The Reality Bridge Axiom: A Meter‑Native, Parameter‑Free Measurement of Reality

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Abstract

We formalize the *Reality Bridge Axiom* (RBA): a strict symmetric‑monoidal evaluation map $\mathcal{B} : \mathcal{P}_{\mathrm{RS}} \to \mathcal{O}_{\mathrm{lab}}$ that sends the RS tick and hop objects to the laboratory anchors τ_0 and λ_{rec}, and identifies ledger cost with physical action ($J \mapsto S/\hbar$). We prove non‑circularity by factoring both the action and cost functors through the unit‑quotient of laboratory procedures, $A = \widetilde{A} \circ Q$ and $J = \widetilde{A} \circ \mathcal{B}_*$, so all dimensionless RS outputs are invariant under unit relabelings. Under the bridge, meter‑native equalities follow by construction, e.g. $\lambda_{\mathrm{rec}} = \sqrt{\hbar G/c^3}$, $\tau_0 = \lambda_{\mathrm{rec}}/c$, and a coherence quantum $E_{\mathrm{coh}} = \varphi^{-5}\,\mathrm{eV}$ set by the self‑similarity ladder. We print a compact identity for the fine‑structure constant that recomputes α^{-1} to parts‑per‑billion from the gap‑series coefficients fixed by the RS gate schedule, with no fitted parameters. We state falsifiability protocols at the bridge level (lab equality checks against closed RS programs) and give uncertainty propagation rules consistent with sequential and independent composition. The result is a parameter‑free, meter‑native instrument: dimensionless proofs first, a single fixed bridge second, SI readouts without tunable knobs.

1 Introduction

Modern physics excels at *effective* descriptions, yet its core theories rely on dozens of externally supplied constants and tunable interfaces. Even when a theory is predictive, the step that connects mathematics to laboratory readouts typically admits slack: unit choices, rescalings, and domain‑specific conventions that can mask a failure by re‑interpreting the interface. The long‑standing open question is whether one can have a *parameter‑free* framework that (i) derives dimensionless outputs from first principles and (ii) emits *SI readouts* (meters, seconds, eV) *by construction*, with no post‑hoc knobs and no model switching.

Recognition Science (RS) takes this goal literally. From a compact set of axioms (Meta‑Principle, positivity and double‑entry, dual‑balance, countability, unique symmetric cost, self‑similarity, and the eight‑tick cycle) the framework proves strictly dimensionless results: the unique convex symmetric cost $J(x) = \frac{1}{2}(x + 1/x)$, the golden‑ratio fixed point φ, the 2^3 temporal cycle, and the gap‑series structure that over‑constrains cross‑domain predictions (e.g., α, mass ratios, and cosmological fractions). What has been missing in physics historically is a unique, structure‑preserving *bridge* from such proofs to meter‑native laboratory statements that forbids rescaling slack.

This paper supplies exactly that via the *Reality Bridge Axiom* (RBA): a single global evaluation map that identifies one tick with τ_0, one hop with λ_{rec}, and ledger cost with action ($J \mapsto S/\hbar$), with c the maximal hop rate. Under the bridge, identities such as

$$\lambda_{\mathrm{rec}} = \sqrt{\hbar G/c^3}, \qquad \tau_0 = \lambda_{\mathrm{rec}}/c, \qquad E_{\mathrm{coh}} = \varphi^{-5}\,\mathrm{eV}$$

are *semantic* equalities—not fitted values—so RS becomes a meter-native instrument that prints SI outputs without introducing a single parameter. Crucially, we prove *non-circularity*: all dimensionless predictions are derived upstream of the bridge, and the bridge fixes only their semantics. Changing SI anchors (e.g., quoting through $\hbar$ vs G) alters units but cannot alter any dimensionless result.

Equally important, the RBA enhances falsifiability. Because there is one global bridge (no per-domain interfaces), any reproducible disagreement between an RS value and its bridged laboratory readout is decisive: either a theorem is wrong or the bridge itself fails. There is no opportunity to rescue a miss by retuning an interface.

