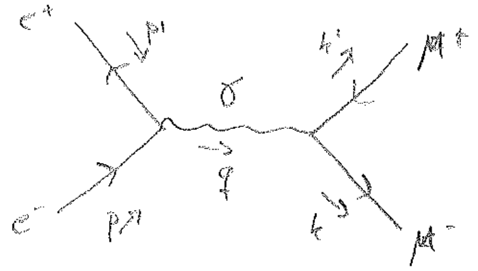
- $\frac{d\sigma}{d\cos\Theta}\,(e^+e^- \to \mu^+\mu^-)$



Vertex: $-ie\gamma^\mu$

γ-propagator: $\dfrac{-ig_{\mu\nu}}{q^2+i\varepsilon}$

$$i\mathcal{M} = \bar{v}^{s'}(p')(-ie\gamma^\mu)u^s(p)\left(\frac{-ig_{\mu\nu}}{\underbrace{q^2+i\varepsilon}_{\text{set back for closure}}}\right)\bar{u}^r(k)(-ie\gamma^\nu)v^{r'}(k')$$

Rewrite this to

$(-i)^3 = -i^3 = -i\cdot i^2$
$= i$

$$i\mathcal{M} = \sum_{s',s,r,r'} \frac{ie^2}{q^2}\,\bar{v}^{s'}(p')\gamma^\mu u^s(p)\bar{u}^r(k)\gamma_\mu v^{r'}(k')$$

$\bar{v} = v^\dagger\gamma^0$

$$(\bar{v}\gamma^\mu u)^* = u^\dagger(\gamma^\mu)^\dagger(\gamma^0)^\dagger v = u^\dagger(\gamma^\mu)^\dagger\gamma^0 v = u^\dagger\gamma^0\gamma^\mu v$$
$$= \bar{u}\gamma^\mu v$$

$$\Rightarrow |\mathcal{M}|^2 = \frac{e^4}{q^4}\left(\bar{v}(p')\gamma^\mu u(p)\bar{u}(p)\gamma^\nu v(p')\right)\left(\bar{u}(k)\gamma_\mu v(k')\bar{v}(k')\gamma_\nu u(k)\right) \quad (1)$$

Spin sums are implicit, however, we want to remain spin information since we don't measure the spin anyway.

Thus we want to find,

$$\boxed{\not{p} = \gamma^\mu p_\mu}$$

$$\frac{1}{2}\sum_s \frac{1}{2}\sum_{s',r,r'} |\mathcal{M}(s,s' \to r,r')|^2.$$

Remember completeness relations:

$$\sum_s u^s(p)\bar{u}^s(p) = \not{p} + m, \quad \sum_s v^s(p)\bar{v}^s(p) = \not{p} - m.$$

Looking at $\bar{v}(p')\gamma^\mu u(p)\bar{u}(p)\gamma^\nu v(p')$:

$$\sum_{s,s'} \bar{v}^{s'}_a(p')\gamma^\mu_{ab} u^s_b(p)\bar{u}^s_c(p)\gamma^\nu_{cd} v^{s'}_d(p')$$

$$= \sum_{s,s'} \underbrace{v^{s'}_d(p')\bar{v}^{s'}_a(p')}_{\not{p}'-m}\gamma^\mu_{ab} \underbrace{u^s_b(p)\bar{u}^s_c(p)}_{\not{p}+m}\gamma^\nu_{cd}$$

$$= (\not{p}'-m)_{da}\gamma^\mu_{ab}(\not{p}+m)_{bc}\gamma^\nu_{cd} = \mathrm{tr}\left((\not{p}'-m_e)\gamma^\mu(\not{p}+m_e)\gamma^\nu\right)$$

why?

Then the other part of (1)

$$\sum_{s,s'} \bar{u}^s_a(k)\gamma_\mu^{ab} v^{s'}_b(k')\bar{v}^{s'}_c(k')\gamma_\nu^{cd} u^s_d(k) = \sum_{s,s'} u^s_d(k)\bar{u}^s_a(k)\gamma_\mu^{ab} v^{s'}_b(k')\bar{v}^{s'}_c(k')\gamma_\nu^{cd}$$

$$= (\not{k}+m_\mu)_{da}\gamma_\mu^{ab}(\not{k}'-m_\mu)_{bc}\gamma_\nu^{cd}$$

$$= \mathrm{tr}\left((\not{k}+m_\mu)\gamma_\mu(\not{k}'-m_\mu)\gamma_\nu\right)$$

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$$\Rightarrow \frac{1}{4}\sum_{s',s,r',r} |\mathcal{M}|^2 = \frac{1}{4}\frac{e^4}{q^4}\, tr\left((\not p' - m_e)\gamma^\mu(\not p + m_e)\gamma^\nu\right) tr\left((\not k + m_\mu)\gamma_\mu(\not k' - m_\mu)\gamma_\nu\right)$$

We can simplify the traces:

$$tr\left((\not p' - m_e)\gamma^\mu(\not p + m_e)\gamma^\nu\right) = tr\left((\gamma^\rho p'_\rho - m_e)\gamma^\mu(\gamma^\sigma p_\sigma + m_e)\gamma^\nu\right)$$

$$= tr\left(\gamma^\rho p'_\rho \gamma^\mu \gamma^\sigma p_\sigma \gamma^\nu + \cancel{\gamma^\rho p'_\rho \gamma^\mu m_e \gamma^\nu} - \cancel{m_e \gamma^\mu \gamma^\sigma p_\sigma \gamma^\nu} - m_e^2 \gamma^\mu \gamma^\nu\right)$$

$$= 4(g^{\rho\mu}g^{\sigma\nu} - g^{\rho\sigma}g^{\mu\nu} + g^{\rho\nu}g^{\mu\sigma})p'_\rho p_\sigma - \underbrace{4g^{\mu\nu}m_e^2}_{\sim 0}$$

$$= 4(g^{\rho\mu}g^{\sigma\nu} - g^{\rho\sigma}g^{\mu\nu} + g^{\rho\nu}g^{\mu\sigma})p'_\rho p_\sigma \qquad (2)$$

$$tr\left((\not k + m_\mu)\gamma_\mu(\not k' - m_\mu)\gamma_\nu\right) = tr\left((\gamma^\rho k_\rho + m_\mu)\gamma_\mu(\gamma^\sigma k'_\sigma - m_\mu)\gamma_\nu\right)$$

$$= tr\left(\gamma^\rho k_\rho \gamma_\mu \gamma^\sigma k'_\sigma \gamma_\nu - m_\mu^2 \gamma_\mu \gamma_\nu\right)$$

$$= 4(g^\rho{}_\mu g^\sigma{}_\nu - g^{\rho\sigma}g_{\mu\nu} + g^\rho{}_\nu g_\mu{}^\sigma)k_\rho k'_\sigma$$

$$- 4g_{\mu\nu}m_\mu^2 \qquad (3)$$

$$(2):\quad 4\left(g^{\rho\mu}p'_\rho g^{\sigma\nu}p_\sigma - g^{\rho\sigma}g^{\mu\nu}p'_\rho p_\sigma + g^{\rho\nu}p'_\rho g^{\mu\sigma}p_\sigma\right)$$

$$= 4\left(p'^\mu p^\nu - g^{\mu\nu}p'^\sigma p_\sigma + p'^\nu p^\mu\right)$$

$$= 4\left(p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p'\cdot p)\right)$$

Similarly for (3) we'll get

$$(3):\ 4\left(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}(k\cdot k' + m_\mu^2)\right)$$

Thus,

$$\frac{1}{4}\sum_{s,s',r,r'} |\mathcal{M}|^2 = \frac{e^4}{4q^4}\, 16\, \left(p'^\mu p^\nu + p'^\nu p^\mu - g^{\mu\nu}(p\cdot p')\right)\left(k_\mu k'_\nu + k_\nu k'_\mu - g_{\mu\nu}(k\cdot k' + m_\mu^2)\right)$$

$$= \frac{4e^4}{q^4}\Big(p'^\mu p^\nu k_\mu k'_\nu + p'^\mu p^\nu k_\nu k'_\mu - p'^\mu p^\nu g_{\mu\nu}(k\cdot k' + m_\mu^2)$$
$$+ p'^\nu p^\mu k_\mu k'_\nu + p'^\nu p^\mu k_\nu k'_\mu - p'^\nu p^\mu g_{\mu\nu}(k\cdot k' + m_\mu^2)$$
$$- g^{\mu\nu}(p\cdot p') k_\mu k'_\nu - g^{\mu\nu}(p\cdot p') k_\nu k'_\mu + g^{\mu\nu}(p\cdot p') g_{\mu\nu}(k\cdot k' + m_\mu^2)\Big)$$

$$= \frac{4e^4}{q^4}\Big((p'\cdot k)(p\cdot k') + (p'\cdot k')(p\cdot k) - (p'\cdot p)(k\cdot k' + m_\mu^2)$$
$$+ (p'\cdot k')(p\cdot k) + (p'\cdot k)(p\cdot k') - (p'\cdot p)(k\cdot k' + m_\mu^2)$$
$$- (p\cdot p')(k\cdot k') - (p\cdot p')(k\cdot k') + 4(p\cdot p')(k\cdot k' + m_\mu^2)\Big)$$

$$= \frac{4e^4}{q^4}\Big(2(p'\cdot k)(p\cdot k') + 2(p'\cdot k')(p\cdot k) + 2(p\cdot p') m_\mu^2\Big)$$

$$= \frac{8e^4}{q^4}\Big((p'\cdot k)(p\cdot k') + (p'\cdot k')(p\cdot k) + (p\cdot p') m_\mu^2\Big) \qquad (4)$$

