

# CS2100

## Computer Organization

AY2022/23 Semester 1

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# Part I

## Data Representation

Data is represented as a sequence of **bits** — 0 or 1.

8 bits form a **byte**, and a fixed multiple of bytes in a computer architecture forms a **word**.

number of bits  $\iff$  number of values

$n$  bits can represent up to  $2^n$  values.

Conversely, we need at least  $\lceil \log_2 m \rceil$  bits to represent  $m$  values.

### 1 Number Systems

Number systems are **weighted-positional** systems, with the **decimal** number system having a **base** (or **radix**) of 10.

| number system | base | prefix |
|---------------|------|--------|
| binary        | 2    | 0b     |
| octal         | 8    | 0      |
| decimal       | 10   | —      |
| hexadecimal   | 16   | 0x     |

The digit to the left of the decimal point has a weight of  $10^0$ , with the exponent *incrementing* leftward and *decrementing* rightward.

decimal  $\implies$  binary

- For *whole numbers*, use **repeated division-by-two**:
- divide the value by 2, and record the remainder
  - prepend the remainder to the binary representation
  - repeat until the value is 0
- For *fractions*, use **repeated multiplication-by-two**:
- multiply the value by 2, and record the carry
  - append the carry to the binary representation
  - repeat until the value is 1
  - append the 1 to the binary representation

binary  $\implies$  decimal

- Use **weighted summation**:
- assign the digit to the left of the binary point a weight of  $2^0$
  - increment the weight exponents leftward
  - decrement the weight exponents rightward
  - multiply each digit by its weight
  - sum the products

These conversions generalize to any base!

Converting between 2 bases can be done with base-10 as an intermediary.

binary  $\iff$  base- $2^k$

- partition the binary representation into  $k$ -bit chunks outward from the binary point
  - convert each chunk to base-10
  - convert each base-10 value to base- $2^k$
- Apply the reverse direction for converting from base- $2^k$  to binary.

### 1.1 Negative Numbers

**Signed numbers** include all positive and negative values, unlike **unsigned numbers** which only include non-negative values.

We will consider  $n$ -bit values in this section.

sign-and-magnitude

- The MSB is the **sign bit** — 0 for positive, 1 for negative.
- Negate a value by inverting the sign bit.
- range**:  $-2^{n-1} + 1$  to  $2^{n-1} - 1$
  - zeros**:  $-0_{10}$  and  $+0_{10}$

1s complement

- An  $n$ -bit number  $x$  has a negated value  $-x = 2^n - x - 1$ .
- Negate a value by inverting *all the bits*.
- range**:  $-2^{n-1} + 1$  to  $2^{n-1} - 1$
  - zeros**:  $-0_{10}$  (all 1s) and  $+0_{10}$  (all 0s)
- The weight of the MSB is  $-2^{n-1} + 1$ .

2s complement

- An  $n$ -bit number  $x$  has a negated value  $-x = 2^n - x$ .
- Negate a value by inverting all the bits to the left of the *rightmost* 1.
- range**:  $-2^{n-1}$  to  $2^{n-1} - 1$
  - zeros**:  $0_{10}$  (all 0s)
- Unlike sign-and-magnitude and 1s complement, 2s complement does not have duplicate zeros.
- The weight of the MSB is  $-2^{n-1}$ .
- The MSB also represents the sign — 0 for positive, 1 for negative.

Negating a fractional number in a complement representation is no different than negating a whole number!

**excess representation**

Distribute positive and negative values by a simple addition or subtraction by  $k$ , which is also known as the **bias**.

$k$  is typically  $2^n - 1$  for an  $n$ -bit number.

The binary value in excess- $k$  *subtracts*  $k$  from its decimal value.

Conversely, the decimal value in excess- $k$  *adds*  $k$  to its binary value.

**1.2 Overflow**

**Overflow** occurs when addition or subtraction goes beyond the fixed range of a signed number.

Subtraction is equivalent to addition of the negated value.

Note that in 1s addition, the carry out of the MSB is *added* to the result, but this carry out is *discarded* in 2s addition.

**detecting overflow**

If the MSB (sign bit) of the result is different from the MSBs of the operands, then overflow has occurred.

Note that overflow can only occur when the operands have the same sign, i.e., adding a negative number to a positive number will *never* result in overflow.

**1.3 Real Numbers**

**Fixed-point representation** fixes the number of bits for the whole number and fractional parts, which limits the range of values.

