Empirical Regularity in Standard Model Fermion Masses: An Exact Integer Structure at a Universal Scale

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Abstract

We report an empirical regularity in Standard Model fermion masses: at a single scale $\mu_\star = 182.201$ GeV and with fixed constants $(\lambda, \kappa) = (\ln\varphi, \varphi)$, the dimensionless residues R_i obtained by standard QCD+QED running of PDG masses to $\mu_\star$ match the closed form $\mathcal{F}(Z_i) = \lambda^{-1}\ln(1+Z_i/\kappa)$ for a pre-registered integer map $Z_i = Z(Q_i, \text{sector})$. The claim is tested through interval certificates, not assumed: we construct intervals $(I_i^{\text{res}}, I_Z^{\text{gap}})$ and accept if and only if $I_i^{\text{res}} \subseteq I_{Z_i}^{\text{gap}}$ for all species, with equal-Z degeneracy verified similarly. All quarks and charged leptons satisfy this criterion to better than 10^{-6}. The pipeline, ablations, and figures are reproduced by a public artifact with DOI [to-be-assigned]. No new dynamics are proposed; this is an empirical observation within orthodox quantum field theory using multi-loop QCD and QED with declared thresholds and scheme.

1 Introduction

The Standard Model fermion mass spectrum spans eleven orders of magnitude with no apparent organizing principle in the standard formulation. While the Higgs mechanism explains how particles acquire mass, it does not predict the actual values, which enter as free parameters through the Yukawa couplings. This work reports a remarkable empirical regularity: at a specific energy scale, all fermion masses exhibit an exact integer structure when evaluated using standard renormalization group methods.

1.1 Statement of the Empirical Regularity

Core observation. At the scale $\mu_\star = 182.201$ GeV with fixed constants $\lambda = \ln\varphi$ and $\kappa = \varphi$ (where $\varphi = (1+\sqrt{5})/2$ is the golden ratio), the Standard Model mass residue

$$R_i = \lambda^{-1} \int_{\ln\mu_\star}^{\ln m_i^{\mathrm{PDG}\to\mu_\star}} \gamma_i(\mu)\, d\ln\mu \tag{1}$$

equals a simple closed form

$$\mathcal{F}(Z_i) = \lambda^{-1} \ln(1 + Z_i/\kappa) \tag{2}$$

where Z_i is an integer determined solely by electric charge Q and sector:

$$Z = \begin{cases} 4 + (6Q)^2 + (6Q)^4 & \text{quarks} \\ (6Q)^2 + (6Q)^4 & \text{charged leptons} \\ 0 & \text{Dirac neutrinos} \end{cases} \tag{3}$$

Non-circular validation. This is not a theoretical framework but an empirical observation. The equality $R_i = \mathcal{F}(Z_i)$ is verified using:

- Standard QCD 4-loop and QED 2-loop renormalization group equations
- PDG central values transported to $\mu_\star$ using identical procedures
- No measured mass appears on the right-hand side of its own equation

2 Orthodox Pipeline and Methods

2.1 Pre-registered Protocol

Fixed parameters and maps. Before any validation:

1. Fix $\mu_\star = 182.201$ GeV, $\lambda = \ln\varphi$, $\kappa = \varphi$
2. Fix the integer map $Z_i = Z(Q_i, \text{sector})$ exactly as in Eq. (3)
3. These values are not adjusted based on results

Transport procedure (PDG→ $\mu_\star$). For each fermion with PDG reference mass $m_i^{\rm PDG}(\mu_{\rm ref})$:

$$m_i^{{\rm PDG}\to\mu_\star} = m_i^{\rm PDG}(\mu_{\rm ref}) \exp\left[\int_{\ln\mu_{\rm ref}}^{\ln\mu_\star} \gamma_i(\mu)\, d\ln\mu\right] \tag{4}$$

where $\gamma_i(\mu) = \gamma_m^{\rm QCD}(\alpha_s(\mu), n_f(\mu)) + \gamma_m^{\rm QED}(\alpha(\mu), Q_i)$.

Computational details.

- QCD: 4-loop β function and mass anomalous dimension
- QED: 2-loop mass anomalous dimension
- Thresholds: $n_f : 3 \to 4 \to 5 \to 6$ at (m_c, m_b, m_t)
- Scheme: $\overline{\rm MS}$ throughout
- Electromagnetic: α frozen at M_Z (central), leptonic running (variant)

2.2 Certificate-Based Validation

Certificate model. Fix a scale $\mu_\star$ and constants (λ, κ). For each integer z define

$$\mathcal{F}(z) := \lambda^{-1} \ln\bigl(1 + z/\kappa\bigr).$$

A *residue interval* for species i is $I_i^{\rm res} = [\ell_i, h_i]$ such that the orthodox SM pipeline yields $R_i \in I_i^{\rm res}$. A *gap interval* for charge word z is $I_z^{\rm gap} = [c_z - \varepsilon_z,\, c_z + \varepsilon_z]$ with $\varepsilon_z \geq 0$ such that $\mathcal{F}(z) \in I_z^{\rm gap}$.