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Mandelstam: $s = (p+p')^2 = (k+k')^2$

$= \underbrace{p \cdot p}_{m_e} + \underbrace{p' \cdot p'}_{m_e} + 2 p \cdot p'$

$= \underbrace{k \cdot k}_{m_\gamma} + \underbrace{k' \cdot k'}_{m_\gamma} + 2 k \cdot k'$

$\Rightarrow \quad p \cdot p' = \frac{1}{2}(s - 2 m_e)$

$p = (E, \vec{p})$

$p^2 = E^2 + \vec{p}^2 = m^2$

$p \cdot k = (E_{e^-}, \vec{p})(E_{\gamma}, \vec{k})$

$= E_{e^-} E_{\gamma} - E_{e^-} \vec{k} - E_{\gamma} \vec{p} \cdot \vec{k} \cdot \vec{p}$

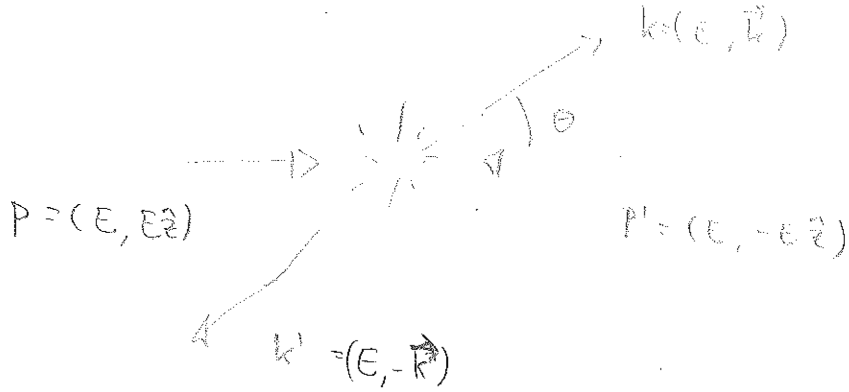
We introduce (by choosing centre of mass as reference frame):

$s = (p+p')^2 = 4E^2$

$p \cdot p' = 2E^2$

$p \cdot k = p' \cdot k' = E^2 - E |\vec{k}| \cos\theta$

$p \cdot k' = p' \cdot k = E^2 + E |\vec{k}| \cos\theta$



$p \cdot k = (E, E\hat{z}) \cdot (E, \vec{k})$

$= E^2 - E \cdot \hat{z} \cdot \vec{k}$

$= E^2 - E \underbrace{|\hat{z}|}_{=1} |\vec{k}| \cos\theta$

We plug the substitutions into (4):

$$\frac{1}{4}\sum_{s,s',r,r'} |M|^2 = \frac{8e^4}{(2E)^4}\left((E^2 + E|\vec{k}|\cos\theta)^2 + (E^2 - E|\vec{k}|\cos\theta)^2 + 2E^2 m_\mu^2 \right)$$

$$= \frac{8e^4}{16E^4}\Big[E^4 + \cancel{2E^3\sqrt{E^2-m_\mu^2}\cos\theta} + E^2(E^2-m_\mu^2)\cos^2\theta$$

$$+ E^4 - \cancel{2E^3\sqrt{E^2-m_\mu^2}\cos\theta} + E^2(E^2-m_\mu^2)\cos^2\theta + 2E^2 m_\mu^2\Big]$$

$$= \frac{\cancel{8}e^4}{\cancel{16}\cancel{E^4}}\left(\cancel{2E^4}\left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta + \cancel{2E^4}\left(1+\frac{m_\mu^2}{E^2}\right)\right)$$

$$= e^4\left[\left(1+\frac{m_\mu^2}{E^2}\right) + \left(1-\frac{m_\mu^2}{E^2}\right)\cos\theta\right]$$

The differential cross section is

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{1}{2E_{cm}^2}\,\frac{|\vec{k}|}{16\pi^2 E_{cm}}\cdot\frac{1}{4}\sum_{s,s',r,r'}|M|^2$$

$$= \frac{1}{2E_{cm}^2}\,\frac{\sqrt{E^2-m_\mu^2}}{16\pi^2 E_{cm}}\, e^4\left[\left(1+\frac{m_\mu^2}{E^2}\right)+\left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta\right]$$

$$= \frac{e^4}{32\pi^2 E_{cm}^2}\, E\cdot\sqrt{\cdots}$$

$$= \frac{e^4}{64\pi^2 E_{cm}^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left[\left(1+\frac{m_\mu^2}{E^2}\right)+\left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta\right]$$

$\alpha^2 = \frac{e^4}{16\pi^2}$

$$= \frac{\alpha^2}{4E_{cm}^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\Big[\qquad\Big]$$

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(VI)

$$\sigma = \int_0^{2\pi}\int_0^{\pi} \frac{\alpha^2}{4E^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left[\left(1+\frac{m_\mu^2}{E^2}\right)+\left(1-\frac{m_\mu^2}{E^2}\right)\cos^2\theta\right]\sin\theta \, d\theta \, d\varphi$$

$$d_\beta = \cos\theta \, d\theta$$

$$= \frac{2\pi\alpha^2}{4E^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\int_0^{\pi}\left[\left(1+\frac{m_\mu^2}{E^2}\right)\sin\theta+\left(1-\frac{m_\mu^2}{E^2}\right)(\sin\theta-\sin^3\theta)\right]d\theta$$

$$\int_0^{\pi}\sin^3\theta \, d\theta = \pi\cdot\sin(\pi/2)\underbrace{\Gamma(3)}_{2}\Big/\Big(2\cdot\underbrace{\Gamma(5/2)}_{\frac{3}{4}\sqrt{\pi}}\underbrace{\Gamma(3/2)}_{\frac{1}{2}\sqrt{\pi}}\Big) = \frac{\cancel{\pi}\cdot 2\cdot\cancel{4}\cdot 2}{\cancel{4}\cdot 3\cdot\cancel{\pi}} = \frac{4}{3}$$

$$= \text{---} \;\; " \;\; \text{---}\left[-\left(1+\frac{m_\mu^2}{E^2}\right)\cos\theta+\left(1-\frac{m_\mu^2}{E^2}\right)\left(-\cos\theta-\frac{4}{3}\right)\right]\Big|_0^{\pi}$$

$$= \text{---} \;\; " \;\; \text{---}\left[\left(1+\frac{m_\mu^2}{E^2}\right)(+2)+\left(1-\frac{m_\mu^2}{E^2}\right)\left(+2-\frac{4}{3}\right)\right]$$

$$= \text{---} \;\; " \;\; \text{---}\left[+2+\cancel{2\frac{m_\mu^2}{E^2}}+2\cancel{-2\frac{m_\mu^2}{E^2}}-\frac{4}{3}+\frac{4}{3}\frac{m_\mu^2}{E^2}\right]$$

$$= \text{---} \;\; " \;\; \text{---}\left[\frac{8}{3}+\frac{4}{3}\frac{m_\mu^2}{E^2}\right]$$

$$= \frac{2\pi\alpha^2}{\cancel{4}E^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\;\frac{\cancel{8}^{\,2}}{3}\left[1+\frac{1}{2}\frac{m_\mu^2}{E^2}\right]$$

$$= \frac{4\pi\alpha^2}{3E^2}\sqrt{1-\frac{m_\mu^2}{E^2}}\left[1+\frac{1}{2}\frac{m_\mu^2}{E^2}\right]$$

$$L = 100\ \mathrm{pb}^{-1} = 100\cdot(10^{-40}\,\mathrm{m}^2)^{-1}$$

(11)

We'll look at each term independently:

$$|\mathcal{M}|^2 = |\mathcal{M}_\gamma|^2 + |\mathcal{M}_{Z^0}|^2 + |\mathcal{M}_\gamma^* \mathcal{M}_{Z^0}|$$

$|\mathcal{M}_\gamma|^2$ is the same as in QED.