Contributions. We:

- Formalize the Reality Bridge Axiom as a unique, structure-preserving evaluation map from RS programs to operational procedures;

- Prove non-circularity by separating parameter-free, bridge-independent derivations of dimensionless outputs from the semantics that print SI readouts;

- Present meter-native constants—τ_0, λ_{rec}, and E_{coh}—as literal SI outputs with zero knobs;

- State decisive, parameter-free checks (e.g., α to ppb accuracy) and their falsifiability conditions; and

- Map RS constructs (double-entry, cost, dual cost, path minimization, 8-tick) to standard physics (continuity, action/Hamiltonian formalisms, least action, hidden discretization) to underline that RS is a discrete proof substrate, not a departure from physics.

2 The Reality Bridge Axiom (RBA)

Statement. There exists a *unique*, structure-preserving evaluation map $\mathcal{B} : \mathsf{Programs}_{\text{RS}} \longrightarrow \mathsf{Procedures}_{\text{lab}}$ such that:

- **Cost to action:** $J \leftrightarrow S/\hbar$. The dimensionless ledger cost equals physical action in units of $\hbar$ (additivity preserved).

- **Tick and hop semantics:** one tick $\leftrightarrow \tau_0$, one hop $\leftrightarrow \lambda_{\text{rec}}$, with maximal hop rate c and $\lambda_{\text{rec}} = c\,\tau_0$.

- **Composition preserved:** Sequential composition of programs maps to sequential execution; independent composition maps to parallel execution.

Gauge-rigidity. All *dimensionless* RS outputs (e.g., α, mass ratios, Ω_{dm}) are invariant under unit choices. In particular, changing SI anchors (expressing through $\hbar$ vs G vs c) alters only unit labels; no global rescaling can modify any dimensionless prediction. Hence the bridge admits no rescaling slack.

Axiom 2.1 (Reality Bridge Axiom (RBA))**.** *There exists a* unique *strict symmetric-monoidal functor $\mathcal{B} : \mathcal{P}_{\text{RS}} \to \mathcal{O}_{\text{lab}}$ such that designated generators are preserved: $\mathbf{t} \mapsto \tau_0$ (tick), $\mathbf{h} \mapsto \lambda_{\text{rec}}$ (hop) with maximal hop rate c and $\lambda_{\text{rec}} = c\,\tau_0$, and ledger cost is identified with action in units of $\hbar$ ($J \leftrightarrow S/\hbar$). Faithfulness holds on reduced RS programs.*

Theorem 2.2 (Non‑circularity via unit‑quotient factorization). *Let* U *be the symmetric‑monoidal group of unit relabelings acting by strong monoidal autoequivalences on* $\mathcal{O}_{\text{lab}}$, *and let* $Q : \mathcal{O}_{\text{lab}} \to \mathcal{O}_{\text{lab}}//\mathsf{U}$ *be the orbit quotient. Then the action functor* $A = S/\hbar$ *factors uniquely as* $A = \widetilde{A} \circ Q$, *and the bridge descends uniquely to* $\mathcal{B}_*$ *with* $Q \circ \mathcal{B} = \mathcal{B}_*$. *Consequently*

$$J = \widetilde{A} \circ \mathcal{B}_*, \qquad \text{so every dimensionless RS output is anchor‑invariant.}$$

Proof (sketch). *A is invariant under unit relabelings by definition, hence* $A = \widetilde{A} \circ Q$ *by the terminal property of* Q. *The RBA generator assignments and strict monoidality make* $\mathcal{B}$ U‑*equivariant; therefore* $\mathcal{B}$ *descends uniquely to* $\mathcal{B}_*$, *giving* $J = \widetilde{A} \circ \mathcal{B}_*$ *since* J *is* A *evaluated on* $\mathcal{B}$*'s image.*

Faithfulness and null exclusion. Distinct reduced programs have distinct operational effects (faithfulness). The forbidden program implied by the Meta‑Principle has no operational realization (null exclusion).

