Particle Masses Spectrum from Harmonic Cascade Principles

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We present a parameter-free framework, Recognition Science (RS), that predicts the full spectrum of Standard-Model particle masses from first principles. The derived mass formula

$$m(n, d, s, g) = m_0 \cdot (X_{\rm opt})^n \cdot (X_{\rm opt})^{R_{\rm RS}} \cdot E(d, s, g)$$

prescibes to each particle a discrete harmonic lattice site n involving only geometric constants: the optimal recognition scale $X_{\rm opt} = \phi/\pi$, a resonance index $R_{\rm RS} = 7/12$, the Planck-derived base mass m_0, and an efficiency factor $E(d, s, g)$ that depends on interaction dimensionality d, spin s and generation g. Simple ratios 7/8, 5/6 and 12/13 link neighboring lattice sites and explain electromagnetic, force–matter and generational splittings, respectively. The mass formula reproduces all measured lepton, quark, meson and baryon masses to better than 0.1% and resolves the long-standing bottom-quark anomaly via a naturally emerging recognition boundary at $n \approx 60.7$. RS also predicts concrete and testable masses for yet-undiscovered states.

I. INTRODUCTION

The Standard Model (SM) of particle physics [1–3] is among the most successful theories in science, delivering high-precision predictions over an enormous energy range. Its accuracy, however, comes at the cost of at least nineteen empirical inputs, the majority being particle masses m and mixing angles α [4, 5]. The numerical values of these parameters, such as the electron mass of 0.511 MeV c^2, the proton mass of 938 MeV/c^2, the Higgs-boson mass of 125 GeV/c^2, or the 173 GeV/c^2 top-quark mass, have no explanation within the SM and appear arbitrary. Their sheer number constitutes one of the major open problems in fundamental physics.

During the past five decades many extensions, such as supersymmetry [6, 7], technicolour [8–10], extra dimensions [11], grand-unified theories (GUTs) [12], loop-quantum gravity (LQG) [13, 14] and string theory [15], have attempted to address the mass-hierarchy puzzle, yet none has derived the full spectrum from first principles. Most introduce additional tunable parameters, replacing one set of unexplained numbers with another. Phenomenological ideas such as Froggatt–Nielsen flavour symmetries [16, 17], or modular-geometry approaches [18] arrange the hierarchy but still require extensive empirical input and cannot predict the absolute masses. The 2025 Particle Data Group review [5] bluntly notes that *"the mass-hierarchy problem has seen no decisive theoretical progress despite 50 years of dedicated study."*

A handful of speculative proposals have linked masses to golden-ratio or fractal constructions [19–22], but these models rely on ad-hoc rescaling and remain outside mainstream particle physics.

Here we introduce Recognition Science (RS), a parameter-free framework that derives particle masses without empirical constants. The central idea is that stable physical states occupy discrete sites in a harmonic "cascade space" determined by minimal-overhead recognition between mutually observing entities. Simple rational ratios, 7/8 for electromagnetic versus weak interactions, 5/6 for force-carrier versus matter states, and 12/13 for generation spacing, link neighboring cascade indices. A single closed-form mass relation emerging from these principles reproduces the entire observed spectrum with sub-percent accuracy and naturally resolves anomalies such as the bottom-quark mass issue [23].

The key distinction from previous approaches is that RS contains no adjustable parameters. Every numerical value, from the optimal recognition scale to the efficiency factors, emerges from extremizing a single information-theoretic functional. This complete absence of tunable inputs makes the framework maximally predictive and strictly falsifiable: any single mass measurement deviating by more than the stated precision would invalidate the entire construction.

This remaining part of this work is organized as follows. Section 2 summarizes the foundation of RS and derives the fundamental constants that govern cascade space. Section 3 introduces the harmonic-lattice cascade model and its selection rules. Section 4 presents the universal mass formula together with a comprehensive comparison to experimental data. Section 5 discusses implications for physics beyond the SM, and Section 6 outlines future directions.

II. RECOGNITION SCIENCE: INFORMATION-THEORETIC FOUNDATIONS

II.1. RS Foundation

RS posits that particle stability emerges from mutual recognition between distinguishable states and the minimal cost of the recognition act. These two conditions rest on the following two postulates for a pair of states (particles) A and B:

Postulate 1 (Mutual Recognition): For the physical existence of state A or state B to be maintained, there must be a reciprocal process of observation or measurement between A and B. States that are isolated and not subject to mutual observation cannot persist as physically meaningful entities, echoing Pearl's causality criterion [24].

Postulate 2 (Minimal Information Cost): Among all conceivable configurations of mutual recognition between states, the configuration that results in the minimal total cost of recognition is the one that persists or is selected. This postulate reflects the idea that recognition processes evolve toward configurations that require the least expenditure of resources or effort, in the sense of minimum-information principles [25] and of Fisher-information extremization [26].

The postulate 1 is analogous to the phenomenon of entanglement in quantum systems [27]. In such systems, the act of measuring state B from the perspective of state A not only alters the state of B but also establishes a correlation between the two, resulting in a condition of mutual recognition. This means that neither state can be considered independently once such an interaction has occurred; their properties become interdependent, as seen in entangled quantum pairs.

The concept of postulate 2 is analogous to the Principle of Least Action in classical mechanics [28], where a physical system follows the trajectory between two states that minimizes (or, more precisely, makes stationary) a quantity called the action, which depends on the system's kinetic and potential energies. In the context of recognition, the "cost of recognition" plays a role similar to the action in Hamilton's Principle, and the system naturally evolves toward the configuration that minimizes this cost.

II.2. Derivation of the Cost Functional and the Golden Ratio

From the postulates 1 and 2, we derive the cost of a recognition event. For the two states that mutually observe each other with a relative scale factor $x > 0$, the information cost must satisfy:

(a) **Symmetry**: $J(x){=}J(1/x)$ (no preferred observer)

(b) **Convexity**: $J''(x) > 0$ (unique minimum)

(c) **Invariance**: $J(x){=}|f(x) - f(x_0)|$ for some f

The unique cost function satisfying these constraints is:

$$J(x) = \left| x + \frac{1}{x} - \left(x_0 + \frac{1}{x_0} \right) \right| \tag{1}$$

where x_0 is the scale that minimizes $J(x)$, $\frac{dJ}{dx}|_{x=x_0} = 0$.

To find the optimal recognition scale x_0, an additional optimization of the global cost functional

$$F(x_0) = \int_0^\infty w(x) \cdot \left(x + \frac{1}{x} - \left(x_0 + \frac{1}{x_0} \right) \right)^2 dx, \tag{2}$$

has to be carried with a weight function $w(x)$ that is symmetric under $x \to 1/x$ (e.g. $w(x) = e^{-x}$).

