Response Note to `hodge-referee-2.tex` (Dec 2025)

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Executive summary

This note answers each referee-critical item in `hodge-referee-2.tex` by pointing to the exact, numbered statements in `hodge-SAVE-dec-12-handoff.tex` that close the issue. The two "bridges" singled out by the referee are now explicit:

- **Template → holomorphic:** uniform C^1 single-sheet control on an entire cell, plus stability of face incidence and face-slice masses.
- **Local → global gluing:** a per-face flat-norm bound that sums to a global estimate, followed by a standard flat-norm filling with vanishing mass.

Key bridge A: template → holomorphic (the core stability verification)

The referee asks for a proved statement that holomorphic realizations preserve the geometric hypotheses used by the mismatch bookkeeping. This is provided by:

- **Uniform holomorphic C^1 control at Bergman scale:** `lem:bergman-control`.
- **Global "graph on the whole cell" criterion:** `lem:global-graph-contraction` and its holomorphic implementation `prop:cell-scale-linear-model-graph`.
- **Realization of a finite translation template with uniform graph control and disjointness:** `prop:finite-template`.
- **No accidental face hits + localized trace + per-face mass comparability:** `prop:holomorphic-corne` and `cor:holomorphic-corner-exit-inherits`.
- **Corner-exit holomorphic slivers (L1):** `prop:holomorphic-corner-exit-L1` (plus vertex-star coherence `rem:vertex-star-coherence`).

Key bridge B: local → global gluing (flat norm + filling)

The referee asks for an explicit derivation of face-by-face mismatch bounds and a global small-boundary conclusion in a clearly specified norm. This is provided by:

- **Per-face mismatch from prefix templates (with $O(h)$ edit regime):** `prop:prefix-template-coheren`

- **Vertex-template route that proves the face-edit regime from boundary-mass control (no-heavy-tail):** `prop:vertex-template-face-edits` (and the tail-vs-prefix reduction `lem:oh-face-edit-regime`).
- **Global summation in the sliver-compatible weighted form:** `cor:global-flat-weighted` and scaling bookkeeping `rem:weighted-scaling`.
- **Microstructure/gluing estimate (now a numbered proposition):** `prop:glue-gap`.
- **Boundary correction with vanishing mass (flat-norm decomposition + Federer–Fleming):** Step 5 in `hodge-SAVE-dec-12-handoff.tex`.

Referee short list: "Missing steps (what a referee would still require)"

The referee isolates four bridge-closure items (Section "Missing steps (what a referee would still require)" in `hodge-referee-2.tex`). They are now closed in `hodge-SAVE-dec-12-handoff.tex` as follows:

1. **(G1)–(G2) verification for the *holomorphic* corner-exit slivers.** This is proved in `prop:holomorphic-corner-exit-g1g2` and packaged in `cor:holomorphic-corner-exit-inherits` (using the corner-exit existence `prop:holomorphic-corner-exit-L1`).
2. **Jet-to-global / "graph on all of Q" lemma (closing the Prop. 8.95-type gap).** The deterministic global-sheet criterion is `lem:global-graph-contraction`; its holomorphic implementation is `prop:cell-scale-linear-model-graph`, and the resulting finite-template realization (disjointness + mass comparison) is `prop:finite-template`.
3. **Global face-cancellation / Thm. 8.46(iv) from mismatch estimates.** Per-face control is `prop:prefix-template-coherence` (with the vertex-template edit regime `prop:vertex-template-face-e` global summation is `cor:global-flat-weighted`, and the combined global conclusion is the numbered estimate `prop:glue-gap`.
4. **Positivity/type hypothesis for the holomorphic-chain theorem.** The promotion is stated and invoked in `thm:realization-from-almost`, with applicability discussed in `rem:hl-applicable`; algebraicity in the projective case is recorded in `rem:chow-gaga`.