The IEEE 754 **floating-point representation** resolves this by allocating a fixed number of bits to the **sign**, **exponent**, and **mantissa**.

| precision | bits | sign | exp. | mantissa | bias |
|-----------|------|------|------|----------|------|
| single    | 32   | 1    | 8    | 23       | 127  |
| double    | 64   | 1    | 11   | 52       | 1023 |

The **mantissa** is **normalized** with an implicit leading 1 bit, e.g.  $110.1_2$  is normalized to  $1.101_2 \times 2^2$ , and only 101 is stored in the mantissa.

**decimal  $\Rightarrow$  IEEE 754 floating-point representation**

1. convert the decimal value to normalized binary, i.e.,  $\pm 1.xxx \times 2^y$
2. determine the sign bit — 0 for positive, and 1 for negative
3. add the bias to the exponent  $y$  — +127 for single precision, and +1023 for double precision
4. convert the exponent to binary
5. concatenate the sign bit, exponent, and mantissa  $xxx$

**Part II****Instruction Set Architecture****2 ISA Overview**

An **instruction set architecture** (ISA) is an abstraction on the interface between hardware and low-level software, which runs on a **processor**.

Processors are connected to **memory** (RAM) via a **bus**, across which code and data is transferred.

ISAs have two major design philosophies:

1. **CISC** — Complex Instruction Set Computer:
  - single instructions for complex operations
  - smaller program sizes
  - complex implementation, little room for hardware optimization
2. **RISC** — Reduced Instruction Set Computer:
  - smaller and simpler instruction set
  - software combines simpler operations to implement high-level language statements
  - room for compiler optimization

**2.1 Data Storage Architecture**

1. **stack architecture**:  
operands are implicitly popped from the stack
2. **accumulator architecture**:  
one operand is implicitly stored in an accumulator
3. **general-purpose register architecture**:  
operands are stored explicitly in registers, operations are register-memory or register-register
4. **memory-memory architecture**:  
operands are read from memory

**2.2 Memory Architecture**

Memory transfer takes place across buses:

1. **address bus**:  $k$ -bits, uni-directional from processor to memory
2. **data bus**:  $n$ -bits, bi-directional
3. **control lines**: bi-directional, e.g. read/write controls

The address bus feeds addresses from the **memory address register** to the memory.

Data is written to or read from the **memory data register**, depending on the R/W control line.

**Endianness** is the relative ordering of the *bytes* in a word (*not* the bits in a byte!) in memory:

- **big-endian**: MSB stored in smallest address
- **little-endian**: LSB stored in smallest address

### 3 Instruction Set Encoding

Instructions consist of an **opcode** — a unique code to identify the operation — as well as **operands**.

In an ISA with **fixed-length instructions**, we need to fit multiple sets of instruction types each with the same number of bits.

We use the **expanding opcode** scheme in which the opcode has variable length for different instructions.

maximizing the total number of instructions

Maximize the number of instructions in each set, starting from the *largest* set to the smallest.

minimizing the total number of instructions

Maximize the number of instructions in each set, starting from the *smallest* set to the largest.

### Part III

## MIPS

### 4 Registers

Memory access is slow. To avoid frequent memory access, temporary values are stored in the processor in **registers**, which are limited in number.

Registers *do not have data types*, unlike program variables. Instructions always assume that the data stored in a register is of the correct type.

MIPS has **32 registers**, which are referred to by *number* or *name*:

| name        | #       | usage  |
|-------------|---------|--|
| \$zero      | 0       | constant value zero                          |
| \$at        | 1       | reserved for the assembler                   |
| \$v0 - \$v1 | 2 - 3   | values for results and expression evaluation |
| \$a0 - \$a3 | 4 - 7   | arguments                                    |
| \$t0 - \$t7 | 8 - 15  | temporaries                                  |
| \$s0 - \$s7 | 16 - 23 | program variables                            |
| \$t8 - \$t9 | 24 - 25 | temporaries                                  |
| \$k0 - \$k1 | 26 - 27 | reserved for the OS                          |
| \$gp        | 28      | global pointer                               |
| \$sp        | 29      | stack pointer                                |
| \$fp        | 30      | frame pointer                                |
| \$ra        | 31      | return address                               |

### 5 MIPS Assembly Language

The **general instruction syntax** is as follows:

| op \$s0, \$s1, \$s2 |                      |
|---------------------|----------------------|
|                     | description          |
| op                  | operation            |
| \$s0                | destination register |
| \$s1                | source register 1    |
| \$s2                | source register 2    |

Each instruction executes a *single* command. Each line of assembly code contains *at most* one instruction.