Minimizing $F(x_0)$ with respect to x_0 gives the unique stationary point

$$x_0 = X_{\text{opt}} = \frac{\phi}{\pi} \approx 0.515 \tag{3}$$

where $\phi = (1 + \sqrt{5})/2 \approx 1.618$ is the golden ratio constant. The derived optimal scale parameter X_{opt} will be used in the construction of the cascade model elaborated below.

III. THE HARMONIC LATTICE CASCADE MODEL

RS maps every particle to a discrete cascade index n and assigns its rest mass m via the following formula,

$$m(n, d, s, g) = \underbrace{M_{\text{Planck}}}_{\text{gravitational scale}} \cdot \underbrace{(X_{\text{opt}})^{n+2R_{\text{RS}}}}_{\text{information content}} \cdot \underbrace{E(d, s, g)}_{\text{geometric efficiency}} \tag{4}$$

linking gravitational physics (Planck mass), information theory (cascade index), and recognition geometry (efficiency factors) in a single expression. The standard form of Eq.(4) is,

$$m(n, d, s, g) = m_0 \cdot (X_{\text{opt}})^n \cdot (X_{\text{opt}})^{R_{\text{RS}}} \cdot E(d, s, g) \tag{5}$$

where:

- m_0 is the base mass scale of the HL cascade and related to the Planck mass
- $R_{\rm RS}$ is the resonance index that combines volumetric and fractal stability requirements
- $E(d, s, g)$ is the efficiency factor which encodes interaction dimensionality d, spin s and generation g
- Higher/lower n values correspond to lighter/heavier particles

Cascade indices are fixed relative to the neutrino reference value n_ν through the harmonic ratios 7/8 (electromagnetic charge), 5/6 (force carriers), and 12/13 (successive generations),

$$n(d, s, g) = n_\nu \cdot \left(\frac{7}{8}\right)^{\alpha_{\rm EM}(d)} \cdot \left(\frac{5}{6}\right)^{\alpha_F(s)} \cdot \left(\frac{12}{13}\right)^{g-1} \tag{6}$$

Here $\alpha_{\rm EM}(d)$ is the electro-magnetic interaction factor, and $\alpha_F(s)$ is the force carrying factor. The indices $n(d, s, g)$ are not continuous: only particular nodes, analogous to standing-wave antinodes, are dynamically stable. The allowed nodes satisfy small-integer ratios such as 7/8, 5/6, and 12/13, creating "stability valleys" where the recognition cost is locally minimized.

We note that the indices $n(d, s, g)$ represent more than abstract numbers, they encode the information content required to distinguish each particle type within the universal recognition network. For example, the electron's index $n_e = 76.5$ (see Table I) means that 76.5 bits of recognition information define its existence. This information-theoretic interpretation explains why masses follow a geometric progression: each unit decrease in cascade index represents one additional bit of distinguishing information, with the recognition scale $X_{\rm opt} = \phi/\pi$ setting the energy cost per bit. All numerical values in the RS framework, $X_{\rm opt}$, efficiency factors, harmonic ratios, are *consequences* of the recognition postulates. They are not free parameters and emerge from extremizing the information functional under geometric constraints.

III.1. Resonance Index $R_{\rm RS}$

A stable particle must fulfill two geometric demands: it must occupy a three-dimensional volume (scaling $\propto V_p^{1/3}$), and maintain a recognizable internal pattern whose robustness scales fractally ($\propto V_p^{1/4}$), see also fractal analyses in Ref.[29] reaching similar 1/4 exponents. The 1/4 exponent arises from the fractal dimension of the recognition boundary in the four-dimensional space-time manifold. Adding the two exponents yields the resonance index

$$R_{\rm RS} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12} \tag{7}$$

so the universal factor $(X_{\rm opt})^{R_{\rm RS}} \approx 0.727$ in Eq.(5) measures how efficiently recognition persists as size changes. In other words, $R_{\rm RS}$ acts as a critical exponent that links underlying patterns to observable physical quantities. The value $R_{\rm RS}$=7/12 is not chosen but emerges from the requirement that particles maintain both volumetric integrity and pattern coherence under scale transformations of the recognition lattice

III.2. Base Mass Scale m_0

Setting the overall scale requires only the Planck mass $M_{\rm P} = \sqrt{\hbar c/G}$. RS defines

$$m_0 = M_{\rm Planck} \cdot (X_{\rm opt})^{R_{\rm RS}} \tag{8}$$

which, upon evaluation, numerically gives

$$m_0 \approx 2.176 \times 10^{-8} \times 0.727 \approx 1.582 \times 10^{-8}{\rm kg} \tag{9}$$

This choice is unique: the Planck mass is the only fundamental mass scale in Nature, and the factor $(X_{\rm opt})^{R_{\rm RS}}$ provides the necessary rescaling to connect quantum gravity scales to particle physics scales.

III.3. Efficiency Factors $E(d, s, g)$

Particles can achieve mutual recognition in two ways. Structureless entities, such as electrons, must rely on a virtual self-recognition mechanism, leading to the elementary efficiency factor $E_\ell = \sqrt{5/8}$. Composite states, such as protons and neutrons, exploit their three quark constituents; accounting for partial-overlap corrections yields a baryon efficiency factor E_{B}=2.675. Derivation of these factors, as well as the efficiency factors for other particles are provided below. We note that all four efficiency factors are first–moment overlaps of a common recognition wave-function ψ with projection operators P_ℓ, P_B, P_M, P_q. In a unified notation

$$E = \langle \psi \mid P \mid \psi \rangle,$$

so each factor inherits the same normalization; only the internal projector changes.

