Response: where SYR / mass convergence is claimed in the manuscript

December 16, 2025

Answer to Amir Rahnama

In the current manuscript `hodge-SAVE-dec-12-handoff.tex`:

1. **The constant c_0 (mass target) is defined and used explicitly.**

   It is introduced in Theorem `thm:realization-from-almost` as

   $$c_0 \;:=\; \langle A, [\psi] \rangle \;=\; \int_X m\, \gamma \wedge \psi, \qquad A = \mathrm{PD}(m[\gamma]), \quad \psi = \omega^{n-p}/(n-p)!.$$

   This is the cohomology–homology pairing lower bound for any cycle in class A.

2. **SYR is defined explicitly.**

   Definition `def:syr` defines Stationary Young–measure Realizability (SYR) for a cone-valued representative β by requiring the existence of a sequence of (stationary) integral cycles whose tangent-plane Young measures converge to a field with barycenter $\widehat{\beta}(x)$, and whose masses converge to the calibration pairing (i.e. to c_0).

3. **Yes: the manuscript claims (and labels) that SYR holds for the cone-valued representatives used in the proof.**

   The explicit global claim is Theorem `thm:automatic-syr` ("Automatic SYR for cone-valued forms"): for every smooth closed cone-valued (p,p)-form β representing a rational Hodge class $[\gamma]$, there exist integral cycles T_k with

   $$\partial T_k = 0, \qquad [T_k] = \mathrm{PD}(m[\gamma]) \text{ (for one fixed } m \text{ independent of } k),$$

   such that

   $$\mathrm{Mass}(T_k) \to m \int_X \beta \wedge \psi \;(= c_0),$$

   and the tangent-plane Young measures of T_k converge a.e. to a measurable field ν_x supported on complex $(n-p)$-planes with barycenter $\int \xi_P \, d\nu_x(P) = \widehat{\beta}(x)$.

Isolation of the mass-convergence estimate (referee-facing). Following your suggestion, the manuscript now isolates the global "almost-calibration" estimate for the *constructed* glued sequence as Proposition `prop:almost-calibration` (in Step 5):

$$0 \;\le\; \mathrm{Mass}(T_\varepsilon) - \langle T_\varepsilon, \psi \rangle \;\le\; 2\,\mathrm{Mass}(U_\varepsilon) \;\to\; 0,$$

and since $\langle T_\varepsilon, \psi \rangle = c_0$ by the fixed homology class and $d\psi = 0$, this gives $\mathrm{Mass}(T_\varepsilon) \to c_0$ as a single clean quantitative argument.

What is not claimed: the manuscript does *not* claim a general principle of the form "stationary $\Rightarrow$ almost-calibrated." Instead, the mass convergence to c_0 is proved for the specific constructed sequences (and recorded in `thm:automatic-syr`), and then Theorem `thm:realization-from-almost` upgrades "almost-calibrated" sequences to a calibrated limit current.