Responses to the referee's "Ten key items"

1. **Core bottleneck trio / uniformity.** The three named bottlenecks correspond (in `hodge-SAVE-dec-12-han` to: `prop:glue-gap` (global gluing), `prop:holomorphic-corner-exit-L1` (corner-exit holomorphic slivers), and `lem:bergman-control` + `lem:global-graph-contraction` (uniform holomorphic control on an entire cell). Uniformity in cell index and label is enforced by the finite direction net package `prop:corner-exit-template-net` and the all-label execution `prop:global-coherence-all-labels`.
2. **Global gluing theorem.** The global small-boundary statement is recorded as `prop:glue-gap` and is proved by combining: `prop:global-coherence-all-labels` (all labels), `thm:sliver-mass-matching` (template bookkeeping), `cor:global-flat-weighted` (global weighted sum), and the standard flat-norm filling argument in Substep 4.2 and Step 5.

3. **Combinatorial bookkeeping conditional on (G1)–(G2).** The needed holomorphic verification of (G1)–(G2) is explicit in `prop:holomorphic-corner-exit-g1g2` and `cor:holomorphic-corner-ex`

4. **Decisive gap: flat templates vs holomorphic realizations.** Closed by the explicit stability statement `prop:holomorphic-corner-exit-g1g2` (no accidental face hits; localized trace; per-face mass comparability), whose hypotheses are produced by `prop:finite-template` and `prop:holomorphic-corner-exit-L1`.

5. **Holomorphic realization of separated planes / global-control gap.** The "single sheet on all of Q" mechanism is formalized by `lem:global-graph-contraction` and implemented for holomorphic sections in `prop:cell-scale-linear-model-graph`. The packaged finite-template realization is `prop:finite-template`.

6. **Corner-exit holomorphic slivers inherit weaknesses of the previous step.** Closed by `prop:holomorphic-corner-exit-L1` together with the inheritance corollary `cor:holomorphic-corner-exi`

7. **"Microstructure/gluing estimate established" must be numbered.** This has been promoted to the numbered proposition `prop:glue-gap` (and is the cited input in the mass-correction Step 5 and in `thm:automatic-syr`).

8. **SYR / promotion step needs exact hypotheses.** The calibrated-limit bridge is stated as `thm:realization-from-almost`. The paper's "automatic SYR" summary is `thm:automatic-syr`, and Harvey–Lawson applicability is discussed in `rem:hl-applicable`.

9. **Flat-norm filling must be parameter-free.** The filling argument used is explicit (flat-norm decomposition + Federer–Fleming isoperimetric inequality) in Step 5 of `hodge-SAVE-dec-12-handoff.` and does not impose extra dimension restrictions beyond standard GMT hypotheses for integral currents.

10. **Structure / dependency chain.** The manuscript already contains an explicit proof-structure list in the Introduction ("Proof structure") and local-to-global steps (Steps 1–6 in `hodge-SAVE-dec-12-handoff.tex`). For the specific activation gate, `rem:activation-hypotheses-status` gives a compact pointer list showing where (i)–(iv) are proved in the corner-exit route.

Responses to the referee's "Missing steps" list

1. **Global microstructure matching (MM/edit-regime gate).** Closed by `thm:sliver-mass-matching-on` together with the verification remark `rem:activation-hypotheses-status` and the all-label package `prop:global-coherence-all-labels`.

2. **"No vanishing sliver mass".** The construction does not assume a uniform lower bound $m_{Q,a} \gtrsim h^{2n-2p}$ as a *formal hypothesis* of the weighted bookkeeping. Instead, the bookkeeping is formulated in the sliver-compatible weighted form `cor:global-flat-weighted`, and the absence of hidden lower bounds is stated in `rem:no-vanishing-piece-mass`. For corner-exit templates specifically, the footprint mass scale is explicit in terms of the template parameter s (and is uniform along the template order): see `lem:corner-exit-mass-scale`. In particular, choosing $s = \theta h$ yields the absolute-scale bound $\mathbf{M}(\text{one piece}) \asymp h^{2n-2p}$ if desired.

3. **Exponent/parameter regime barrier.** The scaling condition is computed in `rem:weighted-scaling` and combined with the standard reduction `rem:lefschetz-reduction` to cover the full Hodge statement.

4. **Slow variation from rounding.** Derived quantitatively in `lem:slow-variation-rounding` and preserved under 0–1 discrepancy rounding in `lem:slow-variation-discrepancy`.

5. **Cohomology quantization / fixed m.** The integrality/period constraints are enforced in `prop:cohomology-match`, and fixed-m (no drift) is built into `thm:global-cohom` and summarized in `thm:automatic-syr` (iii).

6. **Flat-norm filling vs positivity.** The filling current is not required to be positive; only its mass must vanish. This is isolated in `prop:almost-calibration` and explained in `rem:correction-not-pos`

7. **Harvey–Lawson/Siu promotion.** The calibrated-limit theorem is `thm:realization-from-almost`, which cites Harvey–Lawson and uses Federer–Fleming compactness.

8. **Signed decomposition rationality.** Handled in `lem:signed-decomp`, with $N \in \mathbb{Q}_{>0}$ chosen explicitly (see the proof).

9. **Local holomorphic sheets / disjointness / uniform control.** The uniform C^1 control input is `lem:bergman-control`, and the global-sheet criterion is `lem:global-graph-contraction`. Disjointness and mass comparability under separation are handled by `lem:sliver-stability`, `lem:sliver-packing`, and `prop:finite-template`.

10. **Calibration–coercivity constants.** The quantitative coercivity inequality is stated explicitly as `thm:cal-coercivity`, with dependence recorded in the surrounding discussion.

Counterexample / failure-mode stress tests (and where they are excluded)

The referee lists several plausible failure modes. In the present draft they are ruled out as follows:

- **Accidental face hits (a sliver hits a non-designated face).** Excluded by the deterministic face-incidence statement (G1-iff) in `prop:holomorphic-corner-exit-g1g2` and its packaged form `cor:holomorphic-corner-exit-inherits`.

- **Multiplicity/orientation mismatch across an interface.** The activation scheme uses a common labeling/template across interfaces (vertex-star coherence `rem:vertex-star-coherence` and the all-label packaging `prop:global-coherence-all-labels`), so mismatches reduce to controlled prefix edits and are accounted for in the interface bookkeeping (`prop:prefix-template-coherence`
and `prop:vertex-template-face-edits`).

- **Heavy-tail mismatch (few unmatched indices but disproportionately large boundary mass).** Excluded by the "no heavy tail" reduction `lem:oh-face-edit-regime` together with uniform per-face slice comparability (G2) in `prop:holomorphic-corner-exit-g1g2`; for corner-exit templates, uniformity along the order is made explicit in `lem:corner-exit-mass-scale`.

- **Sheet splitting (local control but multi-sheet behavior elsewhere in the cell).** The "graph on the whole cell" criterion is formalized in `lem:global-graph-contraction` and implemented for holomorphic complete intersections in `prop:cell-scale-linear-model-graph`; the finite-template realization is `prop:finite-template`.

- **Positivity gap (cone alignment weaker than the positivity notion needed for the promotion theorem).** The promotion step is invoked only for the *limit* current after almost-calibration has been established (`prop:almost-calibration` and `thm:realization-from-almost`). The limit is a ψ-calibrated *integral* current, so Harvey–Lawson applies (see also `rem:hl-applicable`).

Required references (where they are used)

- **Federer–Fleming / flat norm / isoperimetric filling:** used in Step 5 (boundary correction) and in `prop:glue-gap`.
- **Harvey–Lawson calibrated structure theorem:** used in `thm:realization-from-almost` and discussed in `rem:hl-applicable`.
- **Analytic $\Rightarrow$ algebraic in the projective case:** Chow/GAGA, recorded in `rem:chow-gaga`.
- **Bergman/peak sections and jet control:** used in `lem:bergman-control` (with standard references listed there).

Notes on references

The manuscript cites: Federer–Fleming compactness/isoperimetric filling (Ann. of Math. 72 (1960)), Harvey–Lawson calibrated-geometry structure (Acta Math. 148 (1982)), and standard Bergman kernel/peak-section sources (Tian; Zelditch; Donaldson). The algebraicity of analytic cycles in the projective setting is concluded by Chow/GAGA (standard references include Hartshorne or Griffiths–Harris).