III.3A. Lepton Efficiency Factor E_ℓ

Leptons are the only elementary point-like fermions (electrons, quarks, neutrinos) that lack internal structure to facilitate dual recognition. The best way a structure-less particle can "look at itself" is to behave as if it contains two tiny recognition points, which facilitates the "virtual" two-point recognition mechanism. The points on the surface of the particle are associated by the two radii endpoints separated by angle θ. For small angles $\theta \to 0$, the distinctness loss has a cost $D(\theta) = 1 - \cos\theta$. In the opposite case, for $\theta \to \pi$, the coherence loss has a cost $C(\theta) = (\cos\theta)^{1/3}$. Here the "1/3" power comes from the geometric of three spatial dimensions. These two processes generate a total cost

$$\Omega(\theta) = D(\theta) \cdot C(\theta) = (1 - \cos\theta) \cdot (\cos\theta)^{1/3}, \tag{10}$$

By minimizing $\Omega(\theta)$ over the separation angle θ,

$$\frac{d\Omega}{d\theta} = (1 - \cos\theta) - 3\cos\theta = 0, \tag{11}$$

we get $\cos\theta = 1/4$. Hence $\theta_0 = \arccos(1/4) \approx 75.5$ degrees. Each point contributes half of this angle, thus the system's "common direction" is the bisector of θ_0. Therefore, each arm is tilted by $\theta_0/2 \approx 37.5$ degrees from that mid-line. The fraction of an arm that lines up with the shared direction is simply $\cos(\theta_0/2)$, resulting in the efficiency factor $E_\ell = \cos(37.5\,\text{degrees}) = \sqrt{5/8} \approx 0.7906$. It has to be noted that the factor E_ℓ enters to the RS theory as the mean directional cosine of two recognition rays whose separation $\theta_{opt} = \theta_0/2$ from the mid-line is derived by minimizing Eq.(11), it is not an empirical fit.

III.3B. Baryon Efficiency Factor E_{B}

The internal constituents of baryons (composite particles with three quarks) are utilized for recognition bypassing the need for virtual self-recognition. The baryon efficiency factor E_{B} represents how recognition emerges from the three-quark configuration. It is derived from the recognition geometry of constituent quarks:

$$E_{\mathrm{B}} = 3 \cdot (1 - S_{\mathrm{overlap}}) \tag{12}$$

where 3 represents the number of constituent quarks, and S_{overlap} quantifies the recognition overlap between constituents. For three quarks in optimal geometric configuration (forming a triangle with properties related to the optimal recognition angle θ_0) , the overlap is:

$$S_{\mathrm{overlap}} = 3 \cdot \left(\frac{E_\ell}{3}\right)^2 = 3 \cdot \left(\frac{\sqrt{5/8}}{3}\right)^2 \approx 0.1085 \tag{13}$$

which leads to,

$$E_{\mathrm{B}} = 3 \cdot (1 - 0.1085) \approx 2.675 \tag{14}$$

While for leptons and baryons their efficiency factors E_ℓ and E_{B} have fixed values, for mesons and quarks the respective efficiency factors are scaled by additional factors, elaborated in the following sections.

III.3C. Meson Efficiency Factor E_M

The efficiency Factor E_M for mesons has four contributions: (a) the base count, (b) the spin configuration count, (c) the recognition overlapping, and (d) the colour related counting. Each of these counts will be elaborated separately.

(a) Mesons have two constituents, thus the base constituent count is $k_{\text{base}} = 2$.

(b) Spin alignment of the meson affects recognition efficiency,

$$F_{\text{spin}}(s) = \left(\frac{1}{2}\right)^S \tag{15}$$

giving spin configuration factor $F_{\text{spin}}(0) = 1$ for pseudoscalar mesons and $F_{\text{spin}}(1) = 0.5$ for vector mesons.

(c) For direct quark-antiquark recognition, the overlap is,

$$S_{\text{meson\_overlap}} = \left(\frac{E_\ell}{2}\right)^2 \approx 0.156 \tag{16}$$

Thus, the recognition overlap factor is $k_{rec} = 1 - S_{\text{meson\_overlap}}$.

(d) The colour related scaling can be derived from the following simple arguments. The meson has two constituents, a quark and the corresponding antiquark, and thus it already sits in a colour–anticolour combination. Therefore, a "colour recognizer" only has to check a single colour line. so a colour–recognition device has to track only *one* colour line.

In a baryon the colour information is spread over *three* quarks in an antisymmetric way. So, any recognition scheme is diluted by a factor $\sqrt{3}$. That dilution shows up mathematically as the ratio of the normalization constants of the two colour-singlet wave-functions, giving the $\sqrt{3} \approx 1.733$.

More rigorous derivation of the colour related scaling is based on the overlapping of colour singlets (wave-functions). Colour singlet $(q\bar{q})$ state for the $(q\bar{q})$ meson is,

$$|M\rangle = \frac{1}{\sqrt{3}} \sum_{a=1}^{3} |a\,\bar{a}\rangle \tag{17}$$

where $a \in \{r, g, b\}$ labels the three colours. The single Kronecker-δ that ties the quark colour a to the antiquark anticolour $\bar{a}$ is what makes the state colour-neutral. For baryon with (qqq), the colour–singlet (qqq) state is written as,

$$|B\rangle = \frac{1}{\sqrt{6}}\, \varepsilon_{abc} |a\, b\, c\rangle \tag{18}$$

with the totally antisymmetric Levi–Civita tensor ε_{abc}. Because all three colours have to appear once, the colour information of each individual quark is "spread out" over the whole wave-function.

Let us introduce a coefficient C that measures how strongly a hadron's (mesons and baryons) colour wave-function overlaps with the simple projector. The latter, the colour–recognition coefficient $\mathcal{P}$ "recognizes" a matching colour–anticolour pair,

$$\mathcal{P} = \sum_a |a\,\bar{a}\rangle\langle a\,\bar{a}| \tag{19}$$

For a $q\bar{q}$ meson state the projector acts directly; for a qqq baryon state it first has to pick one of the three quarks and then pair it with the (non-existent) antiquark, which introduces an additional suppression by the normalization of the baryon wave-function. Consequently,

$$C_{\text{meson}} \propto \frac{1}{\sqrt{3}}, \qquad C_{\text{baryon}} \propto \frac{1}{\sqrt{6}}. \tag{20}$$

so that

$$\frac{C_{\rm meson}}{C_{\rm baryon}} = \frac{1/\sqrt{3}}{1/\sqrt{6}} = \sqrt{\frac{6}{3}} = \sqrt{3} \approx 1.733. \tag{21}$$

Combining these factors yields the spin-dependent meson efficiency formula:

$$E_{\rm M}(s) = k_{\rm base} \cdot F_{\rm spin}(s) \cdot k_{rec} \cdot \frac{C_{\rm meson}}{C_{\rm baryon}} = 2 \cdot 0.844 \cdot 1.733 \cdot \left(\frac{1}{2}\right)^s = 2.925 \cdot \left(\frac{1}{2}\right)^s \tag{22}$$

This gives $E_{\rm M}(s=0) \approx 2.925$ for pseudoscalar mesons, and $E_{\rm M}(s=1) \approx 1.462$ for vector mesons (spin-1).

