ACTIVIDAD DE APRENDIZAJE 4. Taller de integrales múltiples

ACTIVIDAD 4

Calculo Multivariado

UNIPANAMERICANA COMPENSAR

JONATHAN CASTILLO GRAJALES

SEMESTRE VII

MODULO I

FACULTAD DE INGENIERÍA

TECNOLOGÍA EN ANÁLISIS Y DESARROLLO

DE SISTEMAS DE INFORMACIÓN

Marzo de 2020

INTEGRALES PARCIALES

1 . Evalúe la integral parcial definida dada.

1. $\int_{-1}^{3} (6xy - 5e^{y})\, dx$

Solución:

$\int_{-1}^{3} 6xy - 5e^{y} dx$

Se aplica la regla de la suma

$= -\int_{-1}^{3} 5e^{y} dx + \int_{-1}^{3} 6yxdx$

$\int_{-1}^{3} 5e^{y} dx$

Integral de una constante

$= \left[5e^{y}x\right]_{-1}^{3}$

$= 20e^{y}$

$\int_{-1}^{3} 6yxdx$

Se saca la constante

$= 6y \cdot \int_{-1}^{3} xdx$

Se aplica la regla de la potencia

$= 6y\left[\frac{x^{1+1}}{1+1}\right]_{-1}^{3}$

Se simplifica

$$= 6y\left[\frac{x^2}{2}\right]_{-1}^{3}$$

$$= 6y \cdot 4$$

$$= 24y$$

$$= -20e^y + 24y$$

2. $\int_1^2 \tan xy\, dy$

Solución:

$$= \int_1^2 \frac{\sin(yx)}{\cos(yx)} dy$$

Se aplica la integración por sustitución

$$= \int_{\cos(x)}^{\cos(2x)} -\frac{1}{xu} du$$

Se saca la constante

$$= -\frac{1}{x} \cdot \int_{\cos(x)}^{\cos(2x)} \frac{1}{u} du$$

Se aplica la regla de integración

$$= -\frac{1}{x}[\ln|u|]_{\cos(x)}^{\cos(2x)}$$

$$= -\frac{1}{x}\left(\frac{1}{2}\ln\left(\cos^2(2x)\right) - \frac{1}{2}\ln\left(\cos^2(x)\right)\right)$$

3. $\int_1^{3x} x^3 e^{xy}\, dy$

Solución:

Sacar la constante

$$= x^3 \cdot \int_1^{3x} e^{xy} dy$$

Se aplica la sustitución por integración u = xy

$$= x^3 \cdot \int_x^{3x^2} \frac{e^u}{x} du$$

Se saca la constante

$$= x^3 \frac{1}{x} \cdot \int_x^{3x^2} e^u du$$

Se aplica la regla de integración

$$= x^3 \frac{1}{x} \left[e^u \right]_x^{3x^2}$$

$$= \frac{1 \cdot x^3 \left[e^u \right]_x^{3x^2}}{x}$$

$$= \frac{x^3 \left[e^u \right]_x^{3x^2}}{x}$$

$$= x^2 \left[e^u \right]_x^{3x^2}$$

$$= x^2 \left(e^{3x^2} - e^x \right)$$

4. $\int_0^{\pi/4}\int_0^{\cos x}(1 + 4y\tan^2 x)\, dy dx$

Solución:

$$\int_0^{\cos(x)} 1 + 4y\tan^2(x)dy$$

$$= \int_0^{\cos(x)} 1dy + \int_0^{\cos(x)} 4\tan^2(x)ydy$$

$$\int_0^{\cos(x)} 1dy$$

$$= [1 \cdot y]_0^{\cos(x)}$$

$$= [y]_0^{\cos(x)}$$

$$= \cos(x)$$

$$\int_0^{\cos(x)} 4\tan^2(x)ydy$$

$$= 4\tan^2(x) \cdot \int_0^{\cos(x)} ydy$$

$$= 4\tan^2(x)\left[\frac{y^{1+1}}{1+1}\right]_0^{\cos(x)}$$

$$= 4\tan^2(x)\left[\frac{y^2}{2}\right]_0^{\cos(x)}$$

$$= 4\tan^2(x)\frac{\cos^2(x)}{2}$$

$$= 2\tan^2(x)\cos^2(x)$$

$$= \cos(x) + 2\tan^2(x)\cos^2(x)$$

$$= \int_0^{\frac{\pi}{4}} \left(\cos(x) + 2\tan^2(x)\cos^2(x)\right) dx$$

$$\int_0^{\frac{\pi}{4}} \cos(x) + 2\tan^2(x)\cos^2(x)\, dx$$

$$\int_0^{\frac{\pi}{4}} \cos(x)\, dx$$

$$= \left[\sin(x)\right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\int_0^{\frac{\pi}{4}} 2\tan^2(x)\cos^2(x)\, dx$$

$$= 2 \cdot \int_0^{\frac{\pi}{4}} \tan^2(x)\cos^2(x)\, dx$$

$$= 2 \cdot \int_0^{\frac{\pi}{4}} \left(\frac{\sin(x)}{\cos(x)}\right)^2 \cos^2(x)\, dx$$

$$= 2 \cdot \int_0^{\frac{\pi}{4}} \sin^2(x)\, dx$$

$$= 2 \cdot \int_0^{\frac{\pi}{4}} \frac{1 - \cos(2x)}{2} dx$$

$$= 2 \cdot \frac{1}{2} \cdot \int_0^{\frac{\pi}{4}} 1 - \cos(2x)\, dx$$

$$\int_0^{\frac{\pi}{4}} 1\, dx$$

$$= \left[1 \cdot x\right]_0^{\frac{\pi}{4}}$$

$$= [x]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{4}} \cos(2x)dx$$

Se aplica la integración por sustitución: u = 2x

$$= \int_0^{\frac{\pi}{2}} \cos(u)\frac{1}{2}du$$

$$= \frac{1}{2} \cdot \int_0^{\frac{\pi}{2}} \cos(u)du$$

$$= \frac{1}{2}[\sin(u)]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2} \cdot 1$$

$$= \frac{1}{2}$$

$$= 2 \cdot \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$2 \cdot \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$= \frac{1 \cdot 2\left(\frac{\pi}{4} - \frac{1}{2}\right)}{2}$$

$$= 1 \cdot \left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$= \left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$= \frac{\pi}{4} - \frac{1}{2} \qquad = \frac{1}{\sqrt{2}} + \frac{\pi}{4} - \frac{1}{2}$$

5. $\int_0^{2x} \frac{xy}{x^2 + y^2}\, dy$

Sacar la constante

$$= x \cdot \int_0^{2x} \frac{y}{x^2 + y^2} dy$$

Integración por sustitución

$$= x \cdot \int_{x^2}^{5x^2} \frac{1}{2u} du$$

Sacar la constante

$$= x\frac{1}{2} \cdot \int_{x^2}^{5x^2} \frac{1}{u} du$$

Aplicar la regla de integración

$$= x\frac{1}{2}\left[\ln|u|\right]_{x^2}^{5x^2}$$

$$= x\frac{1}{2}\ln(5)$$

$$= 0$$

6. $\int_{x^3}^{x} e^{2y/x}\, dy$

Solución:

$$\int_{x^3}^{x} e^{\frac{2y}{x}} dy$$

Aplicar integración por sustitución: u = 2y/x

$= \int_{2x^2}^{2} \frac{xe^u}{2} du$

$= \frac{x}{2} \cdot \int_{2x^2}^{2} e^u du$

$= \frac{x}{2}\left[e^u\right]_{2x^2}^{2}$

Se calculan los limites:

Calcular los limites: $\left[e^u\right]_{2x^2}^{2} = e^2 - e^{2x^2}$

$= \frac{x}{2}\left(e^2 - e^{2x^2}\right)$

2. evalúe la integral iterada dada:

1. $\int_1^2 \int_{-x}^{x^2} (8x - 10y + 2)\, dy\, dx$

Solución:

$\int_1^2 \int_{-x}^{x^2} 8x - 10y + 2dydx$

$\int_{-x}^{x^2} 8x - 10y + 2dy$

$= \int_{-x}^{x^2} 2dy - \int_{-x}^{x^2} 10ydy + \int_{-x}^{x^2} 8xdy$

$\int_{-x}^{x^2} 2dy$

$= \left[2y\right]_{-x}^{x^2}$

$= 2x^2 + 2x$

$$\int_{-x}^{x^2} 10y\,dy$$

Se saca la constante

$$= 10 \cdot \int_{-x}^{x^2} y\,dy$$

Se aplica la regla de la potencia

$$= 10\left[\frac{y^{1+1}}{1+1}\right]_{-x}^{x^2}$$

Se simplifica

$$= 10\left[\frac{y^2}{2}\right]_{-x}^{x^2}$$

$$= 10\left(\frac{x^4}{2} - \frac{x^2}{2}\right)$$

$$\int_{-x}^{x^2} 8x\,dy$$

$$= [8xy]_{-x}^{x^2}$$

$$= 8x^3 + 8x^2$$

$$= 2x^2 + 2x - 10\left(\frac{x^4}{2} - \frac{x^2}{2}\right) + 8x^3 + 8x^2$$

$$2x^2 + 2x - 10\left(\frac{x^4}{2} - \frac{x^2}{2}\right) + 8x^3 + 8x^2$$

$$-10\left(\frac{x^4}{2} - \frac{x^2}{2}\right)$$

$$= -10 \cdot \frac{x^4}{2} - (-10)\frac{x^2}{2}$$

$$= -10 \cdot \frac{x^4}{2} + 10 \cdot \frac{x^2}{2}$$

$$-10 \cdot \frac{x^4}{2} + 10 \cdot \frac{x^2}{2}$$

$$10 \cdot \frac{x^4}{2}$$

$$= 5x^4$$

$$10 \cdot \frac{x^2}{2}$$

$$= \frac{x^2 \cdot 10}{2}$$

$$= 5x^2$$

$$= -5x^4 + 5x^2$$

$$= -5x^4 + 5x^2$$

$$= 2x^2 + 2x - 5x^4 + 5x^2 + 8x^3 + 8x^2$$

$$= -5x^4 + 8x^3 + 2x^2 + 5x^2 + 8x^2 + 2x$$

$$= -5x^4 + 8x^3 + 15x^2 + 2x$$

$$= \int_1^2 \left(-5x^4 + 8x^3 + 15x^2 + 2x\right) dx$$

$$= -\int_1^2 5x^4 dx + \int_1^2 8x^3 dx + \int_1^2 15x^2 dx + \int_1^2 2x dx$$

$$\int_1^2 5x^4 dx$$

$$= 5\left[\frac{x^{4+1}}{4+1}\right]_1^2$$

$$= 5\left[\frac{x^5}{5}\right]_1^2$$

$$= 5 \cdot \frac{31}{5}$$

$$= 31$$

$$= 8 \cdot \int_1^2 x^3 dx$$

$$= 8\left[\frac{x^{3+1}}{3+1}\right]_1^2$$

$$= 8\left[\frac{x^4}{4}\right]_1^2$$

$$= 8 \cdot \frac{15}{4}$$

$$= 30$$

$$\int_1^2 15x^2 dx$$

$$= 15 \cdot \int_1^2 x^2 dx$$

$$= 15\left[\frac{x^{2+1}}{2+1}\right]_1^2$$

$$= 15 \cdot \frac{7}{3}$$

$$= 35$$

$$\int_1^2 2x dx$$

$= 2 \cdot \int_1^2 x dx$

$= 2\left[\frac{x^{1+1}}{1+1}\right]_1^2$

$= 2\left[\frac{x^2}{2}\right]_1^2$

$= 2 \cdot \frac{3}{2}$

$= 3$

$= -31 + 30 + 35 + 3$

$= 37$

2. $\int_{-1}^{1}\int_{0}^{y}(x+y)^2\,dx\,dy$

Solución:

$\int_0^y (x+y)^2 dx$

$(y+x)^2$

$= y^2 + 2yx + x^2$

$= \int_0^y y^2 + 2yx + x^2 dx$

$= \int_0^y y^2 dx + \int_0^y 2yx dx + \int_0^y x^2 dx$

$\int_0^y y^2 dx$

$= \left[y^2 x\right]_0^y$

$$= y^3$$

$$\int_0^y 2yxdx$$

$$= 2y \cdot \int_0^y xdx$$

$$= 2y\left[\frac{x^{1+1}}{1+1}\right]_0^y$$

$$= 2y\left[\frac{x^2}{2}\right]_0^y$$

$$= 2y\frac{y^2}{2}$$

$$= y^3$$

$$\int_0^y x^2dx$$

$$= \left[\frac{x^{2+1}}{2+1}\right]_0^y$$

$$= \frac{y^3}{3}$$

$$= y^3 + y^3 + \frac{y^3}{3}$$

$$= 2y^3 + \frac{y^3}{3}$$

$$= \int_{-1}^1 \left(2y^3 + \frac{y^3}{3}\right)dy$$

$$\int_{-1}^1 2y^3 + \frac{y^3}{3}dy$$

Funcion par: Una función es par si $f(-x) = f(x)$ para todos $x \in \mathbb{R}$
Funcion impar: Una funcion es impar si $f(-x) = -f(x)$ para todos $x \in \mathbb{R}$

$$2y^3 + \frac{y^3}{3}$$

$$= \frac{y^3}{3} + \frac{2y^3 \cdot 3}{3}$$

$$= \frac{y^3 + 2y^3 \cdot 3}{3}$$

$$y^3 + 2y^3 \cdot 3$$

$$= y^3 + 6y^3$$

$$= 7y^3$$

$$= \frac{7y^3}{3}$$

$$= \frac{7(-y)^3}{3}$$

$$7(-y)^3$$

$$= 7\left(-y^3\right)$$

$$= -7y^3$$

$$= \frac{-7y^3}{3}$$

$$= -\frac{7y^3}{3}$$

$$f(y) = \frac{7y^3}{3}, \quad f(-y) = -\frac{7y^3}{3}$$

$$f(y) \neq f(-y)$$

No es una función impar

$-f(y) = -\frac{7y^3}{3}, \ \ f(-y) = -\frac{7y^3}{3}$

$-f(y) = f(-y)$

$\frac{7y^3}{3}$ es una función impar

$= 0$

3. $\int_0^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} (2x - y)\, dx dy$

Solución:

$\int_0^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} (2x - y)^2 dxdy$

$\int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} (2x - y)^2 dx$

$= \left[(-y + 2x)^2 x - \int 4(-y + 2x)xdx \right]_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}}$

$= \left[\frac{1}{3}\left(4x^3 - 6yx^2 + 3y^2x\right) \right]_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}}$

$= 2y^2\sqrt{-y^2 + 2} + \frac{8}{3}\left(-y^2 + 2\right)^{\frac{3}{2}}$

$= \int_0^{\sqrt{2}} \left(2y^2\sqrt{-y^2 + 2} + \frac{8}{3}\left(-y^2 + 2\right)^{\frac{3}{2}}\right)dy$

$\int_0^{\sqrt{2}} 2y^2\sqrt{-y^2 + 2} + \frac{8}{3}\left(-y^2 + 2\right)^{\frac{3}{2}}dy$

$$\int_0^{\sqrt{2}} 2y^2\sqrt{-y^2+2}\,dy$$

$$= 2\cdot\int_0^{\sqrt{2}} y^2\sqrt{-y^2+2}\,dy$$

Se aplica integración por sustitución

$$= 2\cdot\int_0^{\frac{\pi}{2}} 4\sin^2(u)\cos^2(u)du$$

$$= 2\cdot\ 4\cdot\int_0^{\frac{\pi}{2}} \sin^2(u)\cos^2(u)du$$

$$= 2\cdot\ 4\cdot\int_0^{\frac{\pi}{2}} \frac{1-\cos(4u)}{8}du$$

$$= 2\cdot\ 4\cdot\frac{1}{8}\cdot\int_0^{\frac{\pi}{2}} 1-\cos(4u)du$$

$$= 2\cdot\ 4\cdot\frac{1}{8}\left(\int_0^{\frac{\pi}{2}} 1du - \int_0^{\frac{\pi}{2}}\cos(4u)du\right)$$