$$\mathcal{M}_{Z^0} = \frac{-i g_z^2}{2^2 (q_1^2 - M_Z^2)} \bar{v}^{t'}(p_2') \gamma^\rho (c_V^t - c_A^t \gamma^5) u^t(p_2)\, \bar{u}^e(k_2) \gamma_\rho (c_V^e - c_A^e \gamma^5) v^e(k_2')$$

$$\frac{1}{4}\sum_{spins} |\mathcal{M}_{Z^0}|^2 = \frac{1}{4} \left(\frac{g_z^2}{4(q_1^2 - M_Z^2)} \right)^2 \bar{v}_a^{t'}(p_2') \gamma^\rho_{ab} (c_V^t - c_A^t \gamma^5) u_b^t(p_1)\, \bar{u}_c^t(p_2) \gamma^\lambda_{cd} (c_V^t - c_A^t \gamma^5) v_d^{t'}(p_2')$$

$$\cdot \bar{u}_{a'}^{l}(k_2) \gamma_{\rho\, a'b'} (c_V^e - c_A^e \gamma^5) v_{b'}^{e}(k_2')\, \bar{v}_{c'}^{e}(k_2') \gamma_{\nu\, c'd'} (c_V^e + c_A^e \gamma^5) u_{d'}^{e}(k_2).$$

$$= \frac{1}{4} \left(\frac{g_z^2}{4(q^2 - m_Z^2)} \right)^2 \; v_d^{t'}(p_2') \bar{v}_a^{t'}(p_2') \gamma^\rho_{ab} (c_V^e - c_A^e \gamma^5) u_b^t(p_1) \bar{u}_c^t(p_1) \gamma^\lambda_{cd} (c_V^e - c_A^e \gamma^5)$$

$$\cdot u_{d'}^e(k_2) \bar{u}_{a'}^e(k_2) \gamma_{a'b'\rho} (c_V^e - c_A^e \gamma^5) v_{b'}^{e'}(k_2') \bar{v}_{c'}^{e'}(k_2') \gamma_{\nu\, c'd'} (c_V^e + c_A^e \gamma^5)$$

$$= \frac{1}{4} \left(\quad \right)^2 \mathrm{tr}\left[(\not{p}_2' - m_e)_{da} \gamma^\rho_{ab} (c_V^e + c_A^e \gamma^5) (\not{p}_2 + m_e)_{bc} \gamma^\lambda_{cd} (c_V^e - c_A^e \gamma^5) \right] \quad \mathrm{I}$$

$$\cdot \mathrm{tr}\left[(\not{k}_2 + m_\mu)_{d'a'} \gamma_{a'b'\rho} (c_V^e - c_A^e \gamma^5) (\not{k}_2' - m_\mu)_{b'c'} \gamma_{c'd'\lambda} (c_V^e - c_V^e \gamma^5) \right] \quad \mathrm{II}$$

$$\mathrm{I} = \mathrm{tr}\left[(\gamma^\mu p_{2\mu}' - m_e) \gamma^\rho (c_V^e + c_A^e \gamma^5) (\gamma^\mu p_{2\mu} + m_e) \gamma^\lambda (c_V^e - c_A^e \gamma^5) \right]$$

$$= \mathrm{tr}\left[(\gamma^\mu \gamma^\rho p_{2\mu}' - \gamma^\rho m_e)(c_V^e - c_A^e \gamma^5)(\gamma^\mu \gamma^\lambda p_{2\mu} - \gamma^\lambda m_e)(c_V^e - c_A^e \gamma^5) \right]$$

$$= \mathrm{tr}\Big[(\gamma^\mu \gamma^\rho c_V^e p_{2\mu}' - \gamma^\mu \gamma^\rho \gamma^5 c_A^e p_{2\mu}' - \gamma^\rho c_V^e m_e + \gamma^\rho \gamma^5 c_A^e m_e)$$

$$\cdot (\gamma^\mu \gamma^\lambda c_V^e p_{2\mu} - \gamma^\mu \gamma^\lambda \gamma^5 c_A^e p_{2\mu} - \gamma^\lambda c_V^e m_e + \gamma^\lambda \gamma^5 c_A^e m_e) \Big]$$

$$T = \mathrm{tr}\Big(\gamma^\mu\gamma^\rho\gamma^\nu\underset{\wedge\,\gamma^\lambda}{}(c_V^e)^2 p'_{2\mu}p_{2\nu} - \gamma^\mu\gamma^\rho\gamma^\nu\gamma^\lambda\gamma^5 c_V^e c_A^e p'_{2\mu}p_{2\nu}$$

$$\cancel{-\gamma^\mu\gamma^\rho\gamma^\lambda(c_V^e)^2} p'_{2\mu}m_e + \gamma^\mu\gamma^\rho\cancel{\gamma^\lambda\gamma^5} c_V^e c_A^e p'_{2\mu}m_e$$

$$-\gamma^\mu\gamma^\rho\gamma^5\gamma^\nu\gamma^\lambda c_A^e c_V^e p'_{2\mu}p_{2\nu} + \gamma^\mu\gamma^\rho\gamma^5\gamma^\nu\gamma^\lambda\gamma^5 (c_A^e)^2 p'_{2\mu}p_{2\nu}$$

$$\cancel{+\gamma^\mu\gamma^\rho\gamma^5\gamma^\lambda c_A^e c_V^e p'_{2\mu}m_e} \quad \cancel{-\gamma^\mu\gamma^\rho\gamma^5\gamma^\lambda\gamma^5(c_A^e)^2 m_e p'_{2\mu}}$$

$$\gamma^\mu\gamma^\rho\gamma^5\gamma^\lambda = -\gamma\gamma\gamma\gamma^5 = -\gamma^5\gamma\gamma\gamma^\lambda = \gamma^\mu\gamma^5\gamma^\rho\gamma^\lambda = -\gamma^\mu\gamma^\rho\gamma^5\gamma^\lambda = 0 \ !$$

$$-\cancel{\gamma^\rho\gamma^\mu\gamma^\lambda(c_V^e)^2} m_e p_{2\mu} + \cancel{\gamma^\rho\gamma^\mu\gamma^\lambda\gamma^5 c_A^e c_V^e} m_e p_{2\mu}$$

$$+\gamma^\rho\gamma^\lambda(c_V^e)^2 m_e^2 - \gamma^\rho\cancel{\gamma^\lambda\gamma^5 c_V^e c_A^e} m_e^2$$

$$\cancel{+\gamma^\rho\gamma^5\gamma^\mu\gamma^\lambda c_A^e c_V^e m_e p_{2\mu}} - \gamma^\rho\gamma^5\cancel{\gamma^\mu\gamma^\lambda\gamma^5(c_A^e)^2 m_e p_{2\mu}}$$

$$-\cancel{\gamma^\rho\gamma^5\gamma^\lambda c_V^e c_A^e m_e^2} + \gamma^\rho\gamma^5\gamma^\lambda\gamma^5(c_A^e)^2 m_e^2 \Big)$$

Alle ledd med odde antall γ-matriser blir null hvis γ^5 ikke er involvert. $\mathrm{tr}(\gamma^\mu\gamma^\nu\gamma^5)=0$

$$\Downarrow$$

$$= 4(g^{\mu\rho}g^{\nu\lambda} - g^{\mu\nu}g^{\rho\lambda} + g^{\mu\lambda}g^{\rho\nu})(c_V^e)^2 p'_{2\mu}p_{2\nu} + 4i\varepsilon^{\mu\rho\nu\lambda}c_V^e c_A^e p'_{2\mu}p_{2\nu}$$

$$+4i\varepsilon^{\mu\rho\nu\lambda}c_A^e c_V^e p'_{2\mu}p_{2\nu} + 4(g^{\mu\rho}g^{\nu\lambda} - g^{\mu\nu}g^{\rho\lambda} + g^{\mu\lambda}g^{\rho\nu})(c_A^e)^2 p'_{2\mu}p_{2\nu}$$

$$+4g^{\rho\lambda}(c_V^e)^2 m_e^2 - 4g^{\rho\lambda}(c_A^e)^2 m_e^2$$

$$= 4(c_A^{e2}+c_V^{e2})(p_2'^\rho p_2^\lambda - g^{\rho\lambda}p_2'^\nu p_{2\nu} + p_2'^\lambda p_2^\rho) + 8i\varepsilon^{\mu\rho\nu\lambda}c_V^e c_A^e p'_{2\mu}p_{2\nu}$$

$$+4(c_V^{e2} - c_A^2)g^{\rho\lambda}m_e^2$$

IV

$$\mathrm{II} = \mathrm{tr}\Big(\big(\gamma^\mu k_{2\mu} + m_\mu\big)\gamma_\rho\big((c_V^e - c_A^e\gamma^5)\big)\big(\gamma^\nu k'_{2\nu} - m_\mu\big)\gamma_\lambda\big((c_V^e - c_A^e\gamma^5)\big)\Big)$$

$$= \mathrm{tr}\Big[\big(\gamma^\mu k_{2\mu}\gamma_\rho c_V^e - \gamma^\mu k_{2\mu}\gamma_\rho c_A^e\gamma^5 + m_\mu\gamma_\rho c_V^e - m_\mu\gamma_\rho c_A^e\gamma^5\big)$$

$$\big(\gamma^\nu k'_{2\nu}\gamma_\lambda c_V^e - \gamma^\nu k'_{2\nu}\gamma_\lambda c_A^e\gamma^5 - m_\mu\gamma_\lambda c_V^e + m_\mu\gamma_\lambda c_A^e\gamma^5\big)\Big]$$

$$= \mathrm{tr}\Big[\gamma_\mu\gamma_\rho\gamma_\nu\gamma_\lambda k_2^\mu k_2'^\nu (c_V^e)^2 - \gamma_\mu\gamma_\rho\gamma_\nu\gamma_\lambda\gamma^5 c_V^e c_A^e k_2^\mu k_2'^\nu$$

$$\gamma_\mu\gamma_\rho\gamma_\nu\gamma_\lambda(1-\gamma^5)c_V^e c_A^e k_2^\mu k_2'^\nu$$

$$- \cancel{\gamma^\mu\gamma_\rho\gamma_\lambda (c_V^e)^2 k_{2\mu} m_\mu} + \cancel{\gamma^\mu\gamma_\rho\gamma_\lambda\gamma^5 k_{2\mu} c_V^e c_A^e m_\mu}$$