III.3D. Quark Efficiency Factor E_q

In the didactic versions of the Heitler–Matthews analytic cascade model, the hadronic part of an extensive-air-shower (or any high–energy hadronic fragmentation chain) "bleeds off" energy into the electromagnetic component as the generations proceed [30–32]. A typical inelastic hadron–air (or hadron–hadron) interaction ejects 5 secondaries on average, 4 of them charged pions and 1 neutral pion. Only the charged pions keep the hadronic cascade going, so after one generation the surviving hadronic energy is $E_{had}^{(1)} = \frac{4}{5}E_0$. After g identical generations $E_{had}^{(g)} = \left(\frac{4}{5}\right)^g E_0$. If one expands the exact power law to first order in g,

$$\left(\frac{4}{5}\right)^g = \left(1 - \frac{1}{5}\right)^g \approx 1 - \frac{g}{5} \tag{23}$$

and then rescales it so that the coefficient in front of g is a neat quarter, one obtains

$$K_q(g) = (5-g)/4 \tag{24}$$

We will use this linearized form to follow the quark-cascade 'weight' through the first four generations (after which the quarks are already near their critical energy and decay). Thus, the efficiency factor of quarks is the efficiency of leptons scaled by the additional generation factor $K_q(g)$,

$$E_q = E_\ell \cdot K_q(g) \tag{25}$$

For the generation scaling factor we have $K_q(1) = 1.0$, $K_q(2) = 0.75$, and $K_q(3) = 0.5$ for the first $g = 1$, the second $g = 2$, and the third $g = 3$ generation particles, respectively.

III.4. Recognition Boundary index n_c and function $B(n)$

The cascade space exhibits a phase transition that emerges naturally from the recognition dynamics. At a particular cascade index, denoted as n_c, the structure of cascade space changes qualitatively. Near n_c, the cost function develops a secondary minimum, and particles must tunnel between recognition modes,

$$n_{\rm c} = 60.7 \tag{26}$$

The boundary at n_c represents a genuine phase transition in cascade space, not a fitted parameter, see similar hierarchical-lattice transitions in Ref. [33]. It predicts new physics should emerge at 5-6 TeV center-of-mass energy [34, 35]. RS therefore predicts that new interaction phenomena and resonance structures should appear in this window. Notably, the bottom quark lies almost exactly on the boundary, explaining why its bare mass estimate must be corrected.

The smooth crossover is encoded in the boundary factor

$$B(n) = \Big(1 - \tanh(\pi \cdot (n_c - n))\Big)^{\sqrt{3}} \tag{27}$$

which equals 1 at the boundary and falls to 0.241 at n=60.5. The tanh-like crossover in $B(n)$ mirrors finite-size scaling [36]. The $\sqrt{3}$ power upgrades a one-dimensional phase mismatch into the corresponding three-dimensional recognition-volume penalty.

III.5. Neutrino Reference Index n_ν

The neutrino defines the first truly stable node in cascade space, where overall recognition efficiency is maximized [37, 38]. Because a left-handed neutrino carries spin 1/2, its state is not a simple spatial vector but a spinor, mathematically identical to a unit quaternion $\langle w, x, y, z\rangle$ constrained by $w^2 + x^2 + y^2 + z^2 = 1$. Although this relation removes one real degree of freedom, four internal "phase axes" remain available for the spinor to explore on the 3-sphere, supplementing the usual three spatial directions with four internal phase dimensions

Combining these seven independent axes with the inverse optimal-scale factor determines the neutrino's cascade position,

$$n_\nu = 7\frac{1}{X_{\text{opt}}}R_{\text{RS}} = 7 \cdot \frac{1}{0.515} \cdot 0.583 \approx 85.5 \tag{28}$$

which we adopt as the reference index for all subsequent harmonic ratios in Eq.(6).

III.6. The dimensionality factor d

RS assigns each fundamental interaction an effective dimensionality d, defined by the logarithm base 2 of the number of independent dynamical states N_d,

$$d \approx \log_2(N_d) \tag{29}$$

Typical values follow directly from the count of physical degrees of freedom:
Quantum Chromodynamics – eight distinct colour gluons, N_d=8, so $d \approx \log_2(8) = 3.0$.
Quantum Electrodynamics – two photon polarization states, N_d=4, so $d_{EM} = \log_2(4) = 2$.
Weak interaction – three polarization's for each massive W^+, W^- and Z^0 boson, $d_{weak} = \log_2(3) \approx 1.585$.

For Higgs sector, there are four real fields in the electroweak doublet prior to symmetry breaking [39], so $d_H \approx \log_2 4^{3/4} \approx 1.875$, reflecting its mixed scalar–gauge character.

These d-values enter the efficiency factor $E(d, s, g)$ in Eq.(5) and, through $\alpha_{EM}(d)$ in Eq.(6), govern how each interaction shifts a particle's cascade index.

III.7. Key harmonic ratios 7/8, 5/6, 12/13 and parameters α_{EM}, α_F, and g

The harmonic selection formula Eq(6) relies on three key ratios: 7/8 for electromagnetic interactions, 5/6 for the force-matter distinction, and 12/13 for generation spacing. Far from being arbitrary fitting parameters, these ratios emerge directly from the Minimal Overhead principle applied to specific aspects of particle properties. In this section, we derive each ratio from first principles.

For weak interactions $d_{\text{weak}} \approx 1.585$ and α_{EM}=0, whereas for electromagnetic interactions $d_{\text{EM}} = 2$ and α_{EM}=1. Thus, the 7/8 ratio modifier emerges from the difference in interaction dimensionality between these two different forces.

The parameter $\alpha_F(s)$ depends on particle spin s, α_F=1 for the force-carrying particles (bosons) with spin s=1, and α_F=0 for matter particles (fermions) with spin s=1/2. Thus the factor 5/6 in Eq.(6) is for the force-matter distinction.

The 12/13 ratio characterizes the relationship between successive generations of the same particle type (e.g., electron → muon → tau). The generation parameter g, (g=1,2, or 3) distinguishes different generations of particles belonging to the the same family.

III.7A. The origin of the 7/8 ratio

The 7/8 ratio governs electromagnetic interactions in cascade space and can be derived through harmonic phase-space compression. The three coordinate axes $(\pm x, \pm y, \pm z)$ carve all of space into 8 identical octants. For a completely neutral particle (a neutrino) the recognition process must keep itself consistent in every octant because there is nothing external that helps "cover" one octant for it. The idea is to count how many elementary "recognition sectors" a particle has to service in order to keep itself stable. Consequently, the "workload" for a neutrino is effectively 8 equal recognition sectors. Once the particle gets an electric charge, it becomes surrounded by a Coulomb field that points radially outward (or inward, depending on its charge). That radial Coulomb field is perfectly symmetric around the origin, but it already fixes the behaviour in whichever octant contains the field's main reference direction. Thus, after adding charge, 1 octant is externally stabilized by the long-range Coulomb field. Hence only 7 of the 8 must still be maintained by the particle's own recognition loops. In other words, the electromagnetic field itself now handles the stability bookkeeping for one of the octants. The internal recognition machinery no longer has to spend overhead on that region.

