

# CS2102

## Database Systems

AY2022/23 Semester 2

Notes by Jonathan Tay

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# Part I

## Relational and ER Models

Data in **relational databases** are stored in **relations** (tables). Column headers are **attributes**, and rows are **tuples**.

The **degree** is the number of columns, and the **cardinality** is the number of rows.

The **domain** of an attribute  $A_i$ , denoted as  $\text{dom}(A_i)$ , is the set of all possible *atomic* values for  $A_i$ . NULL is an additional special value for unknown or invalid values.

keys

A **superkey** is a subset of attributes that uniquely identifies a tuple. A **key** is a *minimal* superkey.

The **candidate keys** is the set of all keys for a relation, of which one is selected as a **primary key**.

Primary key values must be non-NULL.

foreign keys

A **foreign key** is a subset of attributes of a *referencing relation* that refers to the primary key of a *referenced relation*:

(referencing attributes)  $\rightsquigarrow$  (referenced attributes)

Because the names of the attributes are not necessarily unique, each attribute is prefixed with the name of the relation, like so:

(<relation name> · <attribute name>, ...)  $\rightsquigarrow$  ...

Foreign keys must appear as a primary key in the referenced table, NULL, or a tuple containing NULL.

The key constraints above are not intrinsic properties of a relation; rather, they are specified by the database designer to avoid problematic but otherwise valid data.

### 1 Relational Algebra

Relation are always the **operands** in relational algebra, on which **operators** are applied.

Because relations are closed under *any combination of operators*, the result of an operation is *always* a relation, and no other output is possible (**closure property**).

3-valued logic (false, true, and NULL)

Any operation involving NULL will result in NULL. Hence,  $\equiv$  and  $\neq$  are needed to compare (in)equality of NULL values directly:

NULL = NULL produces NULL, but  $\text{NULL} \equiv \text{NULL}$  produces true.

$a \wedge b$		$a$		
		true	false	NULL
$b$	true	true	false	NULL
	false	false	false	false
	NULL	NULL	false	NULL

$a \vee b$		$a$		
		true	false	NULL
$b$	true	true	true	true
	false	true	false	NULL
	NULL	true	NULL	NULL

#### 1.1 Equivalence and Compatibility

Two *expressions* are **equivalent** if either *both* produce an error, or *both* produce the same result.

Errors occur if attributes are missing (e.g. by projection or renaming), or are incompatible — there is no implicit type conversion.

The order of types in a tuple matters — (int, text) is not equivalent to (text, int).

Two *relations* are **union-compatible** if they have the same number of attributes with the same domain (type).

#### 1.2 Basic Operators

Note that logical conjunction ( $\wedge$ ) has greater precedence than logical disjunction ( $\vee$ ).

selection —  $\sigma_{[c]}(R)$

Filters the rows of relation  $R$ , returning the set of tuples that satisfy the condition  $c$ .

Conditions which evaluate to NULL are excluded from the result.

The condition  $c$  must only specify attributes of  $R$ .

projection —  $\pi_{[l]}(R)$

Maps the relation  $R$ , returning the set of tuples with the attributes in the ordered list  $l$ , in the order specified by  $l$ .

Equivalent to a column filter, but the rows which were previously unique tuples may no longer be unique, and are therefore de-duplicated.

The elements of  $l$  must not be operations, must be attributes of  $R$ , and must be unique.

renaming —  $\rho_{[\mathcal{R}]}(R)$

Renames the attributes of relation  $R$  to the attributes in the unordered list  $\mathcal{R}$ .

Elements of  $\mathcal{R}$  must be in the following form:  
<new name>  $\leftarrow$  <old name>.

New attribute names must be unique, and existing attributes must only be renamed at most once per operation (i.e.  $\rho_{[A \leftarrow B, B \leftarrow C]}$  is invalid).

1.3 Set Operators

The typical set union ( $\cup$ ), set intersection ( $\cap$ ), and set difference ( $-$ ) operators are omitted for brevity.

cross product —  $R \times S$

For every tuple in  $R$ , concatenate it with every tuple in  $S$  to form a new relation, such that the cardinality (number of rows) of the result is  $|R| \times |S|$ .