$$= 2\cdot\ 4\cdot\frac{1}{8}\left(\frac{\pi}{2}-0\right)$$

$$= \frac{\pi}{2}$$

$$\int_0^{\sqrt{2}} \frac{8}{3}\left(-y^2+2\right)^{\frac{3}{2}}dy$$

$$= \frac{8}{3}\cdot\int_0^{\sqrt{2}} \left(-y^2+2\right)^{\frac{3}{2}}dy$$

$$= \frac{8}{3}\cdot\int_0^{2\sqrt{2}} \frac{\left(-u^2+8\right)^{\frac{3}{2}}}{16}du$$

$$= \frac{8}{3}\cdot\frac{1}{16}\left[u\left(-u^2+8\right)^{\frac{3}{2}} - \int -3u^2\left(-u^2+8\right)^{\frac{1}{2}}du\right]_0^{2\sqrt{2}}$$

$$= \frac{8}{3}\cdot\frac{1}{16}\left[u\left(-u^2+8\right)^{\frac{3}{2}} - \left(-u^3\left(-u^2+8\right)^{\frac{1}{2}} + \frac{1}{4}u^3\sqrt{-u^2+8} - 24\arcsin\left(\frac{1}{2\sqrt{2}}u\right) + 12\sin\left(2\arcsin\left(\frac{1}{2\sqrt{2}}u\right)\right)\right)\right]_0^{2\sqrt{2}}$$

$$= \frac{1}{6}\left[\frac{4u\left(-u^2+8\right)^{\frac{3}{2}} - u^3\sqrt{-u^2+8} + 4u^3\sqrt{-u^2+8} - 48\sin\left(2\arcsin\left(\frac{1}{2\sqrt{2}}u\right)\right) + 96\arcsin\left(\frac{1}{2\sqrt{2}}u\right)}{4}\right]_0^{2\sqrt{2}}$$

$$= \frac{1}{6} \cdot 12\pi$$

$$= 2\pi$$

$$= \frac{\pi}{2} + 2\pi$$

$$= \frac{\pi}{2} + 2\pi$$

4. $\displaystyle\int_0^{\pi/4}\int_0^{\cos x}(1 + 4y\tan^2 x)\,dydx$

Solución:

$$\int_0^{\cos(x)} 1 + 4y\tan^2(x)dy$$

$$= \int_0^{\cos(x)} 1dy + \int_0^{\cos(x)} 4\tan^2(x)ydy$$

$$\int_0^{\cos(x)} 1dy$$

$$= [1 \cdot y]_0^{\cos(x)}$$

$$= [y]_0^{\cos(x)}$$

$$= \cos(x)$$

$$\int_0^{\cos(x)} 4\tan^2(x)ydy$$

$$= 4\tan^2(x) \cdot \int_0^{\cos(x)} y dy$$

$$= 4\tan^2(x)\left[\frac{y^{1+1}}{1+1}\right]_0^{\cos(x)}$$

$$= 4\tan^2(x)\left[\frac{y^2}{2}\right]_0^{\cos(x)}$$

$$= 4\tan^2(x)\frac{\cos^2(x)}{2}$$

$$= 2\tan^2(x)\cos^2(x)$$

$$= \cos(x) + 2\tan^2(x)\cos^2(x)$$

$$= \int_0^{\frac{\pi}{4}}\left(\cos(x) + 2\tan^2(x)\cos^2(x)\right)dx$$

$$\int_0^{\frac{\pi}{4}}\cos(x) + 2\tan^2(x)\cos^2(x)dx$$

$$\int_0^{\frac{\pi}{4}}\cos(x)dx$$

$$= [\sin(x)]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\int_0^{\frac{\pi}{4}}2\tan^2(x)\cos^2(x)dx$$

$$= 2 \cdot \int_0^{\frac{\pi}{4}}\tan^2(x)\cos^2(x)dx$$

$$= 2 \cdot \int_0^{\frac{\pi}{4}}\left(\frac{\sin(x)}{\cos(x)}\right)^2\cos^2(x)dx$$