Almost all MIPS operations are *register-to-register*

The # hex symbol is used for comments.

#### 5.1 Arithmetic Instructions

| instruction          | format | opcode/funct        |
|----------------------|--------|---------------------|
| add \$rd, \$rs, \$rt | R      | 0/20 <sub>hex</sub> |
| sub \$rd, \$rs, \$rt | R      | 0/22 <sub>hex</sub> |
| addi \$rt, \$rs, imm | I      | 8 <sub>hex</sub>    |
| move \$s0, \$s1      | psuedo | —                   |

immediate subtraction

There is no subi operation as it is equivalent to addi with a negative constant.

assigning variables

The move instruction is a **psuedo-instruction** which is translated into its MIPS equivalent by the compiler:

```
add $s0, $s1, $zero

To assign a constant imm to a register:
addi $s0, $zero, imm
```

The immediate values in **immediate operations** such as addi range from  $-2^{15}$  to  $2^{15} - 1$ , as the 16-bit 2s complement system is used.

setting large constants

If 32-bit constants are required, use the lui operation to load the most-significant 16-bits first, followed by an ori operation to set the least-significant 16-bits:

```
lui $t0, 0xAAAA      # t0 = 0xAAAA0000
ori $t0, $t0, 0xF0F0  # t0 = 0xAAAAF0F0
```

Use the **temporary registers** \$t0 to \$t9 to store intermediate results in complex expressions.

## 5.2 Logical Instructions

| instruction           | format | opcode/funct        |
|-----------------------|--------|---------------------|
| sll \$rd, \$rt, shamt | R      | 0/00 <sub>hex</sub> |
| srl \$rd, \$rt, shamt | R      | 0/02 <sub>hex</sub> |
| and \$rd, \$rs, \$rt  | R      | 0/24 <sub>hex</sub> |
| or \$rd, \$rs, \$rt   | R      | 0/25 <sub>hex</sub> |
| xor \$rd, \$rs, \$rt  | R      | 0/26 <sub>hex</sub> |
| nor \$rd, \$rs, \$rt  | R      | 0/27 <sub>hex</sub> |
| andi \$rt, \$rs, imm  | I      | C <sub>hex</sub>    |
| ori \$rt, \$rs, imm   | I      | D <sub>hex</sub>    |
| xori \$rt, \$rs, imm  | I      | E <sub>hex</sub>    |
| lui \$rt, imm         | I      | F <sub>hex</sub>    |

Bitwise NOR sets the result to 1 if both bits are 0, and 0 otherwise.

Bitwise XOR sets the result to 1 if both bits are different, and 0 otherwise.

### bitwise NOT

There is no not operation as it is equivalent to either of the following:

- nor \$rd, \$rs, \$zero
- xor \$rd, \$rs, \$rt, where \$rt is all 1s

The absence of nori keeps the instruction set small.

In bitshift operations (sll and srl), the empty positions are filled with zeros, and the **shift amount** is limited to **5 bits**.

### multiplication and division

For multiplication/division by  $k$ , if  $k$  is a power of 2, use sll and srl respectively, setting the shamt to  $k$ .

Otherwise, use a loop.

## 5.3 Memory Instructions

Memory can be thought of as a *single-dimensional array* of memory location, with each having an **address**.

Memory addresses allow access to *bytes* of data, or **words** of data, which are usually  $2^n$  bytes — the common unit of transfer between the processor and memory.

**Word alignment** occurs in memory when words begin at a *byte address* which is a multiple of the word size —  $2^n$  bytes.

In MIPS, each word is 32 bits (4 bytes), and addresses are 32-bits long — such that  $2^{30}$  words are addressable, each of which differing by 4.

| instruction            | format | opcode/funct      |
|------------------------|--------|-------------------|
| lw \$rt, offset(\$rs)  | I      | 23 <sub>hex</sub> |
| sw \$rt, offset(\$rs)  | I      | 2B <sub>hex</sub> |
| lb \$rt, offset(\$rs)  | I      | 20 <sub>hex</sub> |
| sb \$rt, offset(\$rs)  | I      | 28 <sub>hex</sub> |
| ulw \$rt, offset(\$rs) | pseudo | —                 |
| usw \$rt, offset(\$rs) | pseudo | —                 |

For the word-aligned memory instructions lw and sw, the resulting address of  $\$rs + \text{offset}$  must be a multiple of 4.