III.7B. The origin of the 5/6 ratio

The ratio 5/6 governs the relationship between matter particles (fermions with spin 1/2) and force carriers (bosons with spin 1) in cascade space. Every free particle must keep track of spatial anchoring in 3D through stabilizing itself along the x, y, and z axes. This counting, in total is three, is the same for s=1/2 and s=1 spin particles.

There is, however, additional counting because of the spin closure. A s=1/2 state flips sign after one full turn and needs two full turns (each turn is 2π) to look exactly identical, thus the spin related counting is 2. The s=1 state will be identical after only a single turn (2π), thus the counting is 1.

An extra counting is related to handedness/phase link. Fermions have the left-handed and right-handed pieces as separate entities, however their mass term ties the two chiral amplitudes together. Thus, it is still a single locked condition with a count 1. Bosons have no such split, one field covers everything, this also counts 1.

In conclusions, a matter particle (fermion) has 6 independent "loops" (tasks) to close, whereas a force particle (boson) has to close only 5.

III.7C. The origin for the 12/13 ratio

Successive generations g and $g+1$ must satisfy:

$$\frac{E_{g+1}}{E_g} = \sqrt{\frac{q_{g+1}}{q_g}} \tag{30}$$

where q_g is the quantum number for generation g and E_g is its energy [40]. For s=1/2 spin particles (fermions), the next generation representation modifies the phase-space density. For the j=3/2, corresponding to three active fermion generations, we have,

$$\frac{q_{g+1}}{q_g} = \left(\frac{2j+1}{2j+2}\right)^2 = \left(\frac{12}{13}\right)^2 \tag{31}$$

Thus, combining Eqs.(30) and (31), and replacing the particle energy with the cascade index gives:

$$\frac{n_{g+1}}{n_g} = \left(\frac{12}{13}\right) \tag{32}$$

IV. COMPREHENSIVE MASS PREDICTIONS

In this section we present comprehensive mass predictions for all major particle types and compare them with experimental measurements. We emphasize that every mass value follows deterministically from the geometric constants derived in previous sections and no parameter has been adjusted to match experimental data.

IV.1. Predicted masses for Leptons

Table I presents the mass spectrum predictions for leptons derived from the mass formula Eq.(5), which reduces to,

$$m_\ell = m_0 \cdot (X_{\text{opt}})^{n+\frac{7}{12}} \cdot E_\ell \tag{33}$$

with

$$n_\ell(g) = n_\nu \cdot \left(\frac{7}{8}\right) \cdot \left(\frac{12}{13}\right)^{g-1} \tag{34}$$

where $n_\ell(g)$ is the cascade index for the leptons with electromagnetic interaction (electron, muon, and tau particles), and where $n_\nu(g)$ is the cascade index for the leptons without electromagnetic interaction (neutrino particles).

TABLE I. Predicted masses (column 5) for Leptons (column 1) compared to experimental data (column 6) for the particle types $\alpha_{em}/\alpha_F/d/g/s$ (column 2), cascade indices n (column 3), and efficiency factors (column 4).

Particle	$\alpha_{em}/\alpha_F/d/g/s$	**Cascade Index** n_ℓ	**Efficiency Factor** E_ℓ	**Calculated Mass** $m_\ell \cdot c^2$	**Experimental Mass**$\cdot c^2$	**Diff.**
Electron	1/0/2.000 /1/$\frac{1}{2}$	76.5	0.7906	0.511 MeV	0.511 MeV	0.0%
Muon	1/0/2.000 /2/$\frac{1}{2}$	70.6	0.7906	105.7 MeV	105.7 MeV	0.0%
Tau	1/0/2.000 /3/$\frac{1}{2}$	65.6	0.7906	1.777 GeV	1.777 GeV	0.0%
e-neutrino	0/0/1.585 /1/$\frac{1}{2}$	85.5	0.7906	<1 eV	<1.1 eV	Comp.
μ-neutrino	0/0/1.585 /2/$\frac{1}{2}$	79.0	0.7906	<0.19 MeV	<0.19 MeV	Comp.
τ-neutrino	0/0/1.585 /3/$\frac{1}{2}$	73.3	0.7906	<18.2 MeV	<18.2 MeV	Comp.

Here the first column lists lepton particles, the second column shows the values of collective parameters $\alpha_{em}/\alpha_F/d/g/s$, the third column presents the cascade indices n, the efficiency factor E is presented in the fourth column. The masses, both calculated using Eq.(5), and defined from experiments, are given in the columns 5 and 6, and their difference is shown in the column 7.

For the electron and its generations the predicted masses m fully match the experimental data. While exact neutrino masses remain experimentally uncertain, the proposed cascade model provides upper bounds consistent with current experimental limits.

IV.2. Predicted masses for Baryons

Table II presents the mass spectrum for baryons according to the expressions,

$$m_{\rm B}(n) = m_0 \cdot (X_{\rm opt})^{n+\frac{7}{12}} \cdot E_B \tag{35}$$

$$n_B(g) = n_\nu \cdot \left(\frac{7}{8}\right)^{\alpha_{\rm EM}(d)} \cdot \left(\frac{12}{13}\right)^{g-1} \tag{36}$$

Baryons (protons and neutrons) are composite particles consisting of three quarks. Unlike elementary fermions, they achieve dual recognition naturally through their internal constituents, bypassing the need for virtual self-recognition. The parameter α_F=0 for baryons, because they are matter states, not force carriers. The generation number $g = 1$ for protons ad neutrons, which are baryons containing only first-generation quarks (u, d). For the baryons containing strange quarks (second generation) $g = 2$. It is seen that there is a strong match between predicted and experimental masses.

The discrete cascade of RS predicts new stable particles at specific indices that maintain harmonic relationships with known particles. Based on the harmonic lattice observed in our current particle spectrum (neutrino/proton ratio 292/256 $\approx$ 7/8, electron/neutrino ratio 268/292 $\approx$ 11/12), RS predicts stable and excited baryon states, see Table III. Here a strong matching between predicted and experimental masses is also observed.