The set of attributes in  $R$  and  $S$  must be disjoint, such that the degree (number of columns) of the result is  $\text{deg}(R) + \text{deg}(S)$ .

Cross products are associative —  $R \times (S \times T) = (R \times S) \times T$ .

1.4 Join Operators

**Joins** are a composite operator, composing cross product, selection, and projection on a relation.

This concatenates two tables and removes unwanted/redundant rows and columns from the result of a cross product.

theta-join ( $\theta$ -join) —  $R \bowtie_{[\theta]} S$

Cross  $R$  and  $S$ , then filter (by selection) the rows using the condition  $\theta$ .

equi-join —  $R \bowtie_{=} S$

A special case of theta-join, where only equality ( $=$ ) conditions are allowed (c.f.  $\theta$ -join which allows  $\equiv, \leq, <>$ , etc.).

This may be more performant versus a theta-join as hashing can be used internally.

natural inner join —  $R \bowtie S$

First, find the set of attributes that are common to both  $R$  and  $S$ .

Then, cross  $R$  and  $S$ , and retain (by selection) the rows for which the **common attributes** are equal —

this also eliminates rows with any NULL value.

If there are no common attributes, then this is simply the cross product (by vacuous truth).

Finally, de-duplicate (by projection) the columns by their attribute names.

Theta-joins, equi-joins, and natural inner joins are collectively known as **inner joins**.

If we wish to perform a join but still retain *all rows* from the *left* table, *right* table, or even *both* tables and simply pad missing values with NULL, we can use **outer joins**.

full outer join —  $L \bowtie_{[\theta]}^{\text{full}} R$

attributes of $L$		attributes of $R$	
values from $L$	...	values from $R$	...
⋮	⋱	⋮	⋱
values from $L$	...	NULL	...
⋮	⋱	⋮	⋱
NULL	...	values from $R$	...
⋮	⋱	⋮	⋱

Inner joins only retain rows (in red) from which both  $L$  and  $R$  have a value for which the condition  $\theta$  evaluates to true (during the selection).

However, rows (in green and blue) which do not satisfy  $\theta$  may also be desirable, and can be retained by outer joins.

The **full outer join** retains all of the rows above.

left outer join —  $L \bowtie_{[\theta]}^{\text{left}} R$

The left outer join retains the red and green rows.

right outer join —  $L \bowtie_{[\theta]}^{\text{right}} R$

The right outer join retains the red and blue rows.

natural outer joins

The natural keyword can be prefixed to the left, right, or full outer joins, e.g. "natural left outer join".

The equality condition is implicitly defined over the set of attributes that are common to both  $L$  and  $R$ .

## 2 Entity Relationship Model

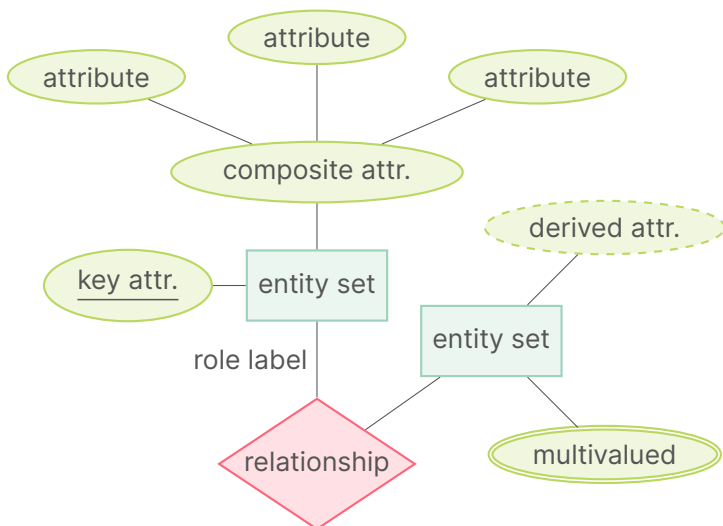
In the ER model, data is described as a collection of:

- **entities:**  
representation of real-world objects that are distinguishable from other objects
- **relationships:**  
associations between one or more entities

Entities and relationships of the same type form **entity sets** and **relationship sets** respectively.