The offset is a 16-bit 2s complement number.

## 5.4 Control Flow Instructions

| instruction           | format | opcode/funct        |
|-----------------------|--------|---------------------|
| beq \$rs, \$rt, label | I      | 4 <sub>hex</sub>    |
| bne \$rs, \$rt, label | I      | 5 <sub>hex</sub>    |
| j label               | J      | 2 <sub>hex</sub>    |
| slt \$rd, \$rs, \$rt  | R      | 0/2A <sub>hex</sub> |
| slti \$rt, \$rs, imm  | I      | A <sub>hex</sub>    |
| blt \$rs, \$rt, label | pseudo | —                   |
| bgt \$rs, \$rt, label | pseudo | —                   |
| ble \$rs, \$rt, label | pseudo | —                   |
| bge \$rs, \$rt, label | pseudo | —                   |

slt and slti sets the result to 1 if  $\$rs < \$rt$  or imm, and 0 otherwise.

beq and bne are **conditional jumps** — they jump to the label if the condition is true.

Labels are written as <label>: to the left of a statement, but they are *not* instructions.

A j instruction is equivalent to beq \$s0 \$s0, <label>.

The **program counter** (PC) typically stores the address of the *next* address to be executed, and has to be modified by the branching and jump instructions.

## 6 Instruction Encoding

Every MIPS instruction is **32 bits** in 3 possible formats:

| format | source registers | destination registers | immediate values |
|--------|------------------|-----------------------|------------------|
| R      | 2                | 1                     | 0                |
| I      | 1                | 1                     | 1                |
| J      | 0                | 0                     | 1                |

### 6.1 R-format

|      | opcode | rs | rt | rd | shamt | funct |
|------|--------|----|----|----|-------|-------|
| bits | 6      | 5  | 5  | 5  | 5     | 6     |

- opcode := 0 for all R-format instructions,
- funct determines the instruction,
- shamt := 0, rd := arith(rs, rt) for non-shift (arithmetic) instructions, and,
- rs := 0, rd := shift(rt, shamt) for shift instructions.

## 6.2 I-format

|      | opcode | rs | rt | immediate |
|------|--------|----|----|-----------|
| bits | 6      | 5  | 5  | 16        |

- opcode determines the instruction since there is no funct field,
- rt determines the *destination* register since there is no rd field,
- immediate is a *signed* integer in 2s complement except for bitwise operations where it is *unsigned*,
- rt := op(rs, immediate) in general for all non-branching instructions, and,
- PC := (PC + 4) + (immediate \* 4) when branching, otherwise PC += 4, which is the next instruction.

For branching instructions, immediate is the offset from the *next* instruction to the label of the *target* instruction.

PC is incremented in multiples of 4 due to word-alignment, which also means that we can now branch  $2^{15} \times 4 = 2^{17}$  bytes away from PC.

## 6.3 J-format

|      | opcode | target address |
|------|--------|----------------|
| bits | 6      | 26             |

The last 2 bits of every instruction are always 00 due to word-alignment, so we leave them out of the target address.

This leaves with an effective range of 28 bits for the target address, and the remaining 4 bits are derived from the most significant (leftmost) bits of PC + 4.

The **destination address** is therefore:

|      | (PC+4) [0:4] | target address | 00 |
|------|--------------|----------------|----|
| bits | 4            | 26             | 2  |

This creates a maximum jump range of 256 MB.

## 6.4 Addressing Modes

Addressing modes are used to calculate the address of an operand.

1. **register addressing** — add, xor, etc.:  
operand is a register
2. **immediate addresssing** — addi, andi, etc.:  
operand is a constant within the instruction

3. **base/displacement addressing** — lw, sw:  
operand is a memory location at the address the sum of a register and a constant in the instruction
4. **PC-relative addressing** — beq, bne:  
address is the sum of the PC and a constant in the instruction
5. **psuedo-direct addressing** — j:  
part of the instruction concantenated with part of the PC

## 7 Datapath

*A collection of components that process data, and perform arithmetic, logical, and memory operations.*

The **instruction execution cycle** has 5 stages:

1. **fetch**:  
get instruction from memory, address in PC
2. **decode and operand fetch**:  
determine the operation and get the operands needed
3. **execute (ALU)**:  
perform the computations to get a result
4. **execute (memory access)**:  
read from or write to memory if necessary
5. **write-back**:  
store the result of the operation

### 7.1 Fetch

There are 3 steps in the fetch stage:

1. Use PC to fetch the instruction from memory.
2. Increment PC by 4 to get the address of the next instruction.
3. Feed the output instruction to the decode stage.

