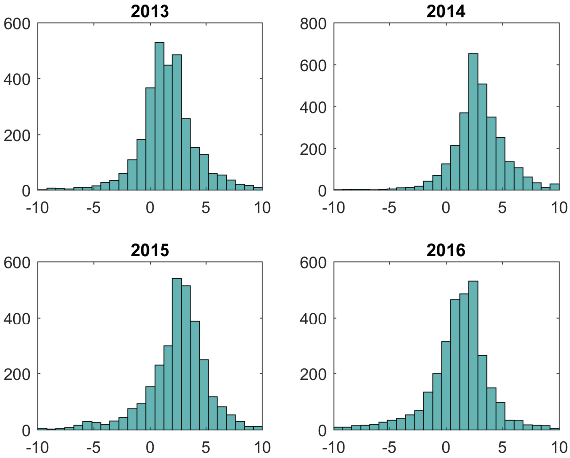


Figure: US county-level average weekly wage oty % change (BLS)

Return

Wage growth dispersion

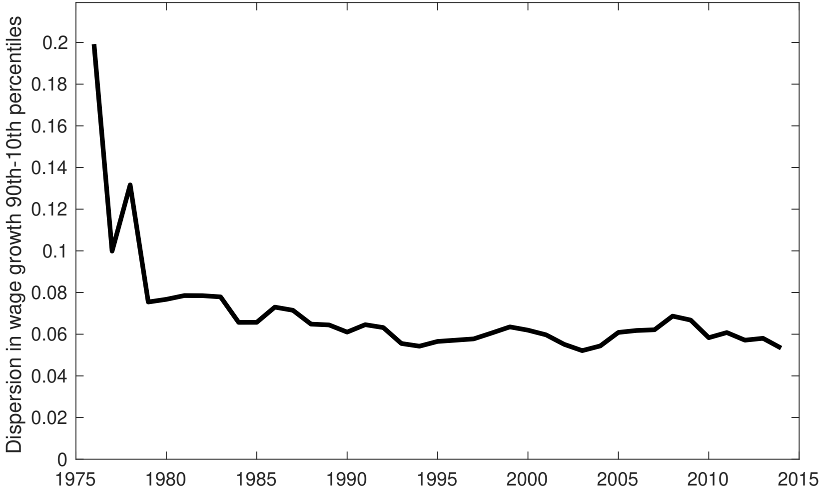


Figure: US county-level wage growth dispersion: 90th-10th percentiles (BLS, unweighted)

Return

Wage growth by rural-urban type

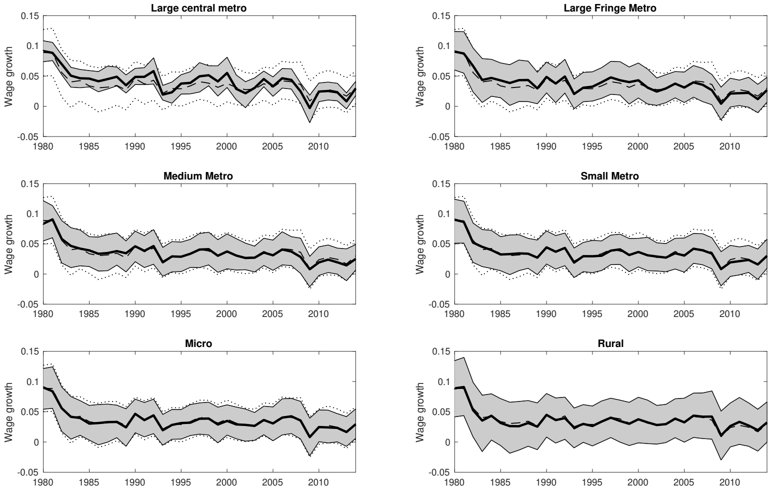
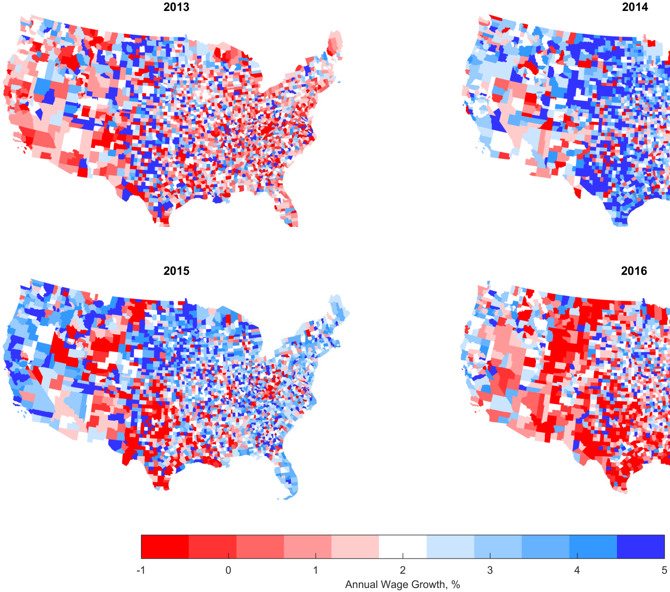


Figure: US county-level wage growth dispersion: 90th-10th percentiles (BLS, unweighted) (dashed=all)

Return

US county-level average weekly wage oty % change (BLS)



Return

New economic geography

Starts with Krugman (1991). See Krugman (1998) and Redding (2013) for reviews.

Branches of the existing literature:

- Stochastic, forward-looking, but few locations, e.g., two-bloc model:
  - E.g., Caselli & Coleman II (2001).
- Stationary equilibria, or purely backward looking decisions, with discreet space:
  - E.g., Michaels, Rauch & Redding (2012), Nagy (2016) and Eckert & Peters (2017).
- Continuous space, dynamic but backward-looking:
  - E.g., Desmet & Rossi-Hansberg (2014) and Desmet, Nagy & Rossi-Hansberg (2015).
- Some dynamic stochastic models in continuous space but with restrictive assumptions:
  - E.g., Quah (2002), Brito (2004), Duranton (2007), Rossi-Hansberg & Wright (2007) and Boucekkine, Camacho & Zou (2009).

Return

Household utility

$$U_{x,t} = \left(\frac{C_{x,t}}{N_{x,t-1}}\right)^{\theta_C} \left(\frac{E_{x,t}}{N_{x,t-1}}\right)^{\theta_F} \left(\frac{1 - L_{x,t}}{N_{x,t-1}}\right)^{\theta_L}$$

$$\left(\frac{1}{1+\nu} - \frac{1}{1+\nu}\left(\frac{H_{x,t}}{N_{x,t-1}}\right)^{1+\nu}\right)^{\theta_H} \left(\frac{1}{2}\Omega^2 - \frac{1}{2}\left(\log\left(\frac{N_{x,t-1}}{N_{t-1}}\right)\right)^2\right)^{\theta_N}$$

$$\left(1 - \frac{\mathcal{N}_{x,t}}{N_{x,t-1}}\right)^{\psi_1} \left(\bar{d} - \frac{\mathcal{D}_{x,t}}{\mathcal{N}_{x,t}}\right)^{\psi_2} \exp\left[\psi_3 \int_X \frac{N_{\tilde{x},t-1}}{N_{t-1}} \log\left(\frac{\mathcal{N}_{x,\tilde{x},t}}{N_{x,t-1}}\right) \mathrm{d}\tilde{x}\right]$$

where

$$\mathcal{D}_{x,t} \equiv \int_X d\left(x, \tilde{x}\right) \mathcal{N}_{x,\tilde{x},t} \, \mathrm{d}\tilde{x}$$

$$\mathcal{N}_{x,t} \equiv \int_X \mathcal{N}_{x,\tilde{x},t} \, \mathrm{d}\tilde{x}$$

$$1 = \theta_C + \theta_F + \theta_L + \theta_H + \theta_N + \psi_1 + \psi_2 + \psi_3$$

Return

Household first order conditions I

- Euler equation:

$$1 = \mathbb{E}_t \left[\Xi_{t,t+1} R_t \right]$$

- where:

$$\Xi_{t,t+1} \equiv \beta_t \frac{N_{x,t} E_{x,t} U_{x,t+1}^{1-\varsigma}}{N_{x,t-1} E_{x,t+1} U_{x,t}^{1-\varsigma}}$$

- and:

$$\frac{E_{x,t}}{N_{x,t-1} U_{x,t}^{1-\varsigma}} = \frac{E_{\tilde{x},t}}{N_{\tilde{x},t-1} U_{\tilde{x},t}^{1-\varsigma}}$$

Return

Household first order conditions II

- Consumption:

$$\theta_C E_{x,t} = \theta_F P_{x,t} C_{x,t}$$

- Land:

$$\theta_L E_{x,t} = \theta_F \mathcal{R}_{L,x,t} \left(1 - L_{x,t}\right)$$

- Labour:

$$\theta_H \left(\frac{H_{x,t}}{N_{x,t-1}} \right)^{\nu} = \theta_F \frac{N_{x,t-1}}{E_{x,t}} W_{x,t} \left(\frac{1}{1+\nu} \Gamma^{1+\nu} - \frac{1}{1+\nu} \left(\frac{H_{x,t}}{N_{x,t-1}} \right)^{1+\nu} \right)$$

Return

Household first order conditions III

- Population:

$$\mu_{N,x,t} = \beta_t \mathbb{E}_t \left[\begin{array}{c} \mu_{N,x,t+1} G_{N,t+1} + U_{x,t+1}^{1-\varsigma} \\ + (1-\varsigma)\, U_{x,t+1}^{1-\varsigma} \left[\begin{array}{c} \theta_H \dfrac{\left(\dfrac{H_{x,t+1}}{N_{x,t}}\right)^{1+\nu}}{\frac{1}{1+\nu}\Gamma^{1+\nu} - \frac{1}{1+\nu}\left(\dfrac{H_{x,t+1}}{N_{x,t}}\right)^{1+\nu}} - \theta_N \dfrac{\log\left(\dfrac{N_{x,t}}{N_t}\right)}{\frac{1}{2}\Omega^2 - \frac{1}{2}\left(\log\left(\dfrac{N_{x,t}}{N_t}\right)\right)^2} \\ + \psi_1 \dfrac{\mathcal{N}_{x,t+1}}{N_{x,t} - \mathcal{N}_{x,t+1}} - (\theta_C + \theta_F + \theta_L + \psi_3) \end{array} \right] \end{array} \right]$$

- Migration:

$$\mu_{N,x,t} = \mu_{N,\tilde{x},t} + (1-\varsigma)\, N_{x,t-1} U_{x,t}^{1-\varsigma} \left[\psi_3 \frac{N_{\tilde{x},t-1}}{N_{t-1}\mathcal{N}_{x,\tilde{x},t}} - \psi_1 \frac{1}{N_{x,t-1} - \mathcal{N}_{x,t}} - \psi_2 \frac{d(x,\tilde{x})\,\mathcal{N}_{x,t} - \mathcal{D}_{x,t}}{\bar{d}\mathcal{N}_{x,t}^2 - \mathcal{N}_{x,t}\mathcal{D}_{x,t}} \right]$$

Return

Auto-covariance function

We recommend (and use) the following auto-covariance function on a circle (identified with $[0,1]$) or torus (identified with $[0,1] \times [0,1]$):

$$\operatorname{cov}\left(\varepsilon_x, \varepsilon_{\tilde{x}}\right) = s\left(\zeta, d\left(x, \tilde{x}\right)\right)$$

where:

$$s\left(\zeta, d\right) = \frac{\exp\left(-\zeta d + \zeta \bar{d}\right) + \exp\left(\zeta d - \zeta \bar{d}\right)}{\exp\left(\zeta \bar{d}\right) + \exp\left(-\zeta \bar{d}\right)}$$

and:

$$\bar{d} \equiv \sup_{x, \tilde{x} \in X} d\left(x, \tilde{x}\right)$$

is the maximum distance between points.

▸ Return

Estimating the spatial shock I

Model:

$$\log w_{i,t} = \alpha_t + \beta_i + \gamma_i t + \sigma_i \varepsilon_{i,t} \tag{7}$$

$$\text{cov}\left(\varepsilon_{i,t}, \varepsilon_{j,s}\right) = \rho^{|t-s|} \exp\left[-\zeta d_{i,j}\right] \tag{8}$$

where $\varepsilon_{i,t}$ is normally distributed and $\mathbb{E}\varepsilon_{i,t} = 0$.
Solve ζ and ρ to maximize the log-likelihood, starting from initial guess of ζ, ρ and $\{\sigma_i\}_{i \in i}$, and for each iteration using:

1. Estimate α_t, β_i and γ_i by GLS
   - I.e., using $\hat{\Phi} = \left(X^{\mathrm{T}}\Omega^{-1}X\right)^{-1} X^{\mathrm{T}}\Omega^{-1}y$ where X is the matrix of indicator variables and the time variable, y the vector of wage observations, Φ is the vector of coefficients containing $\alpha_t, \beta_i, \gamma_i \forall t, i$, and Ω the covariance matrix of $r_{i,t} \equiv \sigma_i \varepsilon_{i,t}$.

Estimating the spatial shock II

2. Estimate σ_i, using $\alpha_t, \beta_i, \gamma_i$ and the current values of ρ and ζ via iterated-feasible GLS. Iterating over:

$$\sigma = \frac{1}{T}\mathrm{diag}\left[EP^{-1}E^{\mathrm{T}}\left(\mathrm{diag}\sigma\right)Z^{-1}\right] \tag{9}$$

where $E = \left[\varepsilon_{i,t}\right]_{\substack{t=1,\cdots,T\\ i=1,\cdots,N}}$, $P = \left[\rho_{A^{\mathrm{T}}}^{|t-s|}\right]_{\substack{t=1,\cdots,T\\ s=1,\cdots,T}}$ and $Z = \left[\exp\left[-\zeta d_{i,j}\right]\right]_{\substack{i=1,\cdots,N\\ j=1,\cdots,N}}$

3. Compute the log-likelihood

To compute the confidence interval, we invert the likelihood ratio test on the profile likelihood for ζ. I.e., holding all other parameters constant.

Return

Calibration detail I

Utility function:

$$
\begin{aligned}
U_{x,t} = & \left(\frac{C_{x,t}}{N_{x,t-1}}\right)^{\theta_C} \left(\frac{E_{x,t}}{N_{x,t-1}}\right)^{\theta_F} \left(\frac{1-L_{x,t}}{N_{x,t-1}}\right)^{\theta_L} \\
& \left(\frac{1}{1+\nu} - \frac{1}{1+\nu}\left(\frac{H_{x,t}}{N_{x,t-1}}\right)^{1+\nu}\right)^{\theta_H} \left(\frac{1}{2}\Omega^2 - \frac{1}{2}\left(\log\left(\frac{N_{x,t-1}}{N_{t-1}}\right)\right)^2\right)^{\theta_N} \\
& \left(1 - \frac{\mathcal{N}_{x,t}}{N_{x,t-1}}\right)^{\psi_1} \left(\bar{d} - \frac{\mathcal{D}_{x,t}}{\mathcal{N}_{x,t}}\right)^{\psi_2} \exp\left[\psi_3 \int_X \frac{N_{\tilde{x},t-1}}{N_{t-1}} \log\left(\frac{\mathcal{N}_{x,\tilde{x},t}}{N_{x,t-1}}\right) d\tilde{x}\right]
\end{aligned}
$$

- U.S. evidence suggests that the average home buyer stays in their house for around 13 years (NAHB/DHUD).
  - Calibrate ψ_1 to hit a proportion of $\frac{1}{12.5\times 4} = \frac{1}{50}$ household members moving each quarter.

- Proportion of households moving each quarter $\mathcal{N}_x / N_x$: choose ψ_2 so to target the distance of 40% of moves being >100 miles and 23% being >500 miles (study by Trulia)

Return

Calibration detail II

Utility function:

$$U_{x,t} = \left(\frac{C_{x,t}}{N_{x,t-1}}\right)^{\theta_C} \left(\frac{E_{x,t}}{N_{x,t-1}}\right)^{\theta_F} \left(\frac{1 - L_{x,t}}{N_{x,t-1}}\right)^{\theta_L}$$

$$\left(\frac{1}{1+\nu} - \frac{1}{1+\nu}\left(\frac{H_{x,t}}{N_{x,t-1}}\right)^{1+\nu}\right)^{\theta_H} \left(\frac{1}{2}\Omega^2 - \frac{1}{2}\left(\log\left(\frac{N_{x,t-1}}{N_{t-1}}\right)\right)^2\right)^{\theta_N}$$

$$\left(1 - \frac{\mathcal{N}_{x,t}}{N_{x,t-1}}\right)^{\psi_1} \left(\bar{d} - \frac{\mathcal{D}_{x,t}}{\mathcal{N}_{x,t}}\right)^{\psi_2} \exp\left[\psi_3 \int_X \frac{N_{\tilde{x},t-1}}{N_{t-1}} \log\left(\frac{\mathcal{N}_{x,\tilde{x},t}}{N_{x,t-1}}\right) d\tilde{x}\right]$$

- Spending on food in U.S. is around 20% of personal consumption expenditure excluding housing (BEA).
  - Set $\theta_F = \frac{1}{4}\theta_C$.

- U.S. population density is $41.5/\text{km}^2$, but ranges between $2.33/\text{km}^2$ for Wyoming and $470/\text{km}^2$ for New Jersey.
  - Correspond to absolute log ratios to the whole U.S. of 2.88 and 2.43 respectively.
  - Set $\Omega = 3$ to allow for such dispersion.

Return

Calibration detail III

Utility function:

$$U_{x,t} = \left(\frac{C_{x,t}}{N_{x,t-1}}\right)^{\theta_C} \left(\frac{E_{x,t}}{N_{x,t-1}}\right)^{\theta_F} \left(\frac{1-L_{x,t}}{N_{x,t-1}}\right)^{\theta_L}$$
$$\left(\frac{1}{1+\nu} - \frac{1}{1+\nu}\left(\frac{H_{x,t}}{N_{x,t-1}}\right)^{1+\nu}\right)^{\theta_H} \left(\frac{1}{2}\Omega^2 - \frac{1}{2}\left(\log\left(\frac{N_{x,t-1}}{N_{t-1}}\right)\right)^2\right)^{\theta_N}$$
$$\left(1 - \frac{\mathcal{N}_{x,t}}{N_{x,t-1}}\right)^{\psi_1} \left(\bar{d} - \frac{\mathcal{D}_{x,t}}{\mathcal{N}_{x,t}}\right)^{\psi_2} \exp\left[\psi_3 \int_X \frac{N_{\tilde{x},t-1}}{N_{t-1}} \log\left(\frac{\mathcal{N}_{x,\tilde{x},t}}{N_{x,t-1}}\right) d\tilde{x}\right]$$

- About 75% U.S. land is in broadly agricultural usage (USDA).
  - Set $\theta_L = \frac{1-0.75}{0.75}\gamma\theta_F$.
- Households on average 41% of waking time working
  - Calibrate θ_H to target steady state hours, H_x
- Calibrate ν to target a standard deviation of hours of 3.89%
- We set θ_N to generate a high degree of persistence in population movements, while ensuring stability of the symmetric steady-state.

Return

Calibration detail IV

All parameters:

$$\alpha = 0.3, \gamma = 0.5, \kappa = 0.5, \nu = 2, \varsigma = 1.5, \zeta = 8, \lambda = 0.1, \delta_j = 0.01, \delta_k = 0.03,$$
$$\Gamma = 1, \Omega = 3, \Phi^{''}(1) = 4,$$
$$\theta_C = \theta_H = 0.2618, \theta_F = \frac{\theta_C}{4}, \theta_L = 0.0109, \theta_N = 0.3338,$$
$$\psi_1 = \psi_2 = \frac{\theta_F}{2}, \psi_3 = 0.007,$$
$$G_A = 1.005, G_N = 1.0025, \tau = 1, \phi = 1, \beta = 0.99,$$
$$\rho_A = 0.9, \rho_{G_A} = 0.8, \rho_{G_N} = 0.5, \rho_\tau = 0.95, \rho_\phi = 0.95, \rho_\beta = 0.95,$$
$$\sigma_A = \sigma_{G_A} = \sigma_{G_N} = \sigma_\tau = \sigma_\phi = \sigma_\beta = 0.001.$$

Return

VAR

1. Instrument for county level population as follows: county by county, total US population (log level) divided by measure of the distance of the county to the nearest coast. Justification of the instrument: immigration has traditionally gone to coasts.
2. County by county, linearly detrend: a) the instrument, b) county level (log level) population and c) county level (log level) wages.
3. For each time period, subtract the mean across counties of each of the three variables (detrended population, detrended wages, detrended instrument) from that variable. (So after this each variable has zero mean both across time and across counties.

VAR

4. Call demeaned detrended population x_t, demeaned detrended wages y_t and the demeaned detrended instrument z_t. In each county, we have the following three equations, where in all cases $\cdots$ covers a sum of lags of x_t, y_t and z_t:

$$x_t = a_{xy} y_t + \cdots + \epsilon_{x,t} \tag{10}$$

$$y_t = a_{yx} x_t + \cdots + \epsilon_{y,t} \tag{11}$$

$$z_t = \cdots + \beta \epsilon_{x,t} + \epsilon_{z,t} \tag{12}$$

   where the structural shocks are uncorrelated. Thus:

$$z_t - \beta x_t + \beta a_{xy} y_t = \cdots + \epsilon_{z,t} \tag{13}$$

   So:

$$\begin{bmatrix} 1 & -a_{xy} & 0 \\ a_{yx} & 1 & 0 \\ -\beta & \beta a_{xy} & 1 \end{bmatrix} \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} = \cdots + \begin{bmatrix} \epsilon_{x,t} \\ \epsilon_{y,t} \\ \epsilon_{z,t} \end{bmatrix} \tag{14}$$

5. Estimate this three variable VAR using data pooled across counties (i.e. assuming all parameters are identical in all counties).

Return