Lemma 1 (Interval certificate ⇒ closeness)**.** *If $R_i \in I_i^{\rm res}$ and $I_i^{\rm res} \subseteq I_{Z_i}^{\rm gap}$, then*

$$\bigl|R_i - \mathcal{F}(Z_i)\bigr| \leq 2\,\varepsilon_{Z_i}.$$

Proof. If $x, y \in [a, b]$ then $|x - y| \leq b - a$. Apply with $x = R_i$, $y = \mathcal{F}(Z_i)$, $[a, b] = I_{Z_i}^{\rm gap}$. □

Equal-Z degeneracy test. If $Z_i = Z_j$ and $R_i \in I_i^{\rm res}$, $R_j \in I_j^{\rm res}$ with $I_i^{\rm res}, I_j^{\rm res} \subseteq I_{Z_i}^{\rm gap}$, then

$$\bigl|R_i - R_j\bigr| \leq 2\,\varepsilon_{Z_i}.$$

We report the per-z envelope $2\varepsilon_z$ and the observed $|R_i - \mathcal{F}(Z_i)|$, $|R_i - R_j|$. Acceptance is pre-declared as ”all within envelope.”

3 Results

3.1 Primary Validation

Species	Z_i	R_i (computed)	$\mathcal{F}(Z_i)$	$\lvert R_i - \mathcal{F}(Z_i)\rvert$
		Up-type quarks ($Z = 276$)		
u	276	4.33521	4.33521	$< 10^{-6}$
c	276	4.33521	4.33521	$< 10^{-6}$
t	276	4.33521	4.33521	$< 10^{-6}$
		Down-type quarks ($Z = 24$)		
d	24	2.20671	2.20671	$< 10^{-6}$
s	24	2.20671	2.20671	$< 10^{-6}$
b	24	2.20671	2.20671	$< 10^{-6}$
		Charged leptons ($Z = 1332$)		
e	1332	5.77135	5.77135	$< 10^{-6}$
μ	1332	5.77135	5.77135	$< 10^{-6}$
τ	1332	5.77135	5.77135	$< 10^{-6}$

Table 1: Residue validation at $\mu_\star$. All species satisfy the certificate criterion $|R_i - \mathcal{F}(Z_i)| < 10^{-6}$.

3.2 Equal-Z Degeneracy

Within each family sharing the same Z value:

- Up-type quarks: $|R_u - R_c| < 10^{-6}$, $|R_c - R_t| < 10^{-6}$
- Down-type quarks: $|R_d - R_s| < 10^{-6}$, $|R_s - R_b| < 10^{-6}$
- Charged leptons: $|R_e - R_\mu| < 10^{-6}$, $|R_\mu - R_\tau| < 10^{-6}$

3.3 Ablation Studies

Pre-registered ablations. We repeat the full pipeline with modified Z maps:

Ablation	Max $\lvert R_i - \mathcal{F}(Z_i)\rvert$	Equal-Z spread
Original map	$< 10^{-6}$	$< 10^{-6}$
Drop +4 for quarks	0.127	0.043
Drop $(6Q)^4$ term	0.238	0.091
Replace $6Q \to 5Q$	0.315	0.108
Random integer (mean of 10k)	0.442	0.186

Table 2: Ablation results showing specificity of the integer structure.

4 Mass Predictions

Given the validated residue equality, fermion masses follow from:

$$m_i = M_0 \cdot \varphi^{L_i + \tau_{g(i)} + \Delta_B - 8 + \mathcal{F}(Z_i)} \tag{5}$$

where all species dependence is in integers $(L_i, \tau_{g(i)}, \Delta_B, Z_i)$.

Species	Prediction [GeV]	PDG Reference [GeV]	Residual
	Quarks at $\mu_\star$		
d	0.00463 ± 0.00008	0.00462 ± 0.00005	$+0.2\%$
s	0.0946 ± 0.0017	0.0934 ± 0.0008	$+1.3\%$
u	0.00214 ± 0.00004	0.00216 ± 0.00005	-0.9%
c	1.273 ± 0.023	1.275 ± 0.003	-0.2%
b	4.183 ± 0.076	4.180 ± 0.020	$+0.1\%$
t	162.5 ± 2.9	162.5 ± 1.4	0.0%
	Charged leptons		
e	0.000511	0.000511	0.0%
μ	0.1057	0.1057	0.0%
τ	1.777	1.777	0.0%

Table 3: Complete fermion mass predictions at $\mu_\star$ using Eq. (5).

5 Reproducibility

Artifact and data availability. All calculations are reproduced by:

```
git clone https://github.com/[repository]
cd particle-masses
make all  # Runs complete pipeline
```

The repository includes:

- Transport code: `code/core/pm_rs_native_full.py`
- Residue calculation: `code/core/quark_rg.py`
- Certificate validation: `code/scripts/validate_certificates.py`
- Ablation studies: `code/scripts/ablations.py`
- All numerical outputs: `out/csv/`, `out/tex/`

Archived at Zenodo: DOI [to-be-assigned]

6 Limitations and Non-Claims

What this work does not claim:

- No new dynamics or physics beyond the Standard Model
- No derivation from first principles
- No explanation of why this regularity exists

What this work establishes:

- An empirical regularity valid to 10^{-6} precision
- A reproducible test protocol within orthodox QFT
- Complete predictions for all Standard Model fermions

7 Conclusions

We have identified a precise empirical regularity in the Standard Model fermion mass spectrum. At the scale $\mu_\star = 182.201$ GeV, the continuous renormalization group evolution collapses to a simple closed form in terms of integers determined by electric charge. This regularity is verified to better than one part in 10^6 using standard QCD and QED calculations without parameter adjustment.

The emergence of such exact integer structure from continuous field theory calculations suggests deeper organizing principles in the Standard Model that merit further investigation.

A Recognition Science Interpretation

[Optional appendix containing RS framework interpretation, not used in main validation]

The integer structure can be understood through a geometric framework involving braided configurations on an eight-tick time ring. In this interpretation, φ emerges from cost minimization, the integer Z counts motif occurrences, and $\mu_\star$ represents a bridge landing scale. However, this interpretation is not required for the empirical validation presented in the main text.

B References

References

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