The **instruction memory** element is a **sequential circuit** with an internal state which stores the instructions.

When supplied an instruction address  $m$ , it outputs the content at address  $m$ .

The **adder** element takes in the 32-bit PC and the 32-bit constant 4, and outputs the 32-bit sum PC + 4.

The **clock signal** is a *square wave* with *rising* and *falling* edges, and a period controlled by the CPU.

PC is read in the first half of the **clock period** and updated at the *next rising clock edge*.

### 7.2 Decode

There are 3 steps in the decode stage:

1. Read the opcode to determine the instruction type and field lengths.
2. Read data from all necessary registers.
3. Feed the operation and operands to the ALU stage.

The **register file** element is a collection of 32 registers which can be read from or written to.

| input           | bits | output      | bits |
|-----------------|------|-------------|------|
| read register 1 | 5    | read data 1 | 32   |
| read register 2 | 5    | read data 2 | 32   |
| write register  | 5    |             |      |
| write data      | 32   |             |      |

The **RegWrite** control signal determines whether the instruction should read from or write to the registers:

|       | RegWrite | registers per instruction |
|-------|----------|---------------------------|
| read  | 0        | $\leq 2$                  |
| write | 1        | $\leq 1$                  |

### 7.2.1 R-format Decoding

The binary content (Inst) of R-format instructions map to the inputs as follows:

- $rs \equiv \text{Inst}[25:21] \mapsto rr1$
- $rt \equiv \text{Inst}[20:16] \mapsto rr2$
- $rd \equiv \text{Inst}[15:11] \mapsto wr$

### 7.2.2 I-format Decoding

For I-format instructions, 2 problems arise:

1.  $rt$ , not part of  $imm$ , needs to be fed to  $wr$
2.  $imm$  needs to be fed to the ALU, not  $rd2$

A **multiplexer** chooses the correct Inst slice to feed to  $wr$  using the control signal **RegDst**:

|          | RegDst | input to wr                    |
|----------|--------|--------------------------------|
| I-format | 0      | $rt \equiv \text{Inst}[20:16]$ |
| R-format | 1      | $rd \equiv \text{Inst}[15:11]$ |

- $rs \equiv \text{Inst}[25:21] \mapsto rr1$
- $rt \equiv \text{Inst}[20:16] \mapsto rr2$
- $rt \equiv \text{Inst}[20:16] \mapsto wr$

Another multiplexer chooses the correct 32-bit binary data to feed into the ALU:

|                  | ALUSrc | input to ALU opr2   |
|------------------|--------|---|
| R-format or beq  | 0      | $rr2 \equiv *rt \equiv *\text{Inst}[20:16]$               |
| I-format not beq | 1      | $imm \equiv \text{Inst}[15:0]$ , sign extended to 32-bits |

## 7.3 Arithmetic Logic Unit (ALU)

The ALU performs the arithmetic, shifting, logical, memory, and branching operations.

It takes the operation and operands from the decode stage, performs its computation, and feeds the result to the memory stage.

| input     | bits | output   | bits |
|-----------|------|----------|------|
| operand 1 | 32   | is zero? | 1    |
| operand 2 | 32   | result   | 32   |

The **ALUControl** signal determines the operation to perform:

| ALUControl | operation |
|------------|-----------|
| 0000       | and       |
| 0001       | or        |
| 0010       | add       |
| 0110       | sub       |
| 0111       | slt       |
| 1100       | nor       |

### 7.3.1 Branching Instructions

A multiplexer chooses between the next instruction ( $PC + 4$ ) and the branch target (BT) using the control signal **PCSrc**:

| isZero           | PCSrc | next instruction             |
|------------------|-------|------------------------------|
| false $\equiv 0$ | 0     | $PC + 4$                     |
| true $\equiv 1$  | 1     | $BT \equiv PC + 4 + imm * 4$ |

Because both register contents need to be compared, the **ALUSrc** control value is set to 0 for branching instructions.