**Attributes** describe information about entities and relationships, and there are various kinds:

- **key:** uniquely identify an entity
- **composite:** collection of multiple *attributes*
- **multivalued:** collection of multiple possible *values*
- **derived:** calculated from other attributes



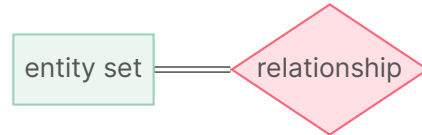
The **degree** (or *arity*) of a relationship set is the number of entity sets involved — binary for  $n = 2$ , ternary for  $n = 3$ .

### 2.1 Relationship Constraints

#### participation constraints

Participation constraints can be **partial** (0 or more) or **total** (1 or more). These are *lower bounds*.

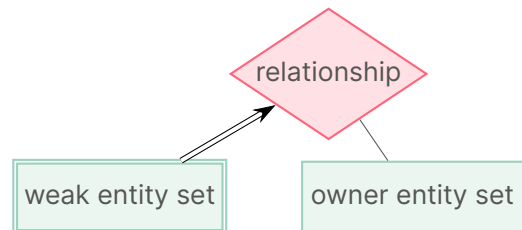
The default lower limit is 0, and setting a lower limit of 1 is depicted in ER diagrams by a double line:



#### dependency constraint

A **weak entity set** does not have its own key, instead having a **partial key** which can only uniquely identify entities with a primary key from its **owner entity set**.

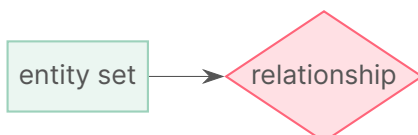
This requires a connection via an **identifying relationship set**:



#### cardinality constraints

Relationships can be **many-to-many**, **many-to-one**, or **one-to-one**. These are *upper bounds*.

The default upper limit is  $\infty$ , and setting an upper limit of 1 is depicted in ER diagrams by an arrowhead:



## Part II

# SQL

**SQL** (Structured Query Language) is case-insensitive, but keywords are uppercase by convention.

The most common data types are `INTEGER`, `VARCHAR(n)`, `BOOLEAN`, and `DATE`.

## 3 Operations and Syntax

### 3.1 Table Operations

---

```
CREATE TABLE table_name (
  -- define attributes (columns)
  <attribute> <type> [<column_constraint>],
  <attribute> <type> [CONSTRAINT <name> <ctr.>],
  ...
  -- define optional table constraints
  [<table_constraint>],
  -- constraints can be named
  [CONSTRAINT <name> <table_constraint>],
  ... /* alternative comment syntax */
  [<table_constraint>] -- no trailing comma
);
```

```
ALTER TABLE <table>
  [ALTER / ADD / DROP] [COLUMN / CONSTRAINT]
  <attribute / name>
  <changes>;
```

```
DROP TABLE
  [IF EXISTS] -- no error if table doesn't exist
  <table>, ... -- drop multiple tables at once
  [CASCADE]; -- also drop referencing tables
```

---

### 3.2 Integrity Constraints

Constraints are specified in the `CREATE TABLE` statement, and reject insertions if the condition evaluates to false (**principle of rejection**).

**Column constraints** are defined on a per-column basis, while **table constraints** are defined on the table as a whole.

Constraints can be *named*, or *unnamed*, in which case the DBMS will generate a name.

