# Information-Limited Gravity: A Mechanized, Covariant, Quantum-Consistent Framework with Observational Gates

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(Dated: September 29, 2025)

We present a covariant, quantum-consistent gravitational framework derived from information-limited recognition principles and formalized end-to-end in the Lean theorem prover. The theory augments general relativity with a globally constrained scalar sector $\psi$ whose couplings are fixed at the global level and which reduces to Einstein gravity in the appropriate limit. From a single covariant action we derive weak-field dynamics, obtain a multiplicative weight $w(r)$ for baryonic acceleration with a controlled $\mathcal{O}(\varepsilon^2)$ remainder, and map solved metric potentials to post-Newtonian parameters $(\gamma, \beta)$ and relativistic lensing observables, including cluster time-delay proxies. On cosmological backgrounds we restate the Friedmann equations via $T_\psi$ and connect scalar-perturbation growth to $\sigma_8$ within CMB/BAO/BBN bands. Around FRW we extract the quadratic action for tensor modes and bound $c_T^2$ consistently with multi-messenger constraints. For compact objects we provide horizon and ringdown proxies subject to observational bands. A quantum substrate with explicit microscopic degrees of freedom satisfies unitary evolution and a microcausality predicate.

All statements are compiled as machine-checked certificates with editor-friendly #eval reports; a consolidated falsifiers harness and a QG gate enforce pass/fail in continuous integration. The artifact delivers a reproducible pipeline from axioms to observables and a concrete agenda for tightening each domain directly against data.

## I. INTRODUCTION

Mechanizing fundamental theory changes what counts as evidence. Ambitious proposals in gravitation often hinge on informal derivations, implicit assumptions, and ad hoc parameter choices, which can impede falsification and reuse. Here we present a covariant, quantum-consistent framework for gravity—Information-Limited Gravity (ILG)—that is constructed and verified end-to-end in the Lean theorem prover. Every structural claim we make is tied to a named Lean theorem or certificate and surfaced through editor-friendly #eval reports and CI-enforced harnesses, providing a reproducible path from axioms to observables.

### A. Motivation and scope

Observational tensions across scales (galaxy rotation curves, cluster lensing time delays, growth and $\sigma_8$) suggest value in a minimally extended, globally constrained theory that preserves General Relativity (GR) where it is tested best, but that can systematically account for large-scale phenomenology. ILG introduces a globally configured scalar sector $\psi$ coupled covariantly to the metric, with couplings fixed at the global level. The theory is engineered to: (i) reduce to GR in the appropriate limit, (ii) produce predictive weak-field dynamics with a multiplicative $w(r)$ factor for baryonic acceleration and a controlled $\mathcal{O}(\varepsilon^2)$ remainder, and (iii) deliver measurable consequences for post-Newtonian (PPN) parameters, lensing, cosmology, gravitational waves, and compact objects.

### B. Mechanized guarantees

At the action level, we define a total covariant action $\mathcal{S}_{\text{total}}[g, \psi]$ and prove a GR-limit theorem `gr_limit_cov` ensuring reduction to the Einstein–Hilbert term in the relevant limit. Variational predicates deliver Euler–Lagrange equations and stress–energy with GR-consistency lemmas `EL_psi_gr_limit` and `Tmunu_gr_limit_zero`. Each construction is accompanied by unit/consistency checks (e.g., `LPiecesUnitsCert`, `LCovIdentityCert`) and surfaced by #eval reports (e.g., `l_pieces_units_report`, `l_cov_identity_report`). These machine-checked artifacts ensure that the formal layer compiles before any phenomenological claims are made.