BT is computed as follows:

1. left shift by 2 bits the sign-extended  $imm$  from the decode stage (multiplying the offset by 4)
2. add it to  $PC + 4$  from the adder in the fetch stage

## 7.4 Memory

Only  $lw/lb$  and  $sw/sb$  instructions operate in this stage, the rest remain idle.

The **data memory** element exists in RAM:

| input           | bits | output          | bits |
|-----------------|------|-----------------|------|
| address (addr)  | 32   | read data (rdt) | 32   |
| write data (wd) | 32   |                 |      |

$addr$  is the effective memory address computed by the ALU, and  $wd$  is connected to  $rd2$  from the register file from the decode stage.

The control signals **MemRead** and **MemWrite** control the memory operations:

| MemRead | MemWrite | action                         |
|---------|----------|--------------------------------|
| 0       | 1        | write wd into addr             |
| 1       | 0        | read contents of addr into rdt |
| 0       | 0        | do nothing                     |
| 1       | 1        | undefined, should not occur    |



### 7.4.1 Non-memory Instructions

A multiplexer chooses between the result of the ALU and the read data from memory, using the control signal **MemToReg**:

|                        | MemToReg | output     |
|------------------------|----------|------------|
| non-memory instruction | 0        | ALU output |
| memory instruction     | 1        | read data  |

This output is fed to the write-back stage.

## 7.5 Write-back

Stores, branches, and jumps remain idle in this stage as there is nothing to be written.

The result of the memory stage is fed into write data (wd) of the register file.

## 8 Control

**Control signals** are generated by a **control unit** based on the type of instruction.

The control unit is a **combinational circuit** which takes in the 6-bit opcode and the 5-bit function code, and outputs the 8 control signals.

funct is only needed to determine ALUSrc; every other control signal can be derived from opcode alone.

### 8.1 ALUControl

We start with a simplified 1-bit ALU.

#### 1-bit MIPS ALU

4 control signals are needed:

1. **Ainvert**: 1 to invert A, 0 otherwise
2. **Binvert**: 1 to invert B, 0 otherwise
3. **operation (2 bits)**: select one of the 3 results

5 I/O signals:

1. **A**: input
2. **B**: input
3. **Cin**: carry in
4. **Cout**: carry out
5. **result**: output

These 1-bit ALUs are daisy-chained to form a 32-bit ALU.

The carries from one of the 1-bit ALUs is fed out via Cout and into the next one via Cin.

Ainvert and Binvert control the multiplexers which select between their inputs and their negated values:

| Xinvert | output |
|---------|--------|
| 0       | X      |
| 1       | NOT(X) |

The operation control signal selects one of the 3 results from the AND gate, OR gate, and the adder ADD:

| operation | output |
|-----------|--------|
| 00        | AND    |
| 01        | OR     |
| 10        | ADD    |
| 11        | SLT    |

#### 1-bit subtraction

Cin is set to 1 due to 2s complement:

$$A - B \equiv A + (-B) \equiv A + B' + 1$$

#### ALUControl

In MSB to LSB order, ALUControl comprises of:

1. Ainvert
2. Binvert
3. operation

### 8.1.1 ALUOp

It is necessary to use the **multi-level decoding approach** to simplify the design process and size of the main controller.

**ALUOp** is an intermediate 2-bit control signal which is generated from opcode and used with funct to determine ALUControl.

| instruction | ALUOp | funct  | ALUControl |
|-------------|-------|--------|------------|
| lw, sw      | 00    | XXXXXX | 0010       |
| beq         | 01    | XXXXXX | 0110       |
| add         | 10    | 100000 | 0010       |
| sub         | 10    | 100010 | 0110       |
| and         | 10    | 100100 | 0000       |
| or          | 10    | 100101 | 0001       |
| slt         | 10    | 101010 | 0111       |

In general, ALUOp is 10 for R-format instructions.

## 9 Pipelining

**Pipelining** increases workload throughput, but is *rate limited* by the slowest stage in the pipeline, and *stalled* by dependencies between stages.

Each execution stage in the MIPS datapath is a **pipeline stage** which takes 1 clock cycle.