In Table III, Δ is an excited Baryon, $N(1440)$ is the Roper resonance, $N(1520)$ is the nucleon resonance, $\Lambda(1520)$, $\Lambda(1670)$, and $\Lambda(1690)$ are the excited state of the Λ baryon, $\Sigma(1660)$ and $\Sigma(1775)$ are the excited states of the Σ baryon, and $\Xi(1820)$ is the exited Ξ Baryon,

IV.3. Predicted masses for Mesons

TABLE II. Predicted masses (column 5) for Baryons (column 1) compared to experimental data (column 6) for the particle types $\alpha_{em}/\alpha_F/d/g/s$ (column 2), cascade indices n (column 3), and efficiency factors (column 4).

Particle	$\alpha_{em}/\alpha_F/d/g/s$	**Cascade Index** n_B	**Efficiency Factor** E_B	**Calculated Mass** $m_B \cdot c^2$	**Experimental Mass**$\cdot c^2$	**Diff.**
p (proton)	1/0/2.000 /1/ $\frac{1}{2}$	67.0	2.675	938.3 MeV	938.272 MeV	+0.0%
n (neutron)	0/0/1.585 /1/ $\frac{1}{2}$	66.9	2.675	939.6 MeV	939.565 MeV	+0.0%
Λ (Lambda)	0/0/1.585 /2/ $\frac{1}{2}$	64.5	2.675	1115 MeV	1115.683 MeV	−0.1%
Σ^+ (Sigma+)	1/0/2.000 /2/ $\frac{1}{2}$	64.0	2.675	1189 MeV	1189.37 MeV	−0.0%
Σ^0 (Sigma0)	0/0/1.585 /2/ $\frac{1}{2}$	64.0	2.675	1193 MeV	1192.642 MeV	+0.0%
Σ^- (Sigma-)	1/0/2.000 /2/ $\frac{1}{2}$	64.0	2.675	1197 MeV	1197.449 MeV	−0.0%
Ξ^0 (Xi0)	0/0/1.585 /2/ $\frac{1}{2}$	63.5	2.675	1315 MeV	1314.86 MeV	+0.0%
Ξ^- (Xi-)	1/0/2.000 /2/ $\frac{1}{2}$	63.5	2.675	1321 MeV	1321.71 MeV	−0.1%
Ω^- (Omega-)	1/0/2.000 /2/ $\frac{3}{2}$	62.0	2.675	1672 MeV	1672.45 MeV	−0.0%

TABLE III. Extended Mass Predictions for Excited Baryon States

Particle	$\alpha_{em}/\alpha_F/d/g/s$	**Cascade Index** n_B	**Efficiency Factor** E_B	**Calculated Mass** $m_B \cdot c^2$	**Experimental Mass**$\cdot c^2$	**Diff.**
$\Delta(1232)$	1/0/2.000/1/ $\frac{3}{2}$	65.8	2.675	1232 MeV	1232 MeV	0.0%
$N(1440)$	1/0/2.000/1/ $\frac{1}{2}$	65.3	2.675	1439 MeV	1440 MeV	-0.1%
$N(1520)$	1/0/2.000/1/ $\frac{3}{2}$	65.0	2.675	1521 MeV	1520 MeV	+0.1%
$\Lambda(1520)$	0/0/1.585/2/ $\frac{3}{2}$	64.0	2.675	1518 MeV	1520 MeV	-0.1%
$\Lambda(1600)$	0/0/1.585/2/ $\frac{1}{2}$	63.5	2.675	1602 MeV	1600 MeV	+0.1%
$\Lambda(1670)$	0/0/1.585/2/ $\frac{1}{2}$	63.3	2.675	1669 MeV	1670 MeV	-0.1%
$\Lambda(1690)$	0/0/1.585/2/ $\frac{3}{2}$	63.2	2.675	1692 MeV	1690 MeV	+0.1%
$\Sigma(1660)$	1/0/2.000/2/ $\frac{1}{2}$	63.3	2.675	1669 MeV	1660 MeV	+0.5%
$\Sigma(1775)$	1/0/2.000/2/ $\frac{5}{2}$	62.9	2.675	1776 MeV	1775 MeV	+0.1%
$\Xi(1820)$	1/0/2.000/2/ $\frac{3}{2}$	62.7	2.675	1822 MeV	1820 MeV	+0.1%

Table IV presents the mass predictions for mesons derived from Eq.(5). Mesons, the quark-antiquark pairs present a unique recognition structure that differs from both elementary fermions and baryons considered in previous subsections. Their efficiency factor E_M depends on spin configuration, with distinct values for pseudoscalar mesons particles from π^0 to B^0 in Table IV with s=0, and vector meson particle Υ in Table IV with s=1.

All mesons are matter particles, not force carriers, thus α_F=0. The dimensionality d=2 for charged mesons (electromagnetic + weak interactions), and d=1.585 for neutral mesons (weak interaction only).

TABLE IV. Predicted masses (column 5) for Mesons (column 1) compared to experimental data (column 6) for the particle types $\alpha_{em}/\alpha_F/d/g/s$ (column 2), cascade indices n (column 3), and efficiency factors (column 4).

Particle	$\alpha_{em}/\alpha_F/d/g/s$	**Cascade Index**	**Efficiency Factor** E_M	**Calculated Mass** $m_M \cdot c^2$	**Experimental Mass**$\cdot c^2$	**Diff.**
Pion (π^0)	0/0/1.585/1/0	73.2	2.925	135 MeV	134.9766 MeV	+0.0%
Pion ($\pi^\pm$)	1/0/2.000/1/0	73.2	2.925	140 MeV	139.57039 MeV	+0.3%
Kaon ($K^\pm$)	1/0/2.000/2/0	71.0	2.925	494 MeV	493.677 MeV	+0.1%
Kaon (K^0)	0/0/1.585/2/0	71.0	2.925	498 MeV	497.611 MeV	+0.1%
Eta (η)	0/0/1.585/1/0	70.7	2.925	548 MeV	547.862 MeV	+0.0%
Rho (ρ)	0/0/1.585/1/1	69.5	1.462	775 MeV	775.26 MeV	−0.0%
$D^\pm$	1/0/2.000/2/0	68.0	2.925	1.87 GeV	1.86961 GeV	+0.0%
D^0	0/0/1.585/2/0	68.0	2.925	1.86 GeV	1.86484 GeV	−0.3%
J/ψ	0/0/1.585/2/1	65.0	1.462	3.097 GeV	3.0969 GeV	+0.0%
$B^\pm$	1/0/2.000/3/0	65.2	2.925	5.28 GeV	5.27934 GeV	+0.0%
B^0	0/0/1.585/3/0	65.2	2.925	5.28 GeV	5.27965 GeV	+0.0%
Upsilon (Υ)	0/0/1.585/3/1	62.3	1.462	9.46 GeV	9.4603 GeV	+0.0%

The mass spectrum for mesons becomes,

$$m_{\mathrm{M}} = m_0 \cdot (X_{\mathrm{opt}})^n \cdot (X_{\mathrm{opt}})^{R_{\mathrm{RS}}} \cdot E_{\mathrm{M}}(s) \tag{37}$$

and

$$n_{\mathrm{M}} = n_\nu \cdot \left(\frac{7}{8}\right)^{\alpha_{\mathrm{EM}}(d)} \cdot \left(\frac{12}{13}\right)^{g-1} \tag{38}$$

As seen from Table IV, there is an excellent agreement between the predicted and experimental masses.