$$-\gamma_\mu\gamma_\rho\gamma^5\gamma_\nu\gamma_\lambda k_2^\mu k_2'^\nu c_A^e c_V^e + \gamma_\mu\gamma_\rho\gamma^5\gamma_\nu\gamma_\lambda\gamma^5 (c_A^e)^2 k_2^\mu k_2'^\nu$$

$$+ \cancel{\gamma^\mu\gamma_\rho\gamma_\lambda c_A^e c_V^e k_{2\mu} m_\mu} - \cancel{\gamma^\mu\gamma_\rho\gamma^5\gamma_\lambda\gamma^5 k_{2\mu}(c_A^e)^2 m_\mu}$$

$$+ \cancel{\gamma_\rho\gamma^\nu\gamma_\lambda k'_{2\nu}(c_V^e)^2 m_\mu} - \cancel{\gamma_\rho\gamma^\nu\gamma_\lambda\gamma^5 m_\mu c_A^e c_V^e k'_{2\nu}}$$

$$- \gamma_\rho\gamma_\lambda (c_V^e)^2 m_\mu^2 \quad + \cancel{\gamma_\rho\gamma_\lambda\gamma^5 m_\mu^2 c_A^e c_V^e}$$

$$- \cancel{\gamma_\rho\gamma^5\gamma^\nu\gamma_\lambda k'_{2\nu} c_A^e c_V^e m_\mu} + \cancel{\gamma_\rho\gamma^5\gamma^\nu\gamma_\lambda\gamma^5 k'_{2\nu}(c_A^e)^2 m_\mu}$$

$$+ \cancel{\gamma_\rho\gamma^5\gamma_\lambda c_A^e c_V^e m_\mu} - \underbrace{\gamma_\rho\gamma^5\gamma_\lambda\gamma^5}_{+\gamma_\rho\gamma_\lambda}(c_A^e)^2 m_\mu^2\Big]$$

$$= 4(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\rho\lambda} + g_{\mu\lambda}g_{\rho\nu})(c_V^e)^2 k_2^\mu k_2'^\nu + 4i\,\varepsilon_{\mu\rho\nu\lambda} c_V^e c_A^e k_2^\mu k_2'^\nu$$

$$+ 4(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\rho\lambda} + g_{\mu\lambda}g_{\rho\nu})(c_A^e)^2 k_2^\mu k_2'^\nu$$

$$- 4 g_{\rho\lambda}(c_V^e)^2 m_\mu^2 + 4 g_{\rho\lambda}(c_A^e)^2 m_\mu^2$$

$$\frac{1}{4}\sum_{spins}|\mathcal{M}_{Z^0}|^2 = \frac{1}{4}\frac{g_Z^4}{16(q^2-m_Z^2)^2}\Big(\bar{v}_a^{t'}(p')\gamma^\rho_{ab}(c_V^e - c_A^e\gamma^5)u_b^t(p)\bar{u}_c^t(p)$$

$$\gamma^\lambda_{cd}(c_V^e - c_A^e\gamma^5)v_d^{t'}(p')\Big)\cdot\Big(\bar{u}_{a'}^\ell(k)\gamma_{\rho\, a'b'}(c_V - c_A\gamma^5)v_{b'}^{\ell'}(k')$$

$$\bar{v}_{c'}^{\ell'}(k')\gamma_{\lambda\, c'd'}(c_V - c_A\gamma^5)u_{d'}^\ell(k)\Big)$$

$$= \frac{1}{4}\frac{g_Z^4}{16(q^2-m_Z^2)^2}\underbrace{tr\big((\not p' - m_e)\gamma^\rho(c_V - c_A\gamma^5)(\not p + m_e)\gamma^\lambda(c_V - c_A\gamma^5)\big)}_{I}$$

$$\cdot\underbrace{tr\big((\not k + m_\mu)\gamma_\rho(c_V - c_A\gamma^5)(\not k' - m_\mu)\gamma_\lambda(c_V - c_A\gamma^5)\big)}_{II}$$

$m_e << m_Z$

$$I = tr\big((\gamma^\mu p'_\mu - \cancel{m_e})\gamma^\rho(c_V - c_A\gamma^5)(\gamma^\nu p_\nu + \cancel{m_e})\gamma^\lambda(c_V - c_A\gamma^5)\big)$$

$$= tr\big(\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\lambda c_V^{e2} p'_\mu p_\nu - \gamma^\mu\gamma^\rho\gamma^\nu\gamma^\lambda\gamma^5 c_V c_A^e p'_\mu p_\nu$$

$$- \gamma^\mu\gamma^\rho\gamma^5\gamma^\nu\gamma^\lambda c_A c_V^e p'_\mu p_\nu + \gamma^\mu\gamma^\rho\gamma^5\gamma^\nu\gamma^\lambda\gamma^5 c_A^{e2} p'_\mu p_\nu\big)$$

$$= 4(g^{\mu\rho}g^{\nu\lambda} - g^{\mu\nu}g^{\rho\lambda} + g^{\mu\lambda}g^{\rho\nu})c_V^{e2}p'_\mu p_\nu$$

$$+ 8i\varepsilon^{\mu\rho\nu\lambda}c_V c_A^e p'_\mu p_\nu + 4(g^{\mu\rho}g^{\nu\lambda} - g^{\mu\nu}g^{\rho\lambda} + g^{\mu\lambda}g^{\rho\nu})c_A^{e2}p'_\mu p_\nu$$

$$II = tr\big((\gamma_\mu k^\mu + m_\mu)\gamma_\rho(c_V^e - c_A\gamma^5)(\gamma_\nu k'^\nu - m_\mu)\gamma_\lambda(c_V - c_V^e\gamma^5)\big)$$

$$= 4(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\rho\lambda} + g_{\mu\lambda}g_{\rho\nu})c_V^{e2}k^\mu k'^\nu$$

$$+ 8i\varepsilon_{\mu\rho\nu\lambda}c_V c_A^e k^\mu k'^\nu + 4(g_{\mu\rho}g_{\nu\lambda} - g_{\mu\nu}g_{\rho\lambda} + g_{\mu\lambda}g_{\rho\nu})c_A^{e2}k^\mu k'^\nu$$

$$+ 4g_{\rho\lambda}(c_A^{e2} - c_V^{e2})m_\mu^2$$

(V)

$$\frac{1}{4}\sum_{spins}|\mathcal{M}_{Z^0}|^2 = \frac{1}{4}\frac{g_Z^4}{16(q_Z^2-m_Z^2)^2}\Big[4(c_A^{e2}+c_V^{e2})(p'^\rho p^\lambda - g^{\rho\lambda}p'^\nu p_\nu + p'^\lambda p^\rho)$$

$$+ 8i\varepsilon^{\mu\rho\nu\lambda}c_V^e c_A^e p'_\mu p_\nu + \cancel{4g^{\rho\lambda}(c_V^{e2}+c_A^{e2})m_e^2}\Big]\cdot\Big[4(c_A^{\mu 2}+c_V^{\mu 2})$$

$$(k_\rho k'_\lambda - g_{\rho\lambda}k^\mu k'_\mu + k_\lambda k'_\rho) + 8i\varepsilon_{\tilde\mu\rho\tilde\nu\lambda}c_V^\mu c_A^\mu k^{\tilde\mu}k'^{\tilde\nu}$$

$$\cancel{+4g_{\rho\lambda}(c_A^{\mu 2}-c_V^{\mu 2})m_\mu^2}\Big]$$

$\varepsilon_{\mu\nu\rho\sigma}$ is antisymmetric, while the other terms are symmetric. We can also neglect the lepton masses since $m_\ell \ll m_Z$.

$$\frac{1}{4}\sum_{spins}|\mathcal{M}_{Z^0}|^2 = \frac{g_Z^4}{4\cdot 16(q_Z^2-m_Z^2)^2}\Big[16(c_A^{e2}+c_V^{e2})^2(p'^\rho p^\lambda(k_\rho k'_\lambda - g_{\rho\lambda}(k^\mu k'_\nu)$$

$$+ k_\lambda k'_\rho) - g^{\rho\lambda}(p'_2\cdot p_2)(k_\rho k'_\lambda - g_{\rho\lambda}(k\cdot k') + k_\lambda k'_\rho)$$

$$+ p'^\lambda p^\rho(k_\rho k'_\lambda - g_{\rho\lambda}(k\cdot k') + k_\lambda k'_\rho) - \underbrace{64\,\varepsilon^{\mu\rho\nu\lambda}\varepsilon_{\tilde\mu\rho\tilde\nu\lambda}}_{\substack{-64\,\varepsilon^{\lambda\mu\rho\nu}\varepsilon_{\lambda\tilde\mu\rho\tilde\nu} \\ =64\,\varepsilon^{\lambda\rho\mu\nu}\varepsilon_{\lambda\rho\tilde\mu\tilde\nu}}}$$

$$c_V^\mu c_A^\mu c_V^e c_A^e p'_\mu p_\nu k^{\tilde\mu}k'^{\tilde\nu}\Big]$$

$$= \frac{1}{4}\frac{g_Z^4}{16(q_Z^2-m_Z^2)^2}\Big[16(c_A^{e2}+c_V^{e2})^2\big((p'\cdot k)(p\cdot k') - (p'\cdot p)(k\cdot k')$$