There are 5 types of constraints that can be placed on a table  $R$  and/or any attribute  $a$ :

1. **NOT NULL**:  $\forall r \in R : r_a \neq \text{NULL}$
2. **UNIQUE**:  $\forall r_1, r_2 \in R : (r_1 \equiv r_2) \vee (\exists r_{1a} \neq r_{2a})$
3. **PRIMARY KEY**: equivalent to **UNIQUE** and **NOT NULL**
4. **FOREIGN KEY** ( $a, \dots$ ) **REFERENCES**  $R'(\alpha', \dots)$ :  
 $\forall r \in R : (\forall a : r_a \in R'_{\alpha'}) \vee (\text{NULL} \in r)$
5. **CHECK**( $c$ ):  $c$  does not evaluate to false

For **UNIQUE**, note that `NULL <> NULL` evaluates to `NULL`, such that duplicate insertions of `NULL` are not rejected.

For **FOREIGN KEY**,  $R'$  must be a valid table name. `SET NULL`, `SET DEFAULT`, or `CASCADE` can be specified as the action to take when a referenced row is deleted.

`CASCADE` will delete all referencing rows, propagating the deletion, which may significantly affect performance.

For **CHECK**,  $c$  must be a boolean expression scoped to the table on individual rows.

### 3.3 Row Operations

---

```
INSERT INTO <table> [(attribute, ...)]
  VALUES -- whitespace and newlines optional
    (value, ...),
    (value, ...),
    ... -- alternatively, all on one line
    (value, ...);
```

---

All values to insert must have the same shape as either the table, or, if specified, the attribute list.

Attributes in the attribute list can be in any order.

Attributes missing from the attribute list will have their values set to `NULL`, or, if specified in the schema, to their default value.

---

```
DELETE FROM <table> [WHERE <condition>];
```

---

The condition only deletes rows which evaluate to `TRUE` (**principle of acceptance**).

If the condition is omitted, all rows will be deleted.

The condition must be a boolean expression scoped to the table on individual rows.

### 3.4 Deferrable Constraints

---

```
BEGIN; -- start a transaction
-- perform operations
COMMIT; -- commit (end) the transaction
```

---

Constraints are checked immediately at the end of every SQL statement (these end with a semicolon) and transaction — violation performs a rollback.

When defining a constraint, three deferments can be specified:

1. **NOT DEFERRABLE:**  
if unspecified, this is the default
2. **DEFERRABLE INITIALLY IMMEDIATE:**  
constraint is checked immediately, but can be deferred later
3. **DEFERRABLE INITIALLY DEFERRED:**  
constraint is deferred by default

DEFERRABLE INITIALLY IMMEDIATE gives the *option* to defer the constraint checking by adding the following line in a transaction, i.e.:

---

```
BEGIN;
SET CONSTRAINTS <name> DEFERRED; -- add this line
-- perform operations
COMMIT;
```

---

Deferring a constraint is necessary when the constraint depends on a row that is inserted later in the transaction, e.g. circular foreign key constraints.

## 4 Queries

SQL by default does not eliminate duplicate rows (without DISTINCT) nor does it have a fixed row ordering (**order-independent**) (without ORDER BY).

The basic query syntax is as follows:

---

```
SELECT [DISTINCT] <attributes> -- * or a1, a2, ...
FROM <tables> -- r1, r2, ...
WHERE <conditions> -- c
ORDER BY <attributes> <ASC / DESC>
OFFSET <n>
LIMIT <n>; -- ; only if nothing else follows
[[UNION / INTERSECT / EXCEPT] [ALL]]
... -- more queries
```

---

DISTINCT uses IS DISTINCT FROM to compare rows, which is different from =. The latter returns NULL if either argument is NULL, while the former treats NULL like any other value.

Condition evaluation is based on the **principle of acceptance** — rows are included if and only if the condition evaluates to TRUE.

The above query up to line 3 is equivalent to the following relational algebra expression:

$$\pi_{[a_1, a_2, \dots]}(\sigma_{[c]}(r_1 \times r_2 \times \dots))$$

### SELECT and FROM clauses — AS expression

Rename columns or tables (within the query scope) with AS, and even operate on entire columns with several functions:

- **mathematical:** + - \* / % || / ^
- **logical:** AND, OR, NOT
- **string:** ||, LOWER(s), UPPER(s)
- **date:** +, NOW()

---

```
SELECT <regular attributes> -- a1, a2, ...
    [<expression> AS <new attribute>], -- ...
FROM <regular tables>,
    [<old table name> [AS] <new table name>];
```

---

Renaming tables is necessary if the same table appears more than once in the FROM clause.