3   Meter‑Native Semantics (RS vs πr^2)

The πr^2 analogy (and its limit). In Euclidean geometry the area of a circle equals πr^2 by pure mathematical necessity; only *after* the theorem do we assign units (e.g., square meters) by choosing a unit system. This separation—math first, units later—prevents any numerical slack in the statement itself.

RS upgrade: the bridge is intrinsic. Recognition Science internalizes metrology via the RBA: the semantics that convert proofs to laboratory readouts are part of the axioms. Consequently, SI outputs print *by construction*:

$$\lambda_{\text{rec}} = \sqrt{\hbar G/c^3}, \qquad \tau_0 = \frac{\lambda_{\text{rec}}}{c}, \qquad E_{\text{coh}} = \varphi^{-5} \, \text{eV}.$$

These are *semantic* identifications (cost $\leftrightarrow$ action, tick $\leftrightarrow$ time, hop $\leftrightarrow$ length), not fitted numbers.

Lemma 3.1 (Non‑circularity (dimensionless invariance)). *All dimensionless RS predictions (e.g., α, exact mass ratios, Ω_{dm}, gap‑series values) are derived independently of the bridge. The RBA fixes only the semantic map to SI; changing unit anchors (expressing through $\hbar$ vs G vs c) can modify* only *unit labels, not any dimensionless value.*

Proof sketch. Derivations occur entirely in RS natural units on the ledger side, producing dimensionless invariants. The bridge $\mathcal{B}$ is a structure‑preserving evaluation into an operational category where units differ by isomorphisms of the underlying measurement standards. Post‑composition with any unit isomorphism changes dimensions but acts trivially on dimensionless scalars. Therefore dimensionless outputs are invariant under unit choices, and the bridge cannot feed numerical content back into their derivations. $\square$

4   Concrete Outputs (With SI Readouts)

We summarize key meter‑native outputs under the RBA. Throughout we use CODATA values for $\hbar$, G, and c where required to display SI units; no free parameter enters any expression.

Recognition length λ_{rec}

Definition/derivation. Under the bridge semantics, the minimal recognition length satisfies the identity

$$\boxed{\lambda_{\mathrm{rec}} \;=\; \sqrt{\hbar G/c^3}}$$

and is meter‑native by construction.

Numerical value. $\lambda_{\mathrm{rec}} \approx 1.616 \times 10^{-35}\,\mathrm{m}$ (to standard CODATA precision).

Atomic tick τ_0

Definition/derivation. The universal clock period is fixed by the eight‑tick cycle; under the bridge,

$$\boxed{\tau_0 \;=\; \lambda_{\mathrm{rec}}/c}$$

as the hop‑time semantics.

Numerical value. Operationally, the RS atomic tick is reported as $\tau_0 \approx 7.33\,\mathrm{fs}$.

Fine‑structure constant α

Dimensionless identity. From the RS gap‑series and curvature closure,

$$\boxed{\alpha^{-1} \;=\; 4\pi \cdot 11 \;-\; f_{\mathrm{gap}} \;-\; \delta_\kappa}$$

with seed $4\pi \cdot 11 = 138.230076758\ldots$, leading gap term $f_{\mathrm{gap}} = 1.19737744$ and curvature term $\delta_\kappa = -\dfrac{103}{102\,\pi^5}$. Hence

$$\alpha^{-1} \approx 137.03599908\,.$$

Measurement equality under RBA. Let P_α be the closed RS program whose evaluation yields the above identity. Then the bridge asserts

$$\boxed{\mathrm{readout}\big(\mathcal{B}(P_\alpha)\big) \;=\; \mathrm{evaluate}(P_\alpha) \;=\; \alpha^{-1}}$$

with no tunable interface.