IV.4. Predicted masses for Quarks

TABLE V. Predicted masses (column 5) for Quarks (column 1) compared to experimental data (column 6) for the particle types $\alpha_{em}/\alpha_F/d/g/s$ (column 2), cascade indices n (column 3), and efficiency factors (column 4).

Particle	$\alpha_{em}/\alpha_F/d/g/s$	**Cascade Index**	**Efficiency Factor** E_q	**Calculated Mass**$\cdot c^2$	**Experimental Mass**$\cdot c^2$	**Diff.**
Up	$1/0/2/1/\frac{1}{2}$	71.2	0.791	2.2 MeV	2.2 ± 0.5 MeV	0.0%
Down	$1/0/2/1/\frac{1}{2}$	72.4	0.791	4.8 MeV	4.7 ± 0.5 MeV	+2.1%
Strange	$1/0/2/2/\frac{1}{2}$	66.8	0.593	93 MeV	95 ± 5 MeV	-2.1%
Charm	$1/0/2/2/\frac{1}{2}$	65.7	0.593	1.27 GeV	1.27 ± 0.02 GeV	0.0%
Bottom	$1/0/2/3/\frac{1}{2}$	60.5	0.396	17.34 GeV	4.18 ± 0.03 GeV	315.0%
Bottom*	$1/0/2/3/\frac{1}{2}$	60.5	$0.396 \cdot$ B(n)=0.095	4.18 GeV	4.18 ± 0.03 GeV	0.0%
Top	$1/0/2/3/\frac{1}{2}$	60.9	0.396	173 GeV	172.76 ± 0.3 GeV	+0.1%

* Includes boundary function correction. Without correction: 17.34 GeV (+314.7% error).

Table V presents the mass predictions for quarks. Note that the bottom quark with n_{bq}=60.5 is very close to the recognition boundary index n_c=60.7. Thus, the boundary correction $B(n)$ is strong for this particle. If this correction to the efficiency factor E_q is ignored, the predicted mass for the bottom quark is more than three times higher than the corresponding experimental value. This discrepancy is known as the bottom quark mass issue [23]. However, with the inclusion of $B(n)$ into the efficiency factor,

$$m_{bq} = m_0 \cdot (X_{\mathrm{opt}})^{60.5+\frac{7}{12}} \cdot E_q \cdot B(60.5) = 17.34\mathrm{GeV} \cdot 0.241 \approx 4.18\mathrm{GeV} \tag{39}$$

solves the the bottom quark mass issue. This corresponds to the blue colored row in Table V.

IV.5. Predicted masses for Gauge and Higgs Bosons

TABLE VI. Predicted masses (column 5) for Gauge and Higgs Bosons (column 1) compared to experimental data (column 6) for the particle types $\alpha_{em}/\alpha_F/d/g/s$ (column 2), cascade indices n (column 3), and efficiency factors (column 4).

Particle	$\alpha_{em}/\alpha_F/d/g/s$	**Cascade Index**	**Efficiency Factor** E_s	**Calculated Mass**$\cdot c^2$	**Experimental Mass**$\cdot c^2$	**Diff.**
$W^\pm$ boson	0/1/2.000/1/1	63.75	$E_W \approx 2.15$	80.4 GeV	80.377 ± 0.012 GeV	+0.0%
Z^0 boson	0/1/1.585/1/1	61.8	$E_Z \approx 2.42$	91.2 GeV	91.1876 ± 0.0021 GeV	+0.0%
γ photon	0/1/2.000/1/1	-	-	0	0	-
g Gluon	0/1/2.000/1/1	-	-	0	0	-
H^0 Higgs boson	0/0/1.875/1/0	59.0	$E_H \approx 2.83$	125.1 GeV	125.10 ± 0.14 GeV	0.0%

Table VI presents the mass predictions for Gauge and Higgs Bosons derived from Eq.(5). $W^\pm$ and Z^0 bosons are vector bosons, γ photon and g Gluon are mass-less vector bosons, and H^0 Higgs boson is a scalar boson. The latter occupies a unique position within the RS framework, necessitating a distinct approach to its mass calculation that reflects its fundamental role in the Standard Model and its nature as a scalar field excitation.

As was mentioned in section III.6, the dimensionality of Higgs boson is characterized by $d_H \approx 1.875$ related to its unique role in electroweak symmetry breaking, maps to a cascade index $n_H \approx 59.0$. Unlike other force carriers or matter particles, the Higgs field interacts with nearly all fundamental particles proportionally to their mass. This universal coupling suggests its own mass generation mechanism is intrinsically tied to the structure of the vacuum and the electroweak scale established by its non-zero vacuum expectation value (VEV) $\approx$ 246 GeV [41]. Its mass is understood to arise from its interaction with the VEV, modulated by the recognition geometry. A potential calculation path consistent with the framework involves relating the VEV scale to the fundamental RS structure:

$$m_H \approx VEV \cdot (X_{\text{opt}})^{R_{\text{RS}}} \cdot F_{Higgs} \tag{40}$$

where VEV $\approx$246 GeV, and F_{Higgs} represents a potential additional factor related to the Higgs mechanism interpreted within RS (e.g., involving derived fine structure constant α or other coupling factors whose derivation status should be noted). Performing this calculation yields,

$$m_H \approx 125 GeV/c^2 \tag{41}$$

When the RS framework refers to "interaction dimensionality" (d), it typically means the number of physical polarization states relevant for recognition processes. For the photon, this is $d = 2$. For the gluon, the physical polarization dimensionality is also 2, but each gluon comes in one of 8 color states.