The AS keyword is optional when used in FROM clauses.

### WHERE clause

Pattern match (or with regex) with LIKE and NOT LIKE as a condition:

- an underscore (\_) matches exactly 1 character,
- % matches 0 or more characters.

---

```
SELECT <attributes>
FROM <tables>
WHERE <attribute> [[NOT] LIKE '<pattern>'];
```

---

The logical operators AND, OR, and NOT can be used to combine/invert conditions.

## 4.1 Set Operations

Without ALL, duplicate rows are eliminated after UNION, INTERSECT, and EXCEPT (set difference).

With ALL, for every row  $x$  in tables  $A$  and  $B$  appearing  $a$  and  $b$  times respectively,  $x$  will appear these many times in the result:

- **UNION:**  $a + b$
- **INTERSECT:**  $\min(a, b)$
- **EXCEPT:**  $\max(a - b, 0)$

## 4.2 Join Operations

---

```
SELECT <attributes>
FROM <r1> [NATURAL] [LEFT / RIGHT / FULL] JOIN
    <r2> [ON <condition>],
... -- more joins, query continues
```

---

If the condition and ON keyword are omitted, the join is implicitly a natural inner or outer join, and the NATURAL keyword is optional.

### 4.3 Composition

A **scalar subquery** is a query that returns a single value (a table with 1 row and 1 column), or an empty table which is treated as NULL.

They are dynamically checked at runtime and can be used as a value in a SELECT, FROM, or WHERE clause.

There are other types of subqueries for dynamically checking if values are in a set or not:

---

```
...
WHERE -- subsequent lines are mutually exclusive
      <expr.> [NOT] IN (<subquery / values>);
      <expr.> <op.> ALL (<subquery / values>);
      <expr.> <op.> ANY (<subquery / values>);
```

---

For ALL and ANY, op can be any comparison operator like =, <, >, <=, >=, <>, while IN uses = and <> implicitly (not IS DISTINCT FROM!).

The subquery must return exactly one column for all the above.

There is also WHERE [NOT] EXISTS (<subquery>), which is true if the subquery returns at least one row, which can lead to unusual queries like this:

---

```
...
WHERE EXISTS (
  SELECT 1 -- any value will do
  FROM <table>
  WHERE <condition>
);
```

---

Unlike the others, the subquery of EXISTS does not need to return a single column.

## 5 Persistent Stored Modules

### 5.1 Functions

Functions in SQL are used to perform operations on data, and must return at least one value.

---

```
CREATE OR REPLACE FUNCTION <name> (<params>)
  RETURNS <return type> AS $$
  <body>
  [LIMIT 1;] -- for RECORDs
  $$ LANGUAGE sql;
```

---

Input parameters (constants) are prefixed with IN, output parameters (uninitialized return values) with OUT, and input/output parameters (initialized variables) with INOUT.

The return type can be any arbitrary type, but returning an existing tuple requires the table name as the return type.

However, if the function returns a new tuple, the return type must be a RECORD or SETOF RECORD, or equivalently, TABLE.

Function bodies cannot commit or rollback transactions.

Functions are called using regular SELECT statements:

---

```
SELECT <name>(<parameters>);
```

---

#### function variables

Variables can be declared in function bodies, and they are local to the function.

---

```
-- ... $$
DECLARE <variable name> <type> [:= <value>];
BEGIN <body> END;
-- $$ ...
```

---

#### function control flow

If statements and loops are available as well.

---

```
-- ... $$
IF <condition> THEN <body>
[ELSEIF <condition> THEN <body>]
[ELSE <body>]
END IF;
LOOP [EXIT WHEN <condition>;]
  <body>;
END LOOP;
-- $$ ...
```

---

#### cursors

Cursors are used to iterate over the result of a query.