$$= 2 \cdot \int_0^{\frac{\pi}{4}} \sin^2(x) dx$$

$$= 2 \cdot \int_0^{\frac{\pi}{4}} \frac{1 - \cos(2x)}{2} dx$$

$$= 2 \cdot \frac{1}{2} \cdot \int_0^{\frac{\pi}{4}} 1 - \cos(2x) dx$$

$$\int_0^{\frac{\pi}{4}} 1 dx$$

$$= [1 \cdot x]_0^{\frac{\pi}{4}}$$

$$= [x]_0^{\frac{\pi}{4}}$$

$$= \frac{\pi}{4}$$

$$\int_0^{\frac{\pi}{4}} \cos(2x) dx$$

Se aplica la integración por sustitución: u = 2x

$$= \int_0^{\frac{\pi}{2}} \cos(u) \frac{1}{2} du$$

$$= \frac{1}{2} \cdot \int_0^{\frac{\pi}{2}} \cos(u) du$$

$$= \frac{1}{2} [\sin(u)]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2} \cdot 1$$

$$= \frac{1}{2}$$

$$= 2 \cdot \frac{1}{2} \left(\frac{\pi}{4} - \frac{1}{2} \right)$$

$2 \cdot \frac{1}{2}\left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{1 \cdot 2\left(\frac{\pi}{4} - \frac{1}{2}\right)}{2}$

$= 1 \cdot \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{\pi}{4} - \frac{1}{2}$

$= \frac{1}{\sqrt{2}} + \frac{\pi}{4} - \frac{1}{2}$

5. $\int_0^{\pi} \int_y^{3y} \cos(2x + y)\, dx dy$

Solución:

$\int_y^{3y} \cos(2x + y) dx$

$= \int_{3y}^{7y} \cos(u) \frac{1}{2} du$

$= \frac{1}{2}[\sin(u)]_{3y}^{7y}$

$= \frac{1}{2}(\sin(7y) - \sin(3y))$

$= \int_0^{\pi} \frac{1}{2}(\sin(7y) - \sin(3y)) dy$

$\int_0^{\pi} \frac{1}{2}(\sin(7y) - \sin(3y)) dy$

$$= \frac{1}{2} \cdot \int_0^{\pi} \sin(7y) - \sin(3y)dy$$

$$\int_0^{\pi} \sin(7y)dy$$

$$= \int_0^{7\pi} \sin(u)\frac{1}{7}du$$

$$= \frac{1}{7}[-\cos(u)]_0^{7\pi}$$

$$= \frac{1}{7} \cdot 2$$

$$= \frac{2}{7}$$

$$\int_0^{\pi} \sin(3y)dy$$

Se aplica la integración por sustitución u = 3y

$$= \int_0^{3\pi} \sin(u)\frac{1}{3}du$$

Sacar la constante

$$= \frac{1}{3} \cdot \int_0^{3\pi} \sin(u)du$$

Se aplica la regla de integración

$$= \frac{1}{3}[-\cos(u)]_0^{3\pi}$$

$$= \frac{1}{3} \cdot 2$$

$$= \frac{2}{3}$$

$$= \frac{1}{2}\left(\frac{2}{7} - \frac{2}{3}\right)$$

$\frac{1}{2}\left(\frac{2}{7}-\frac{2}{3}\right)$

$=\frac{1}{2}\left(-\frac{8}{21}\right)$

$= -\frac{1}{2}\cdot\frac{8}{21}$

$= -\frac{4}{21}$

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6. $\int_1^2 \int_0^{\sqrt{x}} 2y \text{ sen } \pi x^2 \, dy \, dx$