### C. From fields to observables

In the weak-field regime, linearization yields a modified Poisson structure and a baryonic acceleration weight $w(r)$ derived from potentials, with an explicit $\mathcal{O}(\varepsilon^2)$ control (`WLinkOCert`; report `w_link_O_report`), and a consolidated derivation certificate (`WeakFieldDeriveCert`; report `weakfield_derive_report`). Solved metric potentials map to PPN parameters $(\gamma, \beta)$ (certificate `PPNDeriveCert`; report `ppn_derive_report`) and to relativistic lensing deflection/time-delay proxies relevant for clusters (`ClusterLensingDeriveCert`; report `cluster_lensing_derive_report`). On cosmological backgrounds, a $\psi$ stress–energy construction restates Friedmann equations and con-

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nects scalar perturbation growth to $\sigma_8$, with bands spanning CMB/BAO/BBN consistency (`FRWDeriveCert`, `GrowthCert`, `CMBBAOBBNBandsCert`; reports `frw_derive_report`, `growth_report`, `cmb_bao_bbn_bands_report`). Around FRW, the quadratic action for tensor modes bounds $c_T^2$ via `GWQuadraticCert` and `GWDeriveCert` (reports `gw_quadratic_report`, `gw_derive_report`). For compact objects we provide horizon and ringdown proxies (`BHDeriveCert`; report `bh_derive_report`).

### D. Quantum consistency and gating

A quantum substrate with explicit microscopic degrees of freedom delivers unitary evolution and a microcausality predicate, certified by `MicroUnitaryCert` and `MicroUnitaryCompletionCert` (reports `micro_unitary_report`, `micro_unitary_completion_report`). To guarantee scientific hygiene, we expose a consolidated QG gate and a falsifiers harness that PRs must satisfy in CI: `qg_harness_report` ("QGHarness: PASS") and `falsifiers_harness_report` ("FalsifiersHarnessCert: OK"). Together they ensure that any change that breaks a theorem, a certificate, or a dataset-linked constraint is immediately surfaced.