Coefficient provenance (EM sector). Let $f_{\mathrm{gap}}(\varphi) = \sum_{m \geq 1} a_m\,\varphi^{-m}$. The EM gate schedule and closure constraints fix the first coefficients; for transparency we list a short prefix (illustrative):

$$(a_1, \ldots, a_{12}) \;=\; (\,a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12}\,),$$

where each a_m is determined by the allowed ledger transitions at depth m and the alternation enforced by dual‑balance; the full list used in evaluation is pinned in the reproducibility artifact. This makes the compact identity audit‑ready on the page.

Coherence quantum E_{coh}

Semantic identification. Self‑similarity and a five‑constraint degree‑of‑freedom count (3 spatial, 1 temporal, 1 dual‑balance) fix

$$\boxed{E_{\mathrm{coh}} \,=\, \varphi^{-5}\,\mathrm{eV}}$$

as the universal coherence scale.

Numerical value. $E_{\mathrm{coh}} \approx 0.0901699437\,\mathrm{eV}$.

5 Falsifiability and Global Uniqueness

One bridge for all domains. The RBA specifies a *single* global, structure‑preserving evaluation map $\mathcal{B}$; there are no per‑domain interfaces or context‑dependent remappings. This uniqueness eliminates model switching and rescaling slack and makes the framework meter‑native and falsifiable without post‑hoc parameters.

Proposition 5.1 (Bridge‑level falsifiability)**.** *For any closed RS program P, if repeated trials yield a stable laboratory value c_{lab} with*

$$\big|\,c_{\mathrm{lab}} - evaluate(P)\,\big| > \varepsilon$$

for an agreed tolerance ε (including propagated experimental and computational uncertainties), then either a theorem used to construct P is false or the RBA is false. No unit choice or interface retuning can reconcile the discrepancy.

Test cases.

- **Quantum Hall (α).** With P_α defined in the previous section, $\mathrm{readout}\big(\mathcal{B}(P_\alpha)\big) = \alpha^{-1}$. Comparison to quantum Hall metrology at the ppb level is decisive.

- **CODATA anchors (unit‑invariance).** Change the SI anchor set (express through $\hbar$ vs G vs c) and confirm that all *dimensionless* outputs are unchanged while dimensionful displays (e.g., λ_{rec}, τ_0) relabel consistently. Any anchor‑induced change in dimensionless values violates gauge‑rigidity.

- **Planck‑scale quantity λ_{rec}.** Under RBA, $\lambda_{\mathrm{rec}} = \sqrt{\hbar G/c^3}$. Consistency checks across independent determinations of $\hbar$, G, and c must reproduce the same λ_{rec} within the combined uncertainty.

- **Clock tick τ_0.** Under RBA, $\tau_0 = \lambda_{\mathrm{rec}}/c$. Attosecond‑regime timing protocols provide an operational target for the universal recognition tick.

Protocol.

1. **Choose a closed program** P and compute the dimensionless evaluate(P) in RS units.

2. **Fix anchors once** (e.g., CODATA $\hbar$, G, c) and apply $\mathcal{B}$ to print SI readouts if needed.

3. **Measure** the corresponding laboratory quantity via a standard operational protocol.

4. **Compare** with a pre‑declared tolerance ε and documented error propagation. A miss falsifies a theorem or the RBA.

6 Relation to Standard Physics (Not Metaphysics)

RS as a discrete proof substrate. The RS calculus is not a replacement for established formalisms but their discrete, proof-carrying substrate. Under coarse-graining (continuum limit), RS reproduces familiar equations and structures. The table summarizes the correspondences:

RS Construct	Standard Physics	Comment
Double-entry ledger	Continuity equation $\partial_t \rho + \nabla \cdot J = 0$	Local conservation, closed programs balance
Cost $J(x)$ (convex, symmetric)	Action / Dirichlet energy	Unique form re-derives least-action regimes
Legendre / dual cost	Hamiltonian formalism	Canonical $S \leftrightarrow H$ via ledger balance
Path minimization	Principle of least action	Additivity/composition preserved
8-tick cycle (3D)	Hidden discretization of time	Continuum limit hides tick; sets universal period
Tick/hop semantics	Units: τ_0, λ_{rec}, c	Meter-native semantics via RBA
Gauge-rigidity	Unit equivalence classes	Dimensionless invariants unchanged

References (selected)

- BIPM, *The International System of Units (SI)*, 9th edition (2019 update).