$$+(p'\cdot k')(p\cdot k) - (p'\cdot p)(k\cdot k') + 4(p'\cdot p)(k\cdot k')$$

$$-(p'\cdot p)(k\cdot k') + (p'\cdot k')(p\cdot k) - (p'\cdot p)(k\cdot k')$$

$$+(p'\cdot k)(p\cdot k')\big) + 64\cdot 2(\delta^\mu_{\tilde\mu}\delta^\nu_{\tilde\nu} - \delta^\mu_{\tilde\nu}\delta^\nu_{\tilde\mu})$$

$$c_V^{e2}c_A^{e2}p'_\mu p_\nu k^{\tilde\mu}k'^{\tilde\nu}\Big]$$

$$= \frac{g_Z^4}{64(q^2 - M_Z^2)^2} \Big(32 \left(c_A^{e2} + c_V^{e2} \right)^2 \left((p' \cdot k)(p \cdot k') + (p \cdot k)(p' \cdot k') \right)$$

$$+ 128 \left((p' \cdot k)(p \cdot k') - (p \cdot k)(p' \cdot k') \right) c_V^{e2} c_A^{e2} \Big)$$

$$= \frac{g_Z^4}{2(q^2 - M_Z^2)^2} \Big(\left(\left(c_V^{e2} + c_A^{e2} \right)^2 + 4 c_V^{e2} c_A^{e2} \right) (p' \cdot k)(p \cdot k')$$

$$+ \left(\left(c_V^{e2} + c_A^{e2} \right) - 4 c_V^{e2} c_A^{e2} \right) (p' \cdot k')(p \cdot k) \Big)$$

$$\frac{1}{4}\sum_{spins} |\mathcal{M}_Z \mathcal{M}_\gamma^*| = \bar{v}^{s'}(p')\left(\frac{-ig_Z}{2}\gamma^\mu(c_V^e - c_A^e\gamma^5)\right)u^s(p)\left(\frac{-ig_{\mu\nu}}{q^2-m_Z^2}\right)\bar{u}^r(k)\left(\frac{-ig_Z}{2}\gamma^\nu(c_V^\mu - c_A^\mu\gamma^5)\right)v^{r'}(k')$$

$$\cdot\, \bar{u}^s(p)(ie\gamma^\rho)v^{s'}(p')\left(\frac{ig_{\rho\sigma}}{q^2}\right)\bar{v}^{r'}(k')(ie\gamma^\sigma)u^r(k)$$

$$= \frac{1}{4}\frac{g_Z^2\, e^2}{4q^2(q^2-m_Z^2)}\left(\bar{v}^{s'}(p')\gamma^\mu(c_V^e - c_A^e\gamma^5)u^s(p)\bar{u}^s(p)\gamma^\rho v^{s'}(p')\right)$$

$$\cdot\left(\bar{u}^r(k)\gamma_\mu(c_V^\mu - c_A^\mu\gamma^5)v^{r'}(k')\bar{v}^{r'}(k')\gamma_\rho u^r(k)\right)$$

$$= \frac{g_Z^2 e^2}{16 q^2(q^2-m_Z^2)}\underbrace{\mathrm{tr}\left((\not p' - m_e)\gamma^\mu(c_V^e - c_A^e\gamma^5)(\not p + m_e)\gamma^\rho\right)}_{\mathrm{I}}$$

$$\underbrace{\mathrm{tr}\left((\not k + m_\mu)\gamma_\mu(c_V^\mu - c_A^\mu\gamma^5)(\not k' - m_\mu)\gamma_\rho\right)}_{\mathrm{II}}$$

$$\mathrm{I} = \mathrm{tr}\left((\gamma^\sigma p'_\sigma - m_e)\gamma^\mu(c_V^e - c_A^e\gamma^5)(\gamma^\lambda p_\lambda + m_e)\gamma^\rho\right)$$

$$= \mathrm{tr}\left[(\gamma^\sigma\gamma^\mu p'_\sigma c_V^e - \gamma^\sigma\gamma^\mu\gamma^5 p'_\sigma c_A^e \cancel{-\gamma^\mu\gamma^5 m_e - \gamma^\mu c_V^e m_e})(\gamma^\lambda\gamma^\rho p_\lambda \cancel{+\gamma^\rho m_e})\right]$$

$m_e \ll m_Z$

$$= \mathrm{tr}\left[\gamma^\sigma\gamma^\mu\gamma^\lambda\gamma^\rho p'_\sigma p_\lambda c_V^e - \gamma^\sigma\gamma^\mu\gamma^5\gamma^\lambda\gamma^\rho p'_\sigma p_\lambda c_A^e\right]$$

$$= 4\left(g^{\sigma\mu}g^{\lambda\rho} - g^{\sigma\lambda}g^{\mu\rho} + g^{\sigma\rho}g^{\mu\lambda}\right)p'_\sigma p_\lambda c_V^e + 4i\varepsilon^{\sigma\mu\lambda\rho}p'_\sigma p_\lambda c_A^e$$

$$\mathrm{II} = \mathrm{tr}\left[(\gamma_\sigma k^\sigma + m_\mu)\gamma_\mu(c_V^\mu - c_A^\mu\gamma^5)(\gamma_\lambda k'^\lambda - m_\mu)\gamma_\rho\right]$$

$m_\mu \ll m_Z$

$$= \mathrm{tr}\left[(\gamma_\sigma\gamma_\mu k^\sigma c_V^e - \gamma_\sigma\gamma_\mu\gamma^5 k^\sigma c_A^e)(\gamma_\lambda\gamma_\rho k'^\lambda)\right]$$

$$= \mathrm{tr}\left[\gamma_\sigma\gamma_\mu\gamma_\lambda\gamma_\rho k^\sigma k'^\lambda c_V^e - \gamma_\sigma\gamma_\mu\gamma^5\gamma_\lambda\gamma_\rho k^\sigma k'^\lambda c_A^e\right]$$

$$= 4\left(g_{\sigma\mu}g_{\lambda\rho} - g_{\sigma\lambda}g_{\mu\rho} + g_{\sigma\rho}g_{\mu\lambda}\right)k^\sigma k'^\lambda c_V^e + 4i\varepsilon_{\sigma\mu\lambda\rho}k^\sigma k'^\lambda c_A^e$$

$$\frac{1}{4}\sum_{spins} |\mathcal{M}_{Z}\mathcal{M}_\gamma^*| = \frac{g_Z^2 e^2}{16 q^2 (q^2 - m_Z^2)} \Big(16\, c_V^{e\,2} \left(p'^\mu p^\rho - (p'\cdot p) g^{\mu\rho} + p'^\rho p^\mu \right)$$

$$\cdot \left(k_\mu k'_\rho - (k\cdot k') g_{\mu\rho} + k_\rho k'_\mu \right) + 16\, \underbrace{\varepsilon^{\tilde\sigma\mu\tilde\lambda\rho}}_{\substack{= -\varepsilon^{\mu\tilde\sigma\tilde\lambda\rho} \\ = -\varepsilon^{\rho\mu\tilde\sigma\tilde\lambda}}} \underbrace{\varepsilon_{\sigma\mu\lambda\rho}}_{\substack{= -\varepsilon_{\mu\sigma\lambda\rho} \\ = -\varepsilon_{\rho\mu\sigma\lambda}}}$$

$$p'_{\tilde\sigma} p_{\tilde\lambda} k^\sigma k'^\lambda \, c_A^{e\,2} \Big)$$

$$= \frac{g_Z^2 e^2}{16 q^2 (q^2 - m_Z^2)} \Big(16\, c_V^{e\,2} \big(\underline{(p'\cdot k)(p\cdot k')} - (p'\cdot p)(k\cdot k')$$

$$+ (p'\cdot k')(p\cdot k) - (p'\cdot p)(k\cdot k') + 4 (p'\cdot p)(k\cdot k')$$

$$- (p'\cdot p)(k\cdot k') + (p'\cdot k')(p\cdot k) - (k\cdot k')(p\cdot p') + \underline{(p'\cdot k)(p\cdot k')} \big)$$

$$- 16 \cdot \varepsilon^{\rho\mu\tilde\sigma\tilde\lambda} \varepsilon_{\rho\mu\sigma\lambda}\, p'_{\tilde\sigma} p_{\tilde\lambda} k^\sigma k'^\lambda \, c_A^{e\,2} \Big)$$

$$= \frac{32\, g_Z^2 e^2}{16 q^2 (q^2 - m_Z^2)} \Big(c_V^{e\,2} \big((p'\cdot k)(p\cdot k') + (p'\cdot k')(p\cdot k) \big)$$

$$- \left(\delta^{\tilde\sigma}_{\sigma} \delta^{\tilde\lambda}_{\lambda} - \delta^{\tilde\sigma}_{\lambda} \delta^{\tilde\lambda}_{\sigma} \right) p'_{\tilde\sigma} p_{\tilde\lambda} k^\sigma k'^\lambda \, c_A^{e\,2} \Big)$$

$$= \frac{2\, g_Z^2 e^2}{q^2 (q^2 - m_Z^2)} \Big(c_V^{e\,2} \big((p'\cdot k)(p\cdot k') + (p'\cdot k')(p\cdot k) \big)$$

$$+ \big((p'\cdot k)(p\cdot k') - (p'\cdot k')(p\cdot k) \big) c_A^{e\,2} \Big)$$

$$= \frac{2\, g_Z^2 e^2}{q^2 (q^2 - m_Z^2)} \Big((p'\cdot k)(p\cdot k') \left(c_V^{e\,2} + c_A^{e\,2} \right) + (p'\cdot k')(p\cdot k) \left(c_V^{e\,2} - c_A^{e\,2} \right) \Big)$$