---

```
-- $$ ...
DECLARE <cursor name> CURSOR FOR <query>;
r RECORD;
BEGIN
```

---



```

OPEN <cursor name>;
LOOP
    FETCH <cursor name> INTO r;
    EXIT WHEN NOT FOUND;
    <body>;
    [RETURN NEXT;]
END LOOP;
CLOSE <cursor name>;

END
-- $$ ...

```

RETURN NEXT; is used to insert the current value of r into the result of the function.

The function return type must be SETOF RECORD or TABLE for this to work.

Variants of FETCH <variant> FROM with PRIOR, FIRST, LAST, ABSOLUTE <n> and RELATIVE <n> are also available.

## 5.2 Procedures

Unlike functions, procedures do not return any values, but they can commit or rollback transactions as long as they are not called within a transaction.

Procedures have access to the same variable, control flow and cursor constructs as functions.

```

CREATE OR REPLACE PROCEDURE <name> (<params>)
AS $$
    <body>
$$ LANGUAGE sql;

```

Since functions do not return values, their parameters are implicitly (and need not be) prefixed with IN. Nonetheless, INOUT can be used too.

Procedures are called with the CALL keyword:

```
CALL <name>(<parameters>);
```

## 5.3 Triggers

Triggers are used to execute **trigger functions** when a certain event occurs.

As the name implies, trigger functions are functions with a return type of TRIGGER.

To get the database to execute a trigger function, it must be registered as a trigger:

```
CREATE TRIGGER <name>
```

```

-- trigger events, can use multiple with OR
[<AFTER / BEFORE / INSTEAD OF> -- timing
    <INSERT / UPDATE / DELETE>] -- operation
ON <table>
[FOR EACH <ROW / STATEMENT>] -- level
[WHEN <condition>]
EXECUTE FUNCTION <function name>(<params>);

```

Trigger functions can access the inserted or updated tuple using the NEW variable, and the replaced or deleted tuple using OLD.

Other special keywords such as TG\_OP to get the operation type (e.g. INSERT) and TG\_TABLE\_NAME.

### trigger levels

FOR EACH ROW invokes the trigger function for each row affected by the operation.

FOR EACH STATEMENT is used to perform operations on the entire table, and the trigger function is invoked once per operation.

This means that statement-level triggers cannot access the NEW and OLD variables.

Neither can the statement's operation be prevented by returning NULL, unless an exception is raised with RAISE EXCEPTION.

RAISE NOTICE <message> can be used to print a message to the console, but is not equivalent to RAISE EXCEPTION <message>.

### trigger timing - BEFORE

Returning NULL will prevent the operation (insertion, updating, deletion) from happening.

Since the value of OLD is initialized to NULL, RETURN OLD will also prevent the operation from happening *unless* some columns of OLD are modified.

### trigger timing - AFTER

The return value of the trigger function has **no effect** on the operation as it is invoked after the operation has already happened.

### trigger timing - INSTEAD OF (row-level only)

This may only be defined on views, intended to perform the operation on the underlying table instead of the view.

Like BEFORE, returning NULL will prevent the operation from happening.

Triggers are checked after every statement immediately by default, but can be deferred until after the COMMIT of a transaction.

```
CREATE CONSTRAINT TRIGGER <name>
    AFTER <operation> ON <table>
    FOR EACH ROW
    DEFERRABLE INITIALLY <DEFERRED / IMMEDIATE>
    EXECUTE FUNCTION <function name>(<params>);
```

Deferred triggers must be AFTER operations FOR EACH ROW.