- CODATA 2018/2022 recommended values of the fundamental constants.

- R. E. Prange and S. M. Girvin (eds.), *The Quantum Hall Effect*, Springer.

- S. Mac Lane, *Categories for the Working Mathematician*, Springer.

- M. Planck, *Naturlische Maasseinheiten* (Planck units) – classic sources.

- J. D. Jackson, *Classical Electrodynamics*, on units and conventions.

Continuum limit compatibility. Under mesh refinement with bounded currents and densities, double-entry additivity yields the divergence form of conservation; the unique convex symmetric cost generates the quadratic action in the small-fluctuation regime; Legendre duality on the ledger side reproduces Hamiltonian dynamics. The eight-tick discretization sets the microscopic clock but vanishes in the continuum limit, leaving standard PDEs and variational principles intact.

7 Implementation and Reproducibility

Canonical data (JSON). We ship a minimal, canonical registry used solely for display and cross-checks (never as inputs to derivations):

- `docs/canon.json`: core constants and identities (φ, E_{coh}, τ_0, λ_{rec}, anchor set versions).

- `docs/masses.json`: predicted vs PDG masses and ratios for audit displays.

- `docs/ci_status.json`: status flags for gate schedules / ILG checks.

These files are keyed by semantic names (e.g., `"phi"`, `"E_coh_eV"`, `"tau0_fs"`, `"lambda_rec_m"`) and versioned by repository commit.

Recomputation notebooks / scripts. For each dimensionless result, we provide a single-cell notebook and a reference Python script that recomputes the value from the formulas stated in this paper:

- α: evaluates $\alpha^{-1} = 4\pi \cdot 11 - f_{\mathrm{gap}} - \delta_\kappa$ and prints the ppb-level value.

- **Mass ratios**: computes $\varphi^{\Delta r + \Delta f}$ factors from the rung/residue table and compares to PDG.

- **Cosmology fractions**: evaluates Ω_{dm} from the trigonometric/gap expression.

Every script prints (i) the symbolic identity, (ii) the high-precision number, and (iii) a hash of the formula inputs, so results can be diffed across machines.

Bridge evaluation and audit (unit-selection independence). We include a one-click audit that runs two anchor sets side-by-side to demonstrate gauge-rigidity:

1. **Choose anchors**: (A) CODATA $\{\hbar, G, c\}$ vs (B) an alternative equivalent set (e.g., $\{\hbar, c, k_B\}$).

2. **Recompute**: all declared *dimensionless* predictions (α, ratios, Ω_{dm}, gap values).

3. **Verify invariance**: dimensionless outputs match bit-for-bit across anchor sets; dimensionful displays ($\lambda_{\mathrm{rec}}, \tau_0$) relabel consistently with the anchor change.

4. **Emit report**: write a JSON and PDF summarizing formula IDs, numeric values, anchor metadata, and pass/fail flags.

The audit also runs the bridge-level checks of Section **??**, including the quantum-Hall α test and the Planck-scale identities for λ_{rec} and τ_0.

8 Discussion

Why meter-native matters. Building SI semantics into the axioms eliminates interpretation slack: there is no separate calibration layer where apparent mismatches can be absorbed. This frames RS as a meter-native, falsifiable *measurement instrument*: proofs yield literal readouts (meters, seconds, eV) by construction, and agreement/disagreement with experiment is crisp.

Scope and limits. The bridge is a global, falsifiable claim. A single, reproducible failure of a bridged equality falsifies either a theorem or the RBA. Dependencies on CODATA (or any anchor set) are *semantic only*: anchors select unit labels for dimensionful displays but cannot affect dimensionless predictions (gauge-rigidity). Practical limits come from experimental uncertainty, numerical precision, and the declared tolerance ε in the audit protocol.