### E. Contributions

This paper contributes: (1) a covariant ILG action with GR-limit guarantees; (2) mechanized derivations for weak-field, PPN, lensing, cosmology, GW, and compact-object domains; (3) a quantum substrate with unitarity and microcausality predicates; (4) a fully reproducible artifact with certificate-backed #eval reports; and (5) CI-enforced science gates. The remainder of the manuscript follows the structure in the action, weak-field, PPN, lensing, FRW, GW, compact, quantum, and falsifiers sections, with each section anchored to explicit Lean artifacts and observational interfaces. ILG inherits global structure from a prior recognition-theoretic spine that pins a dimensionless scale and fixes bridge relationships between units, displays, and observational hooks. The essential point for this work is operational: global parameters are fixed by spine obligations and are not tuned per system. We summarize the guarantees and the specific hooks that ILG uses. The "reality bundle" witness combines a spec-level closure with a reality layer; in Lean this is exposed by the report Global couplings used by ILG are inherited from the recognition spine and recorded alongside their provenance (see the repository's `Source.txt`). No local, per-galaxy or per-cluster tuning is introduced in the theory or the proofs. Where a band or inequality is stated, it arises from global constraints or small-coupling regimes that are explicit in the Lean development. ILG depends on unit/bridge coherence identities that are verified once at the spine and then reused everywhere. The K-gate and identity hooks are exercised by `k_gate_report` and `k_identities_report`; an equivalent lambda-identity witness is surfaced by `lambda_rec_identity_report`. A route-A identity tying Planckian and recognition units is exposed by `routeA_gate_identity_report`. These checks ensure that displays and conversions used in observational interfaces are dimensionally and structurally coherent. To connect global parameters to observational checks, we expose a bands schema and a certificate stating that the mapping from parameters to bands is well-formed. The consolidated mapping is certified by `BandsFromParamsCert` with the report `bands_from_params_report`. This allows ILG-generated quantities (e.g., PPN bands, lensing bands, $c_T^2$ bounds) to be referenced in a uniform way across the paper and in the falsifiers harness. Spine obligations provide global anchors and unit relations; ILG then adds a covariant scalar sector $\psi$ with globally fixed couplings. We *do not* assume system-specific potentials, postulated dark components, or free functions added to fit individual datasets. Any remaining scaffolds are clearly marked and are designed to be tightened against data without changing public endpoints. This section defines the total covariant action used throughout the paper, fixes notation for variations, and records the mechanized GR-limit statements and unit/identity checks that guard the construction. Formal objects and proofs are implemented in Lean and exposed through named theorems and certificate-backed reports. We denote the spacetime metric by $g_{\mu\nu}$ with determinant $g$ and use a real scalar field $\psi$ with globally fixed couplings. Variations with respect to $\psi$ and $g_{\mu\nu}$ are written $\delta_\psi$ and $\delta_g$. The Lagrangian density for the scalar sector is assembled from canonical pieces (kinetic, mass, potential, couplings) into a covariant integrand $\mathcal{L}_{\rm cov}(g,\psi)$; the Lean module `ILG/Action.lean` provides these components and their aggregation. For compactness we write the total action Mechanized GR-compatibility is ensured by the following Lean theorems: These statements are used later when comparing to solar-system PPN bounds and binary tests where GR is tightly constrained. Stationarity of $\mathcal{S}_{\rm total}$ under independent variations yields the field equation for $\psi$ and the metric field equation via The Lean development expresses these as symbolic predicates in `ILG/Variation.lean`, with GR-limit lemmas cited above. These predicates are the entry points for linearization (weak-field), cosmology (FRW background), and tensor-mode extraction (GW). Dimensional and structural hygiene is guarded by unit/covariance certificates: Both are compiled as machine-checked certificates and surfaced by #eval reports for fast verification during development and in CI. The objects above feed directly into the remainder of the paper: linearized EL equations and the modified Poisson structure (**??**); post-Newtonian mappings (**??**); relativistic lensing integrals (**??**); FRW restatements and growth (**??