Since triggers are constraints, any deferrable triggers which are set to INITIALLY IMMEDIATE can be disabled on a per-transaction basis by changing the trigger constraint.

```
BEGIN TRANSACTION;
SET CONSTRAINTS <trigger name> DEFERRED;
-- ...
COMMIT;
-- trigger is deferred until here
```

If there are multiple triggers on the same operation of a table, the following order applies, ties broken by alphabetical order of the trigger name:

1. BEFORE ... FOR EACH STATEMENT triggers
2. BEFORE ... FOR EACH ROW triggers
3. AFTER ... FOR EACH ROW triggers
4. AFTER ... FOR EACH STATEMENT triggers

If a row-level BEFORE trigger function returns NULL, then all subsequent triggers on the same row are skipped.

## Part III

# Normal Forms

A **normal form** is a definition of minimum requirements to reduce data redundancy and improve data integrity.

Redundancies can lead to **anomalies** where a primary key attribute violates the constraint of being non-NULL.

These anomalies can be resolved by **normalization**.

## 6 Functional Dependencies

A **functional dependency** is a relationship between two sets of attributes  $A$  and  $B$  that occur when the value of  $B$  can always be determined from the value of  $A$ .

That is, when  $A_1A_2 \dots A_m \rightarrow B_1B_2 \dots B_n$ , any tuple with the same values for  $A_1A_2 \dots A_m$  will have the same value for  $B_1B_2 \dots B_n$ .

### determining functional dependencies

Intuitively, the  $\rightarrow$  can be read as "decides" or "determines".

Given attributes  $A$  and  $B$ , if  $A$  is a key of the relation (all unique values), then  $A \rightarrow B$ .

Another method is to construct a counter-example with 2 tuples which would violate the functional dependency.

The attribute with the same value in all rows will be on the LHS of FD, and the other attribute on the RHS.

We can use **Armstrong's axioms** and some additional rules (denoted with  $^\dagger$ ) to determine if a functional dependency is valid.

- **augmentation**: if  $A \rightarrow B$  then  $AC \rightarrow BC$  for any  $C$
- **transitivity**: if  $A \rightarrow B$  and  $B \rightarrow C$  then  $A \rightarrow C$
- **reflexivity**: any set of attributes  $\rightarrow$  subset of the attributes
- **decomposition** $^\dagger$ : if  $A \rightarrow BC$  then  $A \rightarrow B$  and  $A \rightarrow C$
- **union** $^\dagger$ : if  $A \rightarrow B$  and  $A \rightarrow C$  then  $A \rightarrow BC$

Using these rules is pretty tedious, and we can use **closures** to simplify the process.

Given a set of functional dependencies, a closure  $\{A_1, A_2, \dots, A_n\}^+$  is the set of all attributes that can be determined from  $\{A_1, A_2, \dots, A_n\}$ .

### proving functional dependencies $A \rightarrow B$

To prove that  $A \rightarrow B$  holds, we must show that  $B \in \{A\}^+$ .

To prove that  $A \rightarrow B$  does not hold, we must show that  $B \notin \{A\}^+$ .

### finding keys

Given a table  $R$  and set of functional dependencies, to find the keys:

1. Consider every subset of attributes in  $R$ .
2. Derive the closure of each subset.
3. Find the superkeys which are the attributes whose closures contain every attribute in  $R$ .
4. Find the keys which are the minimal superkeys.

Some observations to eliminate some of the tedium:

- If a set of attributes is a key, then any superset cannot be a key (only a superkey).
- If an attribute is in *none* of the RHSs of the FDs, then it must be part of the key.

If an attribute is part of a key, then it is known as a **prime attribute**. Otherwise, it is a **non-prime attribute**.

## 7 Boyce-Codd Normal Form (BCNF)

A table  $R$  is in BCNF if every non-trivial decomposed FD has a superkey on its LHS.

BCNF requires that every non-prime attribute is fully functionally dependent on a superkey.

A **decomposed FD** is an FD with only one attribute on the RHS. Any non-decomposed FD can be decomposed into a set of decomposed FDs.

A **non-trivial FD** is an FD where no attribute in the RHS appears in the LHS.

### finding all non-trivial decomposed FDs

Find the closures of every possible subset of attributes in the table, and then pair the LHS with each attribute in the RHS of the closure.