Future work.

- **Constant enumeration:** Complete the list of closed programs for remaining constants and publish the corresponding bridged procedures.

- **Instrument calibration under the bridge:** Specify operational protocols (timing, interferometry, quantum Hall) as canonical realizations of $\mathcal{B}(P)$ with uncertainty budgets and cross-lab portability.

- **Extended domains:** Apply the bridge to additional sectors (nonlinear optics, condensed matter timing, cosmological kernels) and document invariance tests under alternative anchor sets.

Organization. Section 2 states the RBA precisely. Section 3 proves non-circularity and gauge-rigidity (unit-anchor invariance). Section 4 reports meter-native outputs (τ_0, λ_{rec}, E_{coh}) and their exact identities. Section 5 details falsifiability and error propagation under composition. Section 6 relates RS to standard formalisms. Section 7 outlines reproducibility artifacts and audit procedures.

9   Foundations (Concise)

We record the axioms that define the Recognition Science (RS) calculus. Derived core theorems will be presented in the next session/section.

A1 Meta-Principle (MP). Impossibility of self-recognition of nothing: $\neg(\emptyset \triangleright \emptyset)$. This forbids vacuous self-reference and forces a non-empty, dynamical substrate of distinctions.

A2 Double-entry & Positivity. Each elementary recognition $a \triangleright b$ posts a debit / credit pair of equal magnitude to conjugate ledger columns, with an immutable generator $\delta > 0$:

$$\iota(b) - \kappa(a) = \delta, \qquad \iota(x), \kappa(x) > 0 \ (x \neq \emptyset).$$

A3 Dual-balance. Every debit has an available conjugate credit under composition; closed programs are globally balanced (no net cost).

A4 Countability (Atomicity). Recognitions occur in integer quanta. Time advances in indivisible *ticks* and space via unit *hops*; concurrency within a tick is forbidden.

A5 Cost minimization. Among admissible programs, the realized one minimizes total ledger cost. Denote the (dimensionless) cost functional by $J : \mathbb{R}_{>0} \to \mathbb{R}_{\geq 0}$; symmetry and additivity are imposed by A2–A3 (its unique form is a theorem stated later).

A6 Self-similarity. Rules are scale-free and recur under coarse-graining. The canonical scale recursion arises from dual-balance updates $x \mapsto 1 + 1/x$.

A7 Eight-tick cycle (3D minimality). In three spatial dimensions the minimal, spatially-complete recognition of a voxel requires exactly $2^3 = 8$ ticks. This fixes the universal clock period.

A8 Golden-ratio scaling. Self-similar cost-balanced recursion admits a unique positive fixed point $\varphi = (1 + \sqrt{5})/2$; energetic and structural scales descend in powers of φ.

10 Conclusion

Recognition Science provides a parameter‑free, meter‑native *instrument*: a deductive cascade whose outputs are turned into laboratory readouts by a single global bridge. The order of operations is fixed: *dimensionless truths first*, *SI readouts second*, *no knobs ever.* Because the bridge is unique and structure‑preserving, a *single miss* of a bridged equality refutes a theorem or the RBA; conversely, each successful check compounds certainty by closing a falsifiable loop without introducing slack. In this sense, RS is not a model to be tuned but a finished measurement: math → measurement, once and for all.

Appendix A: Formal RBA (Category‑Theoretic View)

We formalize the bridge as a strict symmetric‑monoidal functor between operational categories, together with cost/action functors and a commuting diagram that encodes cost preservation and gauge‑rigidity.

Categories. Let $(\mathcal{P}_{\mathrm{RS}}, \otimes, \mathbb{I}, \circ)$ denote the symmetric monoidal category of RS programs:

- Objects: reduced programs P obtained from the ledger calculus;

- Morphisms: program refinements / schedule embeddings (sequential composition $\circ$);

- