Empirical Paper

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Things to do:

1. Get access to data
2. Define outside option (and the market share)
3. Define distribution of consumer characteristics (P_{ns})

Evaluation of $G_j(\theta, s^n, P_{ns})$, aka the moments

From the Section 6 of BLP

1. Estimate the market shares implied by the model via simulation
2. Solve for the vector of demand unobservables (i.e. $\xi(\theta, s^n, P_{ns})$) implied by the simulated and observed market shares
3. Calculate the cost side unobservables $\omega(\theta, s^n, P_{ns})$, from the difference between price and the markups computed from the shares
4. Calculate the optimal instruments and interact them with the computed cost and demand side unobservables to produce $G_j(\theta, s^n, P_{ns})$

0.1 Estimating Market Shares

To obtain the market share, it is useful to proceed in two stages. First, we condition on consumer characteristics and integrate out over the extreme value deviates to obtain the conditional market shares:

$$f_j(\nu)i, \delta, p, x, \theta) = \frac{e^{\delta_j + \mu(x_j, p_j, \nu_i, \theta_2)}}{1 + \sum_{j=1}^{J} e^{\delta_j + \mu(x_j, p_j, \nu_i, \theta_2)}}$$

Then we can integrate this to get the market shares conditional only on product characteristics:

$$s_j(p,x,\xi,\theta,P_0) = \int f_j(\nu_i,\delta(x,p,\xi),p,x,\theta)P_0(d\nu) \approx \frac{1}{ns}\sum_{i=1}^{ns} f_j(\nu_i,\delta,p,x,\theta)$$

where $\nu_i, i \in \{1,...,ns\}$ are iid draws from P_0.

Integrating out ϵ analytically reduces the variance of the estimator and ensures that the simulated market shares are positive, sum to one, and are smooth functions of their arguments.

We can further reduce the variance by using importance sampling. We will not implement this on the first pass, but might if there is enough time.

0.2 Demand Unobservables

Recall that δ solves the nonlinear system $s^n = s(p,x,\delta,P_{ns},\theta)$, or:

$$\delta = \delta + \ln(s^n) - \ln(s(p,x,\delta,P_{ns};\theta)$$

which (with some proofs in the appendix) gives us a contraction mapping with modulus less than one. Thus, we can solve for δ recursively by iterating.

Given $\delta_j(\theta,s,P)$, we easily solve for the demand-side unobservable:

$$\xi_j(\theta,s,P) = \delta_j(\theta,s,P) - x_j\beta$$

0.3 Cost Unobservables

First, we solve for the markup using Nash pricing. This requires the derivatives of market share with respect to price:

$$\frac{\partial s_j(p,x,\xi,\theta,P_0)}{\partial p_j} = \int f_j(1-f_j)\frac{\partial \mu_{ij}}{\partial p_j}P_0(d\nu)$$

$$\frac{\partial s_j(p,x,\xi,\theta,P_0)}{\partial p_q} = \int -f_j f_q\frac{\partial \mu_{ij}}{\partial p_q}P_0(d\nu)$$

Any product produced by firm f, i.e. any $j \in \mathcal{J}_f$ must have a price p_j that satisfies the FOCs:

$$s_j(p, x, \xi; \theta) + \sum_{r \in \mathcal{J}_f} (p_r - mc_r) \frac{\partial s_r}{\partial p_j} = 0$$

To obtain the price-cost markups $(p_j - mc_j)$, define a $J \times J$ matrix Δ such that:

$$\Delta_{jr} = \begin{cases} -\frac{\partial s_r}{\partial p_j}, & \text{if } r \text{ and } j \text{ are produced by the same firm} \\ 0, & \text{otherwise} \end{cases}$$

Then prices are given by $p = mc + \Delta^{-1} s(p, x, \xi; \theta)$, and the cost unobservable is:

$$b(p, x, \xi; \theta) = \Delta^{-1} s(p, x, \xi; \theta)$$

0.4 Optimal instruments and interaction