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}_\gamma \mathcal{M}_{Z^0}^*| = \frac{1}{4}\bar{v}^{s'}(p')(-ie\gamma^\mu)u^s(p)\left(\frac{-ig_{\mu\nu}}{q^2}\right)\bar{u}^r(k)(-ie\gamma^\nu)v^{r'}(k')$$

$$\cdot \bar{u}^s(p)\left(\frac{ig_Z}{2}\gamma^\rho(c_V - c_A\gamma^5)v^{s'}(p')\right)\left(\frac{ig_{\rho\lambda}}{q^2-m_Z^2}\right)\bar{v}^{r'}(k')\left(\frac{ig_Z}{2}\gamma^\lambda(c_V - c_A\gamma^5)\right)u^r(k)$$

$$= \frac{1}{4}\bar{v}^{s'}(p')(\cancel{i}e\gamma^\mu)u^s(p)\left(\frac{\cancel{i}}{q_\gamma^2}\right)\bar{u}^r(k)(\cancel{i}e\gamma_\mu)v^{r'}(k')$$

$$\cdot \bar{u}^s(p)\left(\frac{\cancel{i}g_Z}{2}\gamma^\rho((c_V - c_A\gamma^5)v^{s'}(p')\right)\left(\frac{\cancel{i}}{q_Z^2-m_Z^2}\right)\bar{v}^{r'}(k')\left(\frac{\cancel{i}g_Z}{2}\gamma_\lambda(c_V - c_A\gamma^5)\right)u^r(k)$$

$$= \frac{1}{4}\frac{e^2 g_Z^2}{q_\gamma^2(q_Z^2-m_Z^2)4}\underbrace{\bar{v}^{s'}(p')\gamma^\mu u^s(p)\bar{u}^s(p)\gamma^\rho(c_V^e - c_A^e\gamma^5)v^{s'}(p')}_{\mathrm{I}}$$

$$\cdot \underbrace{\bar{u}^r(k)\gamma_\mu v^{r'}(k')\bar{v}^{r'}(k')\gamma_\lambda(c_V^\mu - c_A^\mu\gamma^5)u^r(k)}_{\mathrm{II}},$$

$$\mathrm{I} = \mathrm{tr}\left((\cancel{p}' - m_e)\gamma^\mu(\cancel{p}+m_e)\gamma^\rho((c_V^e - c_A^e\gamma^5)\right)$$

$$\mathrm{II} = \mathrm{tr}\left((\cancel{k}+m_\mu)\gamma_\mu(\cancel{k}'-m_\mu)\gamma_\rho\,(c_V^\mu - c_A^\mu\gamma^5)\right)$$

$$\mathrm{I} = \mathrm{tr}\left[(\gamma^\sigma p'_\sigma - m_e)\gamma^\mu(\gamma^\lambda p_\lambda + m_e)\gamma^\rho(c_V^e - c_A^e\gamma^5)\right]$$

$$= \mathrm{tr}\left[(\gamma^\sigma\gamma^\mu\gamma^\lambda p'_\sigma p_\lambda + \gamma^\sigma\gamma^\mu p'_\sigma m_e - \gamma^\mu\gamma^\lambda p_\lambda m_e + \gamma^\mu m_e^2)(\gamma^\rho c_V^e - c_A^e\gamma^\rho\gamma^5)\right]$$

$$= \mathrm{tr}\left[\gamma^\sigma\gamma^\mu\gamma^\lambda\gamma^\rho p'_\sigma p_\lambda c_V^e - \gamma^\sigma\gamma^\mu\gamma^\lambda\gamma^\rho\gamma^5 p'_\sigma p_\lambda c_A^e + \gamma^\sigma\cancel{\gamma^\mu\gamma^\rho p'_\sigma m_e c_V^e}\right.$$

$$\cancel{-\gamma^\sigma\gamma^\mu\gamma^\rho\gamma^5 p'_\sigma m_e c_A^e} - \gamma^\mu\gamma^\lambda\cancel{\gamma^\rho p_\lambda m_e c_V^e} + \gamma^\mu\gamma^\lambda\cancel{\gamma^\rho\gamma^5 p_\lambda m_e c_A^e}$$

$$\left. + \gamma^\mu\cancel{\gamma^\rho m_e^2 c_V^e} - \gamma^\mu\cancel{\gamma^\rho\gamma^5 m_e^2 c_A^e}\right]$$

$m_e \ll m_Z$

$$\overset{\searrow}{=} 4\left(g^{\sigma\mu}g^{\lambda\rho} - g^{\sigma\lambda}g^{\mu\rho} + g^{\sigma\rho}g^{\mu\lambda}\right) p'_\sigma p_\lambda C_V^e + 4i\varepsilon^{\sigma\mu\lambda\rho} p'_\sigma p_\lambda C_A^e$$

$$= 4\left(p'^\mu p^\rho - p'^\lambda g^{\mu\rho} p_\lambda + p'^\rho p^\mu\right) C_V^e + 4i\varepsilon^{\sigma\mu\lambda\rho} p'_\sigma p_\lambda C_A^e$$

$$\mathrm{II} = \mathrm{tr}\left((\gamma_\delta k^\delta + m_\mu)\gamma_\mu(\gamma_\sigma k'^\sigma - m_\mu)\gamma_\rho (C_V^e - C_A^e\gamma^5)\right)$$

$$= \mathrm{tr}\left[\left(\gamma_\delta\gamma_\mu\gamma_\sigma k^\delta k'^\sigma - \cancel{\gamma_\delta\gamma_\mu k^\delta m_\mu} + \cancel{\gamma_\mu\gamma_\sigma k'^\sigma m_\mu} - \cancel{\gamma_\mu m_\mu^2}\right)\gamma_\rho(C_V^e - C_A^e\gamma^5)\right]$$

$$= \mathrm{tr}\left[\gamma_\delta\gamma_\mu\gamma_\sigma\gamma_\rho k^\delta k'^\sigma C_V^e - \gamma_\delta\gamma_\mu\gamma_\sigma\gamma_\rho\gamma^5 k^\delta k'^\sigma C_A^e\right]$$

$$= 4\left(g_{\delta\mu}g_{\sigma\rho} - g_{\delta\sigma}g_{\mu\rho} + g_{\delta\rho}g_{\mu\sigma}\right)k^\delta k'^\sigma C_V^e + 4i\varepsilon_{\delta\mu\beta\rho} k^\delta k'^\beta C_A^e$$

$$= 4\left(k_\mu k'_\rho C_V^e - k_\sigma k'^\sigma g_{\mu\rho} C_V^e + k_\rho k'_\mu C_V^e\right) + 4i\varepsilon_{\delta\mu\beta\rho} k^\delta k'^\beta C_A^e$$

$$\sum_{\text{spins}} |\mathcal{M}_\gamma \mathcal{M}_{Z^0}^*| = \frac{e^2 g_Z^2}{16 q^2(q^2 - m_Z^2)}\left[4\left(p'^\mu p^\rho - p'^\lambda p_\lambda g^{\mu\rho} + p'^\rho p^\mu\right)C_V^e + 4i\varepsilon^{\tilde\sigma\mu\tilde\lambda\rho} p'_{\tilde\sigma} p_{\tilde\lambda} C_A^e\right]$$

$$\cdot\left[4C_V^e\left(k_\mu k'_\rho - k_\sigma k'^\sigma g_{\mu\rho} + k_\rho k'_\mu\right) + 4i\varepsilon_{\delta\mu\beta\rho} k^\delta k'^\beta C_A^e\right]$$

$$= \frac{1}{4}\frac{e^2 g_Z^2}{4q^2(q^2 - m_Z^2)}\Big[16 C_V^{e\,2}\left(p'^\mu p^\rho - (p'\cdot p)g^{\mu\rho} + p'^\rho p^\mu\right)$$

$$\left(k_\mu k'_\rho - (k\cdot k')g_{\mu\rho} + k_\rho k'_\mu\right) - 16\,\varepsilon^{\tilde\sigma\mu\tilde\lambda\rho}\varepsilon_{\delta\mu\beta\rho}$$

$$p'_{\tilde\sigma} p_{\tilde\lambda} k^\delta k'^\beta C_A^{e\,2}\Big]$$

$$= -\,''\,- \Big[16\,C_V^{e2}\big((p'\cdot k)(p\cdot k') - (p'\cdot p)(k\cdot k') + (p'\cdot k')(p\cdot k)$$

$$- (p'\cdot p)(k'\cdot k) + 4(p'\cdot p)(k'\cdot k) - (p'\cdot p)(k\cdot k')$$

$$+ (p'\cdot k')(p\cdot k) - (k\cdot k')(p\cdot p') + (p'\cdot k)(p\cdot k')\big)$$

$$- 16\,\varepsilon^{\rho\mu\tilde\sigma\tilde\lambda}\varepsilon_{\rho\mu\delta\beta}\, p'_{\tilde\sigma} p_{\tilde\lambda} k^{\delta} k'^{\beta} C_A^{e2}\Big]$$