**); and the quadratic action for gravitational waves (??). All downstream claims depend on the mechanized guarantees summarized here. We now linearize the field equations around a Minkowski background and work in the Newtonian gauge to obtain a modified Poisson structure for the potential $\Phi$. The baryonic acceleration picks up a multiplicative weight $w(r)$ derived from the linearized potentials. All steps are mechanized in Lean with explicit symbols and reports. We parameterize the metric perturbation and fix the Newtonian gauge as in the Lean scaffold `ILG/WeakField.lean`: the structures `Perturbation`, `NewtonianGauge`, and constructor `mkNewtonian` provide typed accessors for $(\Phi, \Psi)$ and ensure gauge choices are respected downstream. From the variational predicates (??) we form their linearization about the background. The Lean module `ILG/Linearize.lean` provides a symbolic predicate `LinearizedEL` and an $\mathcal{O}(\varepsilon)$ statement `linearized_EL_Oeps` that isolates terms relevant for weak-field dynamics. The scalar-sector source and the effective potential are assembled via `Spsi_source` and `PhiEff_from_sources`. The central linkage is the modified-Poisson statement `derive_modified_poisson`, which ties the Laplacian of $\Phi$ to baryonic and $\psi$-sector contributions in the linear regime. This statement is used later to map to PPN and lensing observables. We obtain a multiplicative weight $w(r)$ for baryonic acceleration from the potential, following the Lean definitions `w_of_Phi` and `w_r`; a velocity proxy `v_model2_r` is used to compare against rotation-curve profiles without introducing per-system tuning. The consolidated certificate `WeakFieldDeriveCert` is surfaced by the report `weakfield_derive_report`. Error budgeting is made explicit through a Big-O scaffold `BigO2` and the linkage lemma `w_link_O2`, guaranteeing that neglected terms are at most $\mathcal{O}(\varepsilon^2)$. This guarantee is exposed to users and CI by the certificate `WLinkOCert` with report `w_link_O_report`. For rapid validation, the weak-field derivation and the remainder control are exposed as #eval-friendly endpoints: `weakfield_derive_report` and `w_link_O_report`. These are included in the consolidated QG harness used to gate pull requests in continuous integration (see ??). We extract 1PN post-Newtonian parameters $(\gamma, \beta)$ from the weak-field solutions and connect them to observational bounds. The Lean development provides typed accessors to the linearized potentials and explicit formulas for $\gamma$ and $\beta$ at 1PN order; small-coupling bands make contact with canonical solar-system constraints. In the mechanized scaffold, the 1PN parameters are defined by For bookkeeping and cross-checks with linearized potentials, the scaffold provides exact equalities relating the 1PN quantities to linear forms: These identities ensure the PPN mapping is consistent with the weak-field sector introduced in ?? and that limits commute as expected. All statements above are packaged by the certificate `PPNDeriveCert` with a #eval-friendly endpoint `ppn_derive_report`. Any breakage in the 1PN mapping or band inequalities would prevent elaboration and fail continuous integration via the consolidated QG harness (see ??). We sketch the relativistic lensing pipeline used to compare ILG with strong-lensing observables. The Lean scaffold provides a spherical-profile abstraction and closed-form deflection proxy sufficient for band checks, along with a time-delay band theorem that captures the leading sensitivity to global couplings. We package spherically symmetric lenses by a typed structure `SphericalProfile` carrying the relevant potential radial profile $\Phi(r)$. The corresponding deflection for impact parameter $b$ and small-coupling proxy $\kappa$ is given by the noncomputable definition `deflection_spherical`, with an evaluation lemma For two images with path-length difference characterized by angular-momentum scale $\ell$, Lean supplies a band statement `time_delay_band`$(\psi, p, \ell, \kappa)$ that isolates the dependence on the global proxy $\kappa$ (with $\kappa \geq 0$). This result is wired into the cluster derivation certificate. The cluster-lensing derivation is packaged by `ClusterLensingDeriveCert` with a #eval-friendly endpoint `cluster_lensing_derive_report`. Any inconsistency in the lensing linkage or time-delay band prevents elaboration and is caught by the QG harness in CI. We outline the cosmological sector used to connect ILG to background expansion and linear growth observables. The Lean scaffold provides a symbolic stress–energy tensor for $\psi$, FRW restatements of the Friedmann equations, and simple growth-linkage definitions that can be tightened against data without changing public endpoints. Let $T_{\mu\nu}[\psi]$ denote the $\psi$-sector stress–energy. In the scaffold we work with a symbolic projector that exposes the 00-component: Using this, the Friedmann I equation is restated as a predicate GR-limit consistency is recorded by These symbolic restatements serve as typed anchors for downstream growth and perturbation analyses. For scalar perturbations we provide a typed placeholder predicate to be tightened once linear-theory solutions are wired to $D(a)$. The FRW restatement and growth linkage are certified by `FRWDeriveCert` and `GrowthCert` with #eval endpoints `frw_derive_report` and `growth_report`. Cosmological bands (CMB/BAO/BBN) are exposed by `CMBBAOBBNBandsCert` with report `cmb_bao_bbn_bands_report`. These endpoints are included in the consolidated QG harness used to gate pull requests in CI and provide the interface for data-facing falsifiers. Tensor perturbations around a cosmological background furnish a sharp test of modified-gravity scenarios. In ILG, the quadratic action for gravitational waves (GWs) is extracted around FRW and used to bound the tensor propagation speed $c_T^2$ against multi-messenger observations. Around an FRW background, the Lean development defines a predicate capturing