$\int_0^{\sqrt{x}} 2y\sin(\pi)x^2 dy$

$= \int_0^{\sqrt{x}} 0 dy$

$= [0 \cdot y]_0^{\sqrt{x}}$

$= [0]_0^{\sqrt{x}}$

$= 0$

$= \int_1^2 0 dx$

$\int_1^2 0 dx$

$= [0 \cdot x]_1^2$

$= [0]_1^2$

7. $\int_{1}^{\ln 3}\int_{0}^{x} 6e^{x+2y}\,dy\,dx$

Solución:

$$\int_0^x 6e^{x+2y}dy$$

$$= 6 \cdot \int_0^x e^{x+2y}dy$$

$$= 6 \cdot \frac{1}{2} \cdot \int_x^{3x} e^u du$$

$$= 6 \cdot \frac{1}{2}\left[e^u\right]_x^{3x}$$

$$= 3\left[e^u\right]_x^{3x}$$

$$= 3\left(e^{3x} - e^x\right)$$

$$= \int_1^{\ln(3)} 3\left(e^{3x} - e^x\right)dx$$

$$= 3 \cdot \int_1^{\ln(3)} e^{3x} - e^x dx$$

$$= 3\left(\int_1^{\ln(3)} e^{3x}dx - \int_1^{\ln(3)} e^x dx\right)$$

$$= 3\left(\frac{27 - e^3}{3} - (3 - e)\right)$$

$$= -e^3 + 3e + 18$$

$$8.\quad \int_0^1 \int_0^{2y} e^{-y^2}\,dx\,dy$$

Solución:

$$\int_0^{2y} e^{-y\cdot\ 2} dx$$

$$= \left[e^{-2y}x\right]_0^{2y}$$

$$= 2ye^{-2y}$$

$$= \int_0^1 2ye^{-2y}dy$$

$$= 2\cdot\int_0^1 ye^{-2y}dy$$

Se aplica la integración por sustitución: u = -2y

$$= 2\cdot\int_0^{-2}\frac{e^u u}{4}du$$

$$= 2\left(-\int_{-2}^0\frac{e^u u}{4}du\right)$$

Se aplica la integración por partes: u = u, v’ = e^u

$$= 2\left(-\frac{1}{4}\left[e^u u - \int e^u du\right]_{-2}^0\ \right)$$

$$\int e^u du$$

$$= 2\left(-\frac{1}{4}\left[e^u u - e^u\right]_{-2}^0\ \right)$$

$$= \ -2\cdot\frac{1}{4}\left[e^u u - e^u\right]_{-2}^0$$

$$= -\frac{1 \cdot 2}{4}\left[e^u u - e^u\right]_{-2}^{0}$$

$$\frac{1 \cdot 2}{4}$$

$$= \frac{2}{4}$$

$$= -\frac{1}{2}\left[e^u u - e^u\right]_{-2}^{0}$$

$$= -\frac{1}{2}\left(-1 + \frac{3}{e^2}\right)$$

$$= -\frac{-e^2 + 3}{2e^2}$$

$$= -\frac{-e^2 + 3}{2e^2}$$

9. $\int_0^3 \int_{x+1}^{2x+1} \frac{1}{\sqrt{y-x}}\, dy\, dx$

Solución:

$$\int_{x+1}^{2x+1} \frac{1}{\sqrt{y-x}} dy$$

Se aplica la integración por sustitución

$$= \int_1^{x+1} \frac{1}{\sqrt{u}} du$$

$$= \int_1^{x+1} u^{-\frac{1}{2}} du$$

$$= \left[\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_1^{x+1}$$

$$= \left[2\sqrt{u}\right]_1^{x+1}$$

$$= \int_0^3 \left(2\sqrt{x+1} - 2\right) dx$$

$$= \int_0^3 2\sqrt{x+1}\, dx - \int_0^3 2dx$$

$$\int_0^3 2\sqrt{x+1}\, dx$$

$$= 2 \cdot \int_0^3 \sqrt{x+1}\, dx$$

Se aplica la integración por sustitución

$$= 2 \cdot \int_1^4 \sqrt{u}\, du$$

$$= 2\left[\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^4$$

$$= 2\left[\frac{2}{3}u^{\frac{3}{2}}\right]_1^4$$

$$= 2 \cdot \frac{14}{3}$$

$$= \frac{28}{3} - 6$$

$$\int_0^3 2dx$$

$$= [\,2x\,]_0^3$$

$$= 6$$

$$= \frac{28}{3} - 6$$

$$= -\frac{6\cdot\ 3}{3}+\frac{28}{3}$$

$$= \frac{-6\cdot\ 3+28}{3}$$

$$= \frac{10}{3}$$

10. $\int_0^1 \int_0^y x(y^2 - x^2)^{3/2}\, dx\, dy$

Solución:

$$\int_0^y x\left(y^2-x^2\right)^{\frac{3}{2}}dx$$

$$=\left[\frac{1}{2}x^2\left(y^2-x^2\right)^{\frac{3}{2}}-\int -\frac{3}{2}x^3\sqrt{y^2-x^2}\,dx\right]_0^y$$

$$\int -\frac{3}{2}x^3\sqrt{y^2-x^2}\,dx$$

$$= -\frac{3}{2}\cdot\int x^3\sqrt{y^2-x^2}\,dx$$

$$= -\frac{3}{2}\cdot\int -\frac{\left(-u+y^2\right)u^{\frac{1}{2}}}{2}du$$

$$= -\frac{3}{2}\left(-\frac{1}{2}\cdot\int\left(-u+y^2\right)u^{\frac{1}{2}}du\right)$$

$$= -\frac{3}{2}\left(-\frac{1}{2}\left(-\int u^{\frac{3}{2}}du+\int y^2u^{\frac{1}{2}}du\right)\right)$$

$$\int u^{\frac{3}{2}}du$$

$$= \frac{u^{\frac{3}{2}+1}}{\frac{3}{2}+1}$$

$$= \frac{u^{\frac{3}{2}+1}}{\frac{5}{2}}$$

$$= \frac{u^{\frac{5}{2}}}{\frac{5}{2}}$$

$$= \frac{u^{\frac{5}{2}} \cdot 2}{5}$$

$$= \frac{2}{5}u^{\frac{5}{2}}$$

$$\int y^2 u^{\frac{1}{2}} du$$

$$= y^2 \cdot \int u^{\frac{1}{2}} du$$

$$= y^2 \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}$$

$$\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}$$

$$= \frac{u^{\frac{1}{2}+1}}{\frac{3}{2}}$$

$$u^{\frac{1}{2}+1}$$

$$= u^{\frac{3}{2}}$$

$$= \frac{u^{\frac{3}{2}}}{\frac{3}{2}}$$

$$= \frac{u^{\frac{3}{2}} \cdot 2}{3}$$

$$= y^2 \frac{2u^{\frac{3}{2}}}{3}$$

$$= \frac{u^{\frac{3}{2}} \cdot 2y^2}{3}$$

$$= \frac{2y^2u^{\frac{3}{2}}}{3}$$

$$= -\frac{3}{2}\left(-\frac{1}{2}\left(-\frac{2}{5}u^{\frac{5}{2}} + \frac{2y^2u^{\frac{3}{2}}}{3}\right)\right)$$

$$= -\frac{3}{2}\left(-\frac{1}{2}\left(-\frac{2}{5}\left(y^2 - x^2\right)^{\frac{5}{2}} + \frac{2y^2\left(y^2 - x^2\right)^{\frac{3}{2}}}{3}\right)\right)$$

$$= \frac{1}{10}\left(3x^2 + 2y^2\right)\left(-x^2 + y^2\right)^{\frac{3}{2}}$$

$$= \left[\frac{1}{2}x^2\left(y^2 - x^2\right)^{\frac{3}{2}} - \frac{1}{10}\left(3x^2 + 2y^2\right)\left(-x^2 + y^2\right)^{\frac{3}{2}}\right]_0^y$$

$$= \left[\frac{1}{5}\left(x^2 - y^2\right)\left(-x^2 + y^2\right)^{\frac{3}{2}}\right]_0^y$$

$$= \frac{1}{5}y^5$$

$$= \int_0^1 \frac{1}{5}y^5 dy$$

$$= \frac{1}{5} \cdot \int_0^1 y^5 dy$$

$$= \frac{1}{5}\left[\frac{y^{5+1}}{5+1}\right]_0^1$$

$$= \frac{1}{5}\left[\frac{y^6}{6}\right]_0^1$$

$$= \frac{1}{5} \cdot \frac{1}{6} \qquad = \frac{1}{30}$$