Referee Patch Report (Amir v1)

Critical blockers raised by a hostile referee, and the concrete fixes applied

Auto-generated in `docs/referee/` from the Cursor session

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Scope

This note summarizes two *critical* referee blockers previously identified in `Hodge_REFEREE_Amir-v1.tex`, and documents the corresponding edits that were applied:

- the impossible coverage statement in Proposition `prop:dense-holo`;
- the per-cube mass matching step in Theorem `thm:global-cohom` (and its local precursor `lem:local-bary`), including the quantifier/scaling issue around fixed m.

This report is *not* a proof audit of the full manuscript; it is a surgical "what changed and why" memo.

1 Blocker 1: Proposition `prop:dense-holo` (finite family covering every point)

Referee objection

The manuscript stated (paraphrasing) that for a compact set $K \subset X$ and $\varepsilon > 0$, there exists a *finite* family of proper calibrated submanifolds $Y_1, \ldots, Y_M$ such that for *every* $x \in K$ and *every* calibrated plane $\Pi \subset T_x X$, one can find j with $x \in Y_j$ and $T_x Y_j$ ε-close to Π.

For $p \geq 1$ and K with nonempty interior, a finite union of proper submanifolds cannot contain all points of K. Hence the original formulation is false as stated (even if morally intended as a "finite net at centers" statement).

Correction applied

The statement was replaced by the correct finite-net formulation:

- choose finitely many centers x_α covering K at scale ε;
- at each center x_α, realize a finite $\varepsilon/2$-net of calibrated planes by calibrated complete intersections $Y_{\alpha,j}$ through x_α;
- compare an arbitrary $x \in K$ to a nearby center via a fixed identification of tangent spaces in a coordinate ball (e.g. chart differential or parallel transport).

An explicit "Referee note" was added directly under the proposition explaining why the stronger global coverage formulation cannot hold for a finite family.

Why this resolves the blocker

The revised statement matches what the proof actually constructs and removes an impossible topological/dimension claim. It also aligns with later discretization logic: directions are approximated at finitely many sample points and propagated by continuity estimates.

2 Blocker 2: Theorem `thm:global-cohom` (per-cube mass matching / quantifier swap)

Referee objection

In the per-cube local quantization step, the manuscript previously asserted:

1. "any affine calibrated sheet with tangent plane $P_{Q,j}$ has the same ψ-mass in Q, call it $A_{Q,j}$" (translation-independence in a cube);
2. choose integers $N_{Q,j}$ so that $\sum_j N_{Q,j} A_{Q,j} \approx M_Q$, where $M_Q := m \int_Q \beta \wedge \psi$;
3. justify existence by "m may be taken arbitrarily large."

The referee objection had two parts:

- The "common sheet mass" assertion is false for generic orientations (even in $\mathbb{R}^2$, line–square intersection length depends on offset).
- A scaling obstruction: if $A_{Q,j} \sim h^k$ with $k = 2n - 2p$ and $M_Q \sim mh^{2n}$, then $M_Q/A_{Q,j} \sim mh^{2p} \to 0$ as $h \to 0$ for fixed m. Thus integer matching fails on sufficiently fine meshes unless one lets $m \to \infty$, contradicting the fixed-m SYR definition.

Correction applied

The matching is now routed through the manuscript's *corner-exit template* mechanism:

- introduce a tunable footprint scale $s \ll h := \mathrm{side}(Q)$;
- use the corner-exit template lemmas to choose, for each direction label (Q, j), a family of sheet pieces in Q with *identical* corner-exit footprints (hence equal ψ-mass within the family, up to a common small-slope distortion factor);
- denote this common per-piece mass by $A_{Q,j}$, with scaling $A_{Q,j} \asymp s^k$ where $k = 2n - 2p$;
- choose s small enough so that $M_Q/A_{Q,j} \gg 1$, enabling integer rounding with fixed m (no "$m \to \infty$" quantifier swap).

The same idea was reflected in the local precursor lemma `lem:local-bary`, which was strengthened to explicitly include the cube budget $M_Q := m \int_Q \beta \wedge \psi$ and a quantitative error target $|\,\mathrm{Mass}(S_Q) - M_Q| \le \delta M_Q$.

Why this resolves the blocker

The equal-mass property is no longer a false geometric claim about generic translates; it becomes a *design feature* of the template box. The scaling obstruction is addressed by shrinking the per-piece mass scale $A_{Q,j} \asymp s^k$ with $s \ll h$, allowing many pieces per cube while keeping the total budget M_Q fixed (so fixed-m SYR quantifiers are not violated).

What remains to be checked

These edits remove two hard referee blockers but do not, by themselves, certify the full proof. The remaining highest-risk verification areas include:

- global coherence across all labels and meshes (integer data satisfying face constraints and period constraints simultaneously);
- transport $\Rightarrow$ flat-norm gluing estimates and their parameter dependence;
- exact-class enforcement (`prop:cohomology-match`) under the full parameter schedule;
- ensuring the schedule can satisfy all asymptotic requirements simultaneously (including the new explicit footprint scale $s_j \ll h_j$).