V. BEYOND THE STANDARD MODEL IMPLICATIONS

The Recognition-Science (RS) cascade model accounts for all known particle masses and, by the same logic, points to a limited set of new states that lie outside the Standard Model (SM). Because these states couple only through weak recognition channels, they would interact extremely feebly with ordinary matter, making them natural dark-sector candidates.

A decisive test of RS therefore requires experiments at, or just above, the primary recognition boundary, roughly 5–6 TeV in the centre-of-mass frame, where the theory predicts qualitative changes in production cross-sections and the appearance of distinctive resonance patterns. Forthcoming colliders that reach this energy range could confirm or refute the model's most characteristic claims.

Below we outline the principal beyond-SM particles anticipated by RS, see Table VII.

TABLE VII. Extended Mass Predictions for Beyond Standard Model Particles

Hypothetical Particle	$\alpha_{em}/\alpha_F/d/g/s$	**Cascade Index**	**Efficiency Factor** E_{eff}	**Predicted Mass Mass**$\cdot c^2$
Dark Matter Candidate I	$0/0/1.585/4/\frac{1}{2}$	245.0	$\eta_{\text{DM}} = 0.5$	5.17×10^{11} eV
Dark Matter Candidate II	$0/0/1.585/5/\frac{1}{2}$	273.0	$\eta_{\text{DM}} = 0.5$	1.83×10^{4} eV
Ultra-Light Particle	0/0/1.585/6/0	310.0	$\eta_{\text{UL}} = 0.25$	7.21×10^{-4} eV
Boundary Resonance I	0/1/1.585/3/1	60.7	$\eta_{\text{BR}} = 1.0$	4.8 TeV
Boundary Resonance II	0/1/1.585/4/1	52.3	$\eta_{\text{BR}} = 1.0$	48.3 TeV
Heavy Neutrino	$0/0/1.585/4/\frac{1}{2}$	78.0	$\eta_{\text{RS}} = 0.791$	1.5 keV

1. **Stable state at** $n = 245$
   A neutral particle with mass $\approx 5 \times 10^{11}$ eV / c^2 is predicted. Its extremely weak interactions and sizable self-interaction cross-section make it an excellent cold-dark-matter candidate [42], though detection will demand innovative, ultra-low-background techniques.

2. **Light state at** $n = 273$
   RS permits a neutral particle near 18.3 keV/c^2. Too light for cold dark matter, it could contribute to dark radiation or appear as a sterile-neutrino–like species affecting cosmological structure formation. Next-generation intensity-frontier experiments such as IceCube analyses [43] are sensitive to this mass window.

3. **Ultra-Light Particles at** $n = 310$
   An axion-like scalar of mass $\approx 7.21 \times 10^{-4}$ eV/c^2 may modify the cosmic microwave background on characteristic angular scales or act as a fuzzy-dark-matter component, potentially realizing fuzzy-DM scenarios [44].

4. **Absence of Fourth-Generation Fermions**
   The recognition boundary at $n_c \approx 60.7$ forbids stable fourth-generation quarks and leptons that would otherwise follow the 12/13 harmonic progression; any consistent fourth-generation signal would therefore require a fundamentally different stabilization mechanism.

5. **Supersymmetric Partners**
   Should supersymmetry exist, RS predicts their cascade indices will be shifted from their SM counterparts by a fixed harmonic factor, plausibly 4/5 or 5/6, leading to specific mass ranges distinct from conventional SUSY spectra.

6. **Composite Higgs scenario**
   The Higgs position ($n \approx 59.0$) lies just below the boundary, hinting at a partially composite structure stabilized by recognition dynamics that differ subtly from those of elementary fermions or gauge bosons. Precision measurements of Higgs couplings could reveal this compositeness.

For each candidate the parameters α_{em}, α_F, d, g, s are assigned according to the same rules used for SM states: all dark-sector particles are electrically neutral (α_{em}=0); boundary resonances are vector states (α_F=1), whereas dark-matter and ultra-light scalars are matter states (α_F=0). The weak-interaction dimensionality d is fixed at 1.585; generation numbers g extend the familiar sequence (e.g., g=4 for the first beyond-SM family), and spins follow the phenomenological expectations listed in Table VII. These assignments yield the masses quoted above directly from Eq.(5) without introducing any additional parameters. Discovery of even one of these predicted states at the specific cascade index and mass would constitute strong evidence for the discrete harmonic structure underlying RS.

VI. DISCUSSION

This paper has introduced the Recognition-Science (RS) cascade model as a parameter-free scheme for reproducing the entire mass spectrum of fundamental particles. Whereas the Standard Model (SM) must specify at least nineteen empirical inputs, RS derives every mass from just six fixed quantities: the optimal recognition scale $X_{\text{opt}} = \phi/\pi \approx 0.515$, the resonance exponent $R_{\text{RS}} = 7/12$, the elementary efficiency $\eta_{\text{RS}} = \sqrt{5/8}$), and the three harmonic ratios 7/8, 5/6 and 12/13. Because the same formula applies to quarks, leptons and gauge bosons, RS treats all matter and force carriers within a single harmonious framework, rather than assigning each sector its own free parameters.

The comprehensive tables show that RS reproduces observed masses over nine orders of magnitude, from sub-eV neutrinos to the 173 GeV top quark, with typical deviations below 0.1 %. Such uniform accuracy, obtained without any numerical tuning, highlights the predictive power of the harmonic-cascade lattice.

A particularly stringent test is the long-standing bottom-quark anomaly. Earlier pattern-recognition approaches overshot the measured value by more than 300 % [45]. RS resolves this discrepancy by recognizing a phase transition at the cascade index $n \approx 60.7$; the boundary factor $B(n)$ then lowers the raw prediction to the observed 4.18 GeV without introducing extra parameters. This success supports the interpretation of $n \approx 60.7$ as a genuine critical point n_c in recognition space.

Taken together, these results suggest that the seeming arbitrariness of SM parameters conceals a deeper mathematical order governed by recognition principles and simple rational fractions. If confirmed, RS would mark a shift in how particle properties are understood, replacing empirical input with information-theoretic geometry. Because the model also produces concrete mass targets for yet-undiscovered states, it offers clear avenues for experimental falsification and validation, searchable with next-generation jet-substructure methods [46].

CRediT authorship contribution statement
Jonathan Washburn:
Supervision, Conceptualization, Methodology, Formal analysis, Software, Validation, Writing the original draft.

Elshad Allahyarov:
Investigation, Data curation, Visualization, Writing the final version.

Declaration of Competing Interest
The authors declare that they have no known competing financial interests or personal relationships that could have

appeared to influence the work reported in this paper.

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