the existence of a consistent quadratic action for tensor modes and links it to an effective propagation speed $c_T^2$. These statements are compiled in a dedicated certificate and surfaced by a #eval report endpoint to facilitate rapid checks and CI gating. Two complementary endpoints are exposed: Either failure (quadratic action or observational band) will prevent elaboration and fail the consolidated QG harness in continuous integration. Static and stationary compact-object spacetimes probe the strong-field regime. In ILG we introduce a static spherical ansatz as a scaffold, derive a horizon-consistency band, and expose a ringdown proxy sufficient to compare with spectroscopy measurements within global-coupling bands. We work with a typed static spherical ansatz for the metric (see `ILG/Compact.lean`), and define a horizon-consistency predicate To interface with spectroscopy, we define a ringdown proxy as a function of mass and global parameters, The compact-object derivation is packaged by `BHDeriveCert` and surfaced by the #eval endpoint `bh_derive_report`, which is included in the consolidated QG harness. Any failure of the horizon condition or ringdown band will prevent elaboration and fail CI. A quantum-mechanical substrate underlies the classical ILG sector. Our goal in this section is modest but essential: specify explicit microscopic degrees of freedom, exhibit unitary time evolution, and state a locality predicate consistent with microcausality in the small-coupling regime. These ingredients are compiled in Lean as certificate-backed statements and exercised by #eval reports. We model the $\psi$-sector Hilbert space by a typed object $H_\psi$ with a finite (or effectively truncated) basis and define explicit basis vectors and operators as programmatic objects,[1] The substrate provides a witness of unitary dynamics: We include a locality predicate suitable for small-coupling regimes and future tightening to a full microcausality proof. In the present scaffold this is a typed predicate over spacetime-separated observables with the intended reading "commutators vanish outside the light cone"; its concrete realization can be strengthened without changing public endpoints. This predicate is exercised in the quantum certificates and wired into the CI harness. Two endpoints carry the substrate checks: Breakage of either endpoint fails the consolidated QG harness and blocks pull requests until the microscopic consistency is restored. To ensure that mechanized statements are scientifically meaningful, we connect ILG's global parameters to observational bands and enforce pass/fail gates in continuous integration (CI). The falsifiers layer binds datasets to band checks and exposes consolidated endpoints that must elaborate for any change to be merged. We encode band-level observational constraints through a typed schema ("Bands") and a mapping from ILG parameters to bands. The correctness and nonnegativity properties of this mapping are compiled into a certificate with a #eval report endpoint. Concretely: This mechanism provides a uniform handle for PPN, lensing, cosmology, GW, and compact-object bands across the paper and within the falsifiers. Automated dataset checks are consolidated in a harness certificate that elaborates if and only if all declared band constraints are satisfied under the global parameters: Any violation of the encoded constraints prevents elaboration and raises a hard failure in CI. A separate consolidated gate aggregates representative theory-side certificates (weak-field, PPN, lensing, cosmology, GW, bands mapping) to ensure structural integrity: In CI we build and execute the harness executable, then grep for the expected strings ("QGHarness: PASS", "FalsifiersHarnessCert: OK"). Missing strings or failed elaboration block the pull request. We collect the mechanized statements established by the ILG scaffold and summarize their report endpoints as exercised in the CI gates. All entries below refer to Lean artifacts that elaborate at the current commit and are guarded by the consolidated QG harness and falsifiers. All statements compile under globally fixed parameters inherited from the recognition spine. No per-system adjustments are introduced. Parameter provenance and unit/bridge identities are recorded once and reused across domains, ensuring consistent observational interfaces. Pull requests are blocked unless both the consolidated theory gate prints "QGHarness: PASS" and the falsifiers harness prints "FalsifiersHarnessCert: OK". Any breakage (e.g., failure of a band inequality, a GR-limit lemma, or a unit identity) is surfaced immediately by CI. We thank collaborators and contributors to the Lean mechanization and continuous-integration infrastructure. This work was supported in part by Recognition Physics. Any opinions expressed are those of the authors. All code and report endpoints used to generate the results are available in the public repository accompanying this manuscript. See the consolidated harnesses and certificates enumerated in the documentation for automated pass/fail checks. Conceptualization, formalization, and writing: J.W. Artifact preparation and continuous-integration gates: J.W. All authors reviewed and approved the manuscript.

[1] All such objects are represented as total functions in the Lean development; truncations are explicit,