$$\varepsilon^{\tilde\sigma\mu\tilde\lambda\rho} = -\varepsilon^{\mu\tilde\sigma\tilde\lambda\rho} = -\varepsilon^{\rho\mu\tilde\sigma\tilde\lambda}$$

$$\varepsilon_{\delta\mu\beta\rho} = -\varepsilon_{\mu\delta\beta\rho} = -\varepsilon_{\rho\mu\delta\beta}$$

$$= -\,''\,- \Big[16\,C_V^{e2}\big(2(p'\cdot k)(p\cdot k') + 2(p'\cdot k')(p\cdot k)\big)$$

$$+ 16\cdot 2\big(\delta^{\tilde\sigma}_{\delta}\delta^{\tilde\lambda}_{\beta} - \delta^{\tilde\sigma}_{\beta}\delta^{\tilde\lambda}_{\delta}\big) p'_{\tilde\sigma} p_{\tilde\lambda} k^{\delta} k'^{\beta} C_A^{e2}\Big]$$

$$= -\,''\,- \Big[32\,C_V^{e2}\big((p'\cdot k)(p\cdot k') + (p'\cdot k')(p\cdot k)\big)$$

$$+ 32\cdot\big((p'\cdot k)(p\cdot k') - (p'\cdot k')(p\cdot k)\big)C_A^{e2}\Big]$$

$$= -\,''\,- 32\Big[(p'\cdot k)(p\cdot k')\big(C_V^{e2} + C_A^{e2}\big) + (p'\cdot k')(p\cdot k)\big(C_V^{e2} - C_A^{e2}\big)\Big]$$

$$= \frac{1}{4}\,\frac{\cancel{32}\,8\,e^2 g_z^2}{\cancel{4}\,q^2\,(q^2 - M_Z^2)}\Big((p'\cdot k)(p\cdot k')\big(C_V^{e2} + C_A^{e2}\big) + (p'\cdot k')(p\cdot k)\big(C_V^{e2} - C_A^{e2}\big)\Big)$$

Total amplitude:

$$|\mathcal{M}| = |\mathcal{M}_\gamma|^2 + |\mathcal{M}_{Z^0}|^2 + |\mathcal{M}_\gamma \mathcal{M}_{Z^0}^*| + |\mathcal{M}_{Z^0}\mathcal{M}_\gamma^*|$$

$$= \frac{8e^4}{q^4}\Big((p'\cdot k)(p\cdot k') + (p'\cdot k')(p\cdot k)\Big)$$

$$+ \frac{2g_Z^4}{2(q^2-m_Z^2)^2}\Big[(p'\cdot k)(p\cdot k')\cdot\Big(\big((c_V^e)^2+(c_A^e)^2\big)^2 + 4(c_A^e)^2(c_V^e)^2\Big)$$
$$+ (p'\cdot k')(p\cdot k)\Big(\big((c_V^e)^2+(c_A^e)^2\big)^2 - 4(c_A^e)^2(c_V^e)^2\Big)\Big]$$

$$+ \frac{2e^2g_Z^2}{q^2(q^2-m_Z^2)}\Big((p'\cdot k)(p\cdot k')\big((c_V^e)^2+(c_A^e)^2\big) + (p'\cdot k')(p\cdot k)\big((c_V^e)^2-(c_A^e)^2\big)\Big)$$

$$+ \frac{2e^2g_Z^2}{q^2(q^2-m_Z^2)}\Big((p'\cdot k)(p\cdot k')\big((c_V^e)^2+(c_A^e)^2\big) + (p'\cdot k')(p\cdot k)\big((c_V^e)^2-(c_A^e)^2\big)\Big)$$

$$= \frac{8e^4}{q^4}\Big((p'\cdot k)(p\cdot k') + (p'\cdot k')(p\cdot k)\Big)$$

$$+ \frac{2g_Z^4}{2(q^2-m_Z^2)^2}\Big[\Big(\big((c_V^e)^2+(c_A^e)^2\big)^2 + 4(c_A^e)^2(c_V^e)^2\Big)(p'\cdot k)(p\cdot k')$$
$$+\Big(\big((c_V^e)^2+(c_A^e)^2\big)^2 - 4(c_A^e)^2(c_V^e)^2\Big)(p'\cdot k')(p\cdot k)\Big]$$

$$+ \frac{4g_Z^2e^2}{q^2(q^2-m_Z^2)}\Big[\big((c_V^e)^2+(c_A^e)^2\big)(p'\cdot k)(p\cdot k') + \big((c_V^e)^2-(c_A^e)^2\big)(p'\cdot k')(p\cdot k)\Big]$$

$$|\mathcal{M}| = (p'\cdot k)(p\cdot k')\left[\frac{8e^4}{q^4} + \frac{2g_z^4}{(q^2-m_z^2)^2}\left((c_V^{e2}+c_A^{e2})^2 - 4c_A^{e2}c_V^{e2}\right) + \frac{16e^2g_z^2}{q^2(q^2-m_z^2)}(c_V^{e2}+c_A^{e2})\right] + (p\cdot k)(p'\cdot k')\left[\frac{8e^4}{q^4} + \frac{2g_z^4}{(q^2-m_z^2)^2}\left((c_V^{e2}+c_A^{e2})^2 + 4c_V^{e2}c_A^{e2}\right) + \frac{16g_z^2e^2}{q^2(q^2-m_z^2)}(c_V^{e2}-c_A^{e2})\right]$$

$$(p'\cdot k)(p\cdot k') = (E^2 + E|\vec{k}|\cos\theta)^2$$
$$= (E^2 + E\sqrt{E^2-\underbrace{m_\mu^2}_{\approx 0}}\cos\theta)^2 \cong (E^2+E^2\cos\theta)^2$$
$$= E^4(1+\cos\theta)^2 = E^4(1+2\cos\theta+\cos^2\theta)$$

$$(p\cdot k)(p'\cdot k') = (E^2 - E|\vec{k}|\cos\theta)^2$$
$$\cong (E^2 - E^2\cos\theta)^2$$
$$= E^4(1-\cos\theta)^2 = E^4(1+\cos^2\theta-2\cos\theta)$$

$$q^2 = 4E^2 = s$$
$$s = E_{CM}^2 = (2E)^2 = 4E^2$$
$$s^2 = 16E^4$$

$$|\mathcal{M}| = \frac{1}{2}\frac{8e^4\cancel{E^4}}{\cancel{16E^4}}(2+2\cos^2\theta) + \frac{g_z^4}{2(s-m_z^2)^2}\left[\left((c_V^{e2}+c_A^{e2})^2+4c_A^{e2}c_V^{e2}\right)E^4(1+\cos\theta)^2 + \left((c_V^{e2}+c_A^{e2})^2 - 4c_V^{e2}c_A^{e2}\right)E^4(1-\cos\theta)^2\right] + \frac{4g_z^2e^2}{s(s-m_z^2)}\left[(c_V^{e2}+c_A^{e2})E^4(1+\cos\theta)^2 + (c_V^{e2}-c_A^{e2})E^4(1-\cos\theta)^2\right]$$

$$\mathbb{I} = (C_V^{e2}+C_A^{e2})(1+2\cos\Theta+\cos^2\Theta)+(C_V^{e2}-C_A^{e2})(1-2\cos\Theta+\cos^2\Theta)$$

$$= C_V^{e2} + \cancel{2C_V^{e2}\cos\Theta} + C_V^{e2}\cos^2\Theta \cancel{+C_A^{e2}} + 2\cos\Theta C_A^{e2} + \cancel{\cos^2\Theta C_A^{e2}}$$

$$+ C_V^{e2} - \cancel{2C_V^{e2}\cos\Theta} + C_V^{e2}\cos^2\Theta \cancel{-C_A^{e2}} + 2\cos\Theta C_A^{e2} - \cancel{C_A^{e2}\cos^2\Theta}$$

$$= 2C_V^{e2} + 2C_V^{e2}\cos^2\Theta + 4\cos\Theta C_A^{e2}$$

$$= 2C_V^{e2}(1+\cos^2\Theta) + 4C_A^{e2}\cos\Theta$$

$$\mathrm{I} = \left((C_V^{e2}+C_A^{e2})^2+4C_A^{e2}C_V^{e2}\right)(1+2\cos\Theta+\cos^2\Theta)$$

$$+\left((C_V^{e2}+C_A^{e2})^2-4C_A^{e2}C_V^{e2}\right)(1-2\cos\Theta+\cos^2\Theta)$$

$$= 2(C_V^{e2}+C_A^{e2})^2 + 2(C_V^{e2}+C_A^{e2})^2\cos^2\Theta + 16C_A^{e2}C_V^{e2}\cos\Theta$$

$$\Rightarrow |\mathcal{M}| = e^4(1+\cos^2\Theta) + \frac{g_Z^4 e^4}{(s-M_Z^2)^2}\left[(1+\cos^2\Theta)(C_V^{e2}+C_A^{e2})^2 + 8C_A^{e2}C_V^{e2}\cos\Theta\right]$$

$$+\frac{8g_Z^2e^2e^4}{s(s-M_Z^2)}\left[C_V^{e2}(1+\cos^2\Theta)+2C_A^{e2}\cos\Theta\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{1}{2E_A 2E_B|v_A-v_B|}\;\frac{|\vec{p}_1|}{(2\pi)^2 4E_{CM}}\;|\mathcal{M}(p_A p_B \to p_1, p_2)|^2$$

