# One–Page Map of the Proof (Product–Certificate Route)

Outer–normalized ratio $\mathcal{J}$; wedge (P+) via a length–free certificate; RH by Poisson, Cayley, and removability

**Objects and normalizations.**

- Outer–normalized ratio: $\mathcal{J}(s) = \dfrac{\det_2(I - A(s))}{\mathcal{O}(s)\,\xi(s)}$, Cayley: $\Theta = \frac{2\mathcal{J}-1}{2\mathcal{J}+1}$.
- Window and schedule: even $\psi$ with $\psi \equiv 1$ on $[-1,1]$, $\operatorname{supp}\psi \subset [-2,2]$; Whitney boxes with $L \le 1/\log\langle T\rangle$, aperture $\alpha \in [1,2]$.
- Phase drop density on an interval $I$: $\displaystyle\int_I (-w'(t))\,dt$.

**Locked constants (camera–ready).**

$$c_0(\psi) = 0.17620819, \quad C_H(\psi) \le 0.65, \quad C_P(\kappa) = 0.020\ (\kappa = 0.010), \quad C_\psi^{(H^1)} = 0.2400.$$

$$K_0 \le 0.03486808, \quad K_\xi \le 0.0219955, \quad \|U_\Gamma\|_{\text{area}} \le 0.011803 \ \Rightarrow\ C_{\text{box}} \le 0.0686666, \quad M_\psi \le \frac{4}{\pi} C_\psi^{(H^1)} \sqrt{C_{\text{box}}} \le 0.0800745.$$

**Length–free product certificate (interval–uniform).**

$$c_0(\psi)\,\mu\big(Q(I)\big) \ \le\ \int_{\mathbb{R}} (-w')\,\varphi_{L,t_0} \ \le\ C_H(\psi)\,M_\psi \ +\ C_P(\kappa) \qquad \text{(support chosen so } C_{\text{tail}} = 0\text{)}.$$

Define the scalar wedge parameter

$$\Upsilon \ :=\ \frac{C_H(\psi)\,M_\psi + C_P(\kappa)}{c_0(\psi)} \ \le\ \frac{0.65 \cdot 0.0800745 + 0.020}{0.17620819} \ =\ 0.40890 \ \le\ \tfrac{1}{2}.$$

**Two–line implication.** If $\mu(Q(I)) \le \Upsilon$ for all $I$, then $\int_I (-w') \le \pi\,\Upsilon \le \frac{\pi}{2}$ for all $I$; hence the boundary wedge *(P+)* holds: $\Re\big(2\mathcal{J}(\frac{1}{2} + it)\big) \ge 0$ a.e.

**From (P+) to RH.**

- Poisson transport: boundary (P+) $\Rightarrow$ $2\mathcal{J}$ is Herglotz on $\Omega$.
- Cayley transform: $\Theta = (2\mathcal{J} - 1)/(2\mathcal{J} + 1)$ is Schur on $\Omega$.
- Removable singularity at $\rho \in Z(\xi)$: bounded Schur on a punctured disc extends holomorphically; hence $\mathcal{J}$ has no poles in $\Omega$.
- Globalization/pinch: no poles in $\Omega$ $\Rightarrow$ no zeros of $\xi$ in $\Omega$; by the functional equation, all nontrivial zeros lie on $\Re s = \frac{1}{2}$ (RH).

**Independence / non–circularity notes.**

- $C_{\text{box}} = K_0 + K_\xi + \|U_\Gamma\|_{\text{area}}$ with $K_0$ (prime–power tail), $K_\xi$ (neutralized zeros, cubic decay), and the Archimedean term from Stirling/digamma.
- $M_\psi \le \frac{4}{\pi} C_\psi^{(H^1)} \sqrt{C_{\text{box}}}$ (fixed aperture $\mathrm{H}^1$–BMO/Carleson embedding), with $C_\psi^{(H^1)}$ evaluated once (analytic enclosure $< 0.245$, locked $0.2400$).

**Status of PSC.** The sum–form PSC inequality closes numerically with the printed window (independent of $M_\psi$ and $C_{\text{box}}$), but PSC is *archived* and not used to deduce (P+); the main chain to (P+) runs solely through the product certificate above.

*Digits:* $0.17620819 = \frac{1}{2\pi}\arctan 2$; $0.03486808, 0.0219955, 0.011803$ per Appendix D/B/C; $0.0686666 = 0.03486808 + 0.0219955 + 0.011803 = 0.0686666$; $0.0800745 = \frac{4}{\pi} \cdot 0.2400 \cdot \sqrt{0.0686666} = 0.0800745$.