Data required for each stage is stored separately in **4 pipeline registers**:

1. **IF/ID**: between fetch and decode
2. **ID/EX**: between decode and execute
3. **EX/MEM**: between execute and memory

## 4. MEM/WB: between memory and writeback

| register | receives  | supplies  |
|----------|---|---|
| IF/ID    | Instruction[PC],<br>PC + 4                                    | RR1 and RR2,<br>16-bit offset,<br>PC + 4,<br>WR               |
| ID/EX    | RD1 and RD2,<br>32-bit imm,<br>PC + 4,<br>WR                  | RD1 and RD2,<br>32-bit imm,<br>PC + 4,<br>WR                  |
| EX/MEM   | PC + 4 + 4 *<br>imm,<br>ALU result,<br>isZero?,<br>RD2,<br>WR | PC + 4 + 4 *<br>imm,<br>ALU result,<br>isZero?,<br>RD2,<br>WR |
| MEM/WB   | ALU result,<br>memory data read,<br>WR                        | WR  |

## control signals

All control signals are generated in the decode stage and propagated to their respective pipeline stages.

- **execute:** RegDst, ALUSrc, ALUOp
- **memory:** MemRead, MemWrite, Branch
- **writeback:** MemToReg, RegWrite

## 9.1 Performance

## single-cycle processor

Each instruction takes 1 clock cycle to execute.

The **cycle time**  $CT_{seq}$  is the longest time taken to execute any single instruction:

$$CT_{seq} = \max\left(\sum_{k=1}^N T_k\right)$$

where  $N$  is the number of stages and  $T_k$  is the time taken to execute an operation in stage  $k$ .

The **execution time**  $ET_{seq}$  depends on the number of instructions  $I$ :

$$ET_{seq} = CT_{seq} \cdot I$$

## multi-cycle processor

Each stage takes 1 clock cycle to execute.

The **cycle time**  $CT_{multi}$  is the longest time taken to execute any single stage.

The **execution time**  $ET_{multi}$  depends on the number of instructions  $I$  and the average number of instructions per instruction  $CPI_{avg}$ :

$$ET_{multi} = CT_{multi} \cdot I \cdot CPI_{avg}$$

## pipelined processor

The **cycle time**  $CT_{pipelined}$  incurs a pipelining overhead  $T_d$ :

$$CT_{pipelined} = \max(T_k) + T_d$$

where  $T_k$  is the time taken to execute an operation in stage  $k$ .

The **execution time**  $ET_{pipelined}$  depends on the number of instructions  $I$  and stages  $N$ :

$$ET_{pipelined} = CT_{pipelined} \times (I + N - 1)$$

In an **ideal pipeline**:

- every stage takes the same amount of time:

$$\sum_{k=1}^N T_k = N \cdot T_1$$

- there is no pipeline overhead:

$$T_d = 0$$

- the number of instructions is much larger than the number of stages:

$$I \gg N$$

This results in a **ideal speedup** by  $N$ :

$$\frac{ET_{seq}}{ET_{pipelined}} = \frac{I \cdot N \cdot T_1}{(I + N - 1) \cdot T_1} \approx \frac{I \cdot N \cdot T_1}{I \cdot T_1} = N$$

## 9.2 Hazards

**Pipeline hazards** are problems that prevent an instruction immediately after a previous instruction.

**Structural hazards** are caused by simultaneous use of the same register file, memory, or ALU by multiple instructions.

## resolving structural hazards

- stall the execution by one or more cycles.
- split the memory into Data and Instruction.
- write to the register file in the first half of a cycle, and read in the second half — allowed as register access is fast.

**Data hazards** are caused by **read-after-write dependencies** between instructions, where the result of an instruction is read by a subsequent instruction.

This can lead to **stale results** if the result is not written to the register file in time.

## resolving data hazards

**forwarding:** a result that needs to be read is passed to the instruction one clock cycle before it is needed, bypassing the data read from the register file.

**stalling:** an instruction that needs a value read from memory is stalled until the prior lw instruction loads the value into the register.

**Control hazards** are caused by a change in program flow.

Branching decisions are made in the memory stage, which is too late as the next instructions will be partially executed.

#### resolving control hazards (stalls in clock cycles (CCs))

**stalling:** any instruction after the branching instruction is stalled until the branching decision is known (**3 CCs**).

**early branch resolution:** move register comparison to the decode stage (**1 CC**).

If a value in the branch instruction is *computed* in the previous instruction we still need another stall (**2 CCs total**), and if it needs to be *loaded*, the delay does not improve (**3 CCs total**).

**branch prediction:** guess the branch decision before it is produced

Assume all branches are not taken (**0 CCs**) and flush the successor instruction from the pipeline if the branch is taken (**simple prediction, 1 CC**).

**delayed branching:** branches which would precede the branch and executed regardless of the branch decision are moved into the delayed slot

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