For example, if  $\{A\}^+ = \{ABC\}$ , then the FDs are  $A \rightarrow B$  and  $A \rightarrow C$ .

### determine if a table is in BCNF

If there exists a non-trivial decomposed FD  $A_1A_2\dots A_k \rightarrow B_1$  on  $R$  such that  $\{A_1A_2\dots A_k\}$  is not a superkey, then the table is not in BCNF.

This is the **"more but not all"** condition, since the closure  $\{A_1A_2\dots A_k\}^+$  will contain  $B_1$  but not all attributes in  $R$ .

<sup>†</sup> Any table with 2 or fewer attributes is in BCNF.

### table decomposition into BCNF

First find a subset of the attributes  $X$  in  $R$  such that its closure  $X^+$  contains more attributes than  $X$  but not all attributes in  $R$ .

Repeatedly perform binary splits on  $R$  into smaller tables  $R_1$  and  $R_2$  until it is in BCNF.

$R_1$  will contain all attributes in  $X^+$ .

$R_2$  will contain all attributes in  $X$  and the attributes in  $R$  but not in  $X^+$ .

On the smaller tables, project the original closures of  $R$  onto the smaller tables, eliminating the attributes which are no longer in the smaller tables.

After decomposition, the original table  $R$  can be reconstructed via **lossless joins** as long as there are common attributes between  $R_1$  and  $R_2$  which are superkeys on *either* table.

The downside of such BCNF decomposition is that dependencies may not be preserved.

## 8 Third Normal Form (3NF)

3NF is less restrictive than BCNF, and requires that every non-trivial decomposed FD satisfies either of the following conditions:

1. The LHS is a superkey.
2. The RHS is a prime attribute, appearing in a key (not allowed by BCNF).

### determine if a table is in 3NF

Find all the keys of  $R$ , and check if every non-trivial decomposed FD satisfies either of the conditions above.

<sup>†</sup> Any table with 2 or fewer attributes or where every attribute appears in a key is in 3NF.

Given a set of FDs  $S$ , a **minimal basis** of  $S$  is a simplified set of FDs that retains all the dependencies in  $S$ .

### finding a minimal basis

1. Decompose the given FDs so that every FD has only 1 attribute on the RHS.
2. We now remove extraneous attributes on the LHS of every FD with at least 2 attributes on the LHS in the form  $XY \rightarrow Z$ .
  - 2.1. Find the closure  $X^+$ .
  - 2.2. If  $Y \in X^+$  and  $Z \in X^+$ , then remove  $Y$  from the LHS.
  - 2.3. Otherwise, find the closure  $Y^+$ . If  $X \in Y^+$  and  $Z \in Y^+$ , then remove  $X$  from the LHS.
3. If an FD has more than 2 attributes, e.g.  $ABC \rightarrow D$ , then check every possible subset of the LHS for redundancy.
4. With extraneous attributes removed, we now eliminate FDs that are redundant.
  - 4.1. For each FD, compute the closure  $FD^+$  of the LHS as if the FD does not exist.
  - 4.2. If the RHS of the FD is in  $FD^+$ , then the FD is removed.

<sup>†</sup> For every attribute in  $R$ , if one and only one FD contains that attribute on the RHS, then that FD is not redundant (concerning step 4.).

### table decomposition into 3NF

Given a table  $R$  and a set of FDs  $S$ , first find a minimal basis.

Merge FDs with the same LHSs using the union rule, i.e.  $A \rightarrow B$  and  $A \rightarrow C$  becomes  $A \rightarrow BC$ .

Create a table for each remaining FD using the

attributes of both sides of the FD, i.e.  $ABC \rightarrow D$  yields  $R'(A, B, C, D)$ .

If none of the tables contain a key of  $R$ , create a table that contains any key of  $R$ , i.e. for a table  $R(A, B, C, D)$  with key  $AC$  decomposed into  $R_1(A, B)$  and  $R_2(C, D)$ , create a new table  $R_3(A, C)$ .

Remove redundant tables where the attributes of a table are a subset of another table.

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