We have $E_A = E_B = \frac{E_{CM}}{2}$, $|v_A - v_B| = 2$

$|\vec{p}_1| = |\vec{k}| = \sqrt{E^2 - m_\mu^2} \triangleq E = \frac{E_{CM}}{2}$

$$\Rightarrow 2E_A 2E_B |v_A - v_B| = E_{CM}^2 \cdot 2$$

$$\Longrightarrow \left(\frac{d\sigma}{d\Omega}\right)_{CM} = \frac{1}{2E_{CM}^2}\,\frac{\overset{=\frac{1}{2}E_{CM}}{E}}{4\pi^2\, 4E_{CM}} \cdot \Big[e^4\,(1+\cos^2\Theta)$$

$$+ \frac{g_Z^4\, \overset{=\frac{s^2}{16}}{E^4}}{(s-m_Z)^2}\Big((1+\cos^2\Theta)\,(C_V^{e2}+C_A^{e2}) + 8\,C_A^{e2}C_V^{e2}\cos\Theta\Big)$$

$$+ \frac{8\,g_Z^2\,e^2\,\overset{=\frac{s^2}{16}}{E^4}}{s\,(s-m_Z^2)^2}\Big(C_V^{e2}\,(1+\cos^2\Theta) + 2\,C_A^{e2}\cos\Theta\Big)\Big]$$

$$= \frac{e^4}{64\pi^2 s}\Big[(1+\cos^2\Theta) + \frac{s^2}{16\,(s-m_Z^2)^2\,(s_W c_W)^4}\Big($$

$$\alpha = \frac{e^2}{4\pi\hbar c}$$

$$\Rightarrow \frac{e^4}{64\pi^2} = \frac{\alpha^2(\hbar c)^2}{4}$$

$$(1+\cos^2\Theta)\,(C_V^{e2}+C_A^{e2}) + 8\,C_A^{e2}C_V^{e2}\cos\Theta\Big)$$

$$+ \frac{s}{2\,(s-m_Z^2)\,(s_W c_W)^2}\Big(C_V^{e2}\,(1+\cos^2\Theta) + 2\,C_A^{e2}\cos\Theta\Big)\Big]$$

$$\boxed{\begin{aligned} S_W &= \sin\Theta_W \\ C_W &= \cos\Theta_W \end{aligned}}$$

$\hbar = c = 1.$

$$\sigma = \int_0^{2\pi}\int_0^{\pi} \frac{1}{4}\frac{\alpha^2}{s}\left\{\left[1+\cos^2\theta + \frac{s^2}{4(\sin\theta_W\cos\theta_W)^4(s-m_Z^2)^2 4}\left[(1+\cos^2\theta)((c_V^e)^2+(c_A^e)^2)^2 + 8 (c_V^e)^2 (c_A^e)^2 \cos\theta\right]\right.\right.$$

$$\left.+\frac{2s}{(\sin\theta_W\cos\theta_W)^2(s-m_Z^2)}\left((c_V^e)^2(1+\cos^2\theta) + 2 (c_A^e)^2\cos\theta\right)\right]$$

$$\left.\sin\theta \, d\theta \, d\varphi\right\}$$

$$= \frac{2\pi\alpha^2}{4s}\left[-\cos\theta - \frac{1}{3}\cos^3\theta + \frac{s^2}{16(s_W c_W)^4(s-m_Z^2)^2}\left((-\cos\theta - \frac{1}{3}\cos^3\theta)((c_V^e)^2+(c_A^e)^2)^2\right.\right.$$

$$\left.-4 (c_V^e)^2 (c_A^e)^2 \cos^2\theta\right) + \frac{-s}{2(s_W c_W)^2(s-m_Z^2)}\left((c_V^e)^2(\cos\theta - \frac{1}{3}\cos^3\theta)\right.$$

$$\left.\left.-(c_A^e)^2\cos^2\theta\right)\right]\Bigg|_0^{\pi}$$

$$= \frac{\pi\alpha^2}{2s}\left[\underbrace{(\cos\pi - \cos 0)}_{-2} - \frac{1}{3}\underbrace{(\cos^3\pi - \cos^3 0)}_{-2} + \frac{s^2}{16(s_W c_W)^4(s-m_Z^2)^2}\left[\left(2+\frac{2}{3}\right)\left((c_V^e)^2+(c_A^e)^2\right)^2\right.\right.$$

$$\left.+4 (c_V^e)^2 (c_A^e)^2 \cancel{(\underset{1}{\cos^2\pi} - \underset{1}{\cos^2 0})}\right] + \frac{s}{(s_W c_W)^2(s-m_Z^2)16}\left(\left(2+\frac{2}{3}\right)(c_V^e)^2 - \cancel{((-1)^2-(-1)^2)}(c_A^e)^2\right)\Bigg]$$

$$= \frac{\cancel{8}\,4\pi\alpha^2}{3\,\cancel{2}\,s}\left[\cancel{\frac{8}{3}} + \frac{s^2}{16(s_W c_W)^2(s-m_Z^2)^2}\cancel{\frac{8}{3}}\left((c_V^e)^2+(c_A^e)^2\right)^2\right.$$

$$\left.+\frac{s}{2(s_W c_W)^2(s-m_Z^2)}\cancel{\frac{8}{3}}(c_V^e)^2\right]$$

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$$\sigma_F = \frac{\pi\alpha^2}{2s}\Bigg[-\cos\theta - \frac{1}{3}\cos^3\theta + \frac{s^2}{16\left((c_w s_w)^4(s-M_Z^2)\right)}\Bigg(\left(-\cos\theta - \frac{1}{3}\cos^3\theta\right)$$

$$\left(C_V^{e2}+C_A^{e2}\right)^2 - 4C_V^{e2}C_A^{e2}\cos^2\theta\Bigg) + \frac{s}{2\left((c_w s_w)^2(s-M_Z^2)\right)}$$

$$\left(C_V^{e2}\left(-\cos\theta - \frac{1}{3}\cos^3\theta\right) - C_A^{e2}\cos^2\theta\right)\Bigg]\Bigg|_0^{\pi/2}$$

$$= \frac{\pi\alpha^2}{2s}\Bigg[-(\underbrace{\cos\tfrac{\pi}{2} - \cos 0}_{-1}) - \frac{1}{3}(\underbrace{\cos^3\tfrac{\pi}{2} - \cos^3 0}_{-1}) + \frac{s^2}{16(c_w s_w)^4(s-M_Z^2)^2}$$

$$\Bigg(\left(1+\tfrac{1}{3}\right)\left(C_V^{e2}+C_A^{e2}\right)^2 + 4C_V^{e2}C_A^{e2}(\underbrace{\cos^2\pi/2 - \cos^2 0}_{-1})\Bigg)$$

$$+ \frac{s}{2(c_w s_w)^2(s-M_Z^2)}\left(C_V^{e2}\left(1+\tfrac{1}{3}\right) + C_A^{e2}\right)\Bigg]$$

$$= \frac{\cancel{4}^{\,2}\,\pi\alpha^2}{3\,\cancel{2}\,s}\Bigg[\cancel{\tfrac{4}{3}}^{\,1} + \frac{s^2}{16(c_w s_w)^4(s-M_Z^2)^2}\left(\cancel{\tfrac{4}{3}}\left(C_V^{e2}+C_A^{e2}\right)^2 + \cancel{4}^{\,1/3}\,C_V^{e2}C_A^{e2}\right)$$

$$+ \frac{s}{2(c_w s_w)^2(s-M_Z^2)}\left(\cancel{\tfrac{4}{3}}C_V^{e2} + \frac{3}{4}C_A^{e2}\right)\Bigg]$$

$$\sigma_B = \frac{\pi\alpha^2}{2s}\Bigg[-(\underbrace{\cos\pi - \cos\pi/2}_{-1}) - \frac{1}{3}(\underbrace{\cos^3\pi - \cos^3\pi/2}_{-1})$$

$$+ \frac{s^2}{16(c_w s_w)^4(s-M_Z^2)^2}\Bigg(\left(1+\tfrac{1}{3}\right)\left(C_V^{e2}+C_A^{e2}\right)^2 - 4C_V^{e2}C_A^{e2}(\underbrace{\cos^2\pi - \cos^2\pi/2}_{1})\Bigg)$$

$$+ \frac{s}{2(c_w s_w)^2(s-M_Z^2)}\left(C_V^{e2}\left(1+\tfrac{1}{3}\right) - C_A^{e2}\right)\Bigg]$$

$$= \frac{4}{3}\frac{\pi\alpha^2}{2s}\Bigg[\cancel{\tfrac{4}{3}} + \frac{s^2}{16\left((c_w s_w)^4(s-M_Z^2)\right)^2}\left(\cancel{\tfrac{4}{3}}\left(\left(C_V^{e2}+C_A^{e2}\right)^2 - \cancel{4}^{\,1/3}\,C_A^{e2}C_V^{e2}\right)\right)$$

$$+ \frac{s}{2\left((c_w s_w)^2(s-M_Z^2)\right)}\left(\cancel{\tfrac{4}{3}}C_V^{e2} - \frac{3}{4}C_A^{e2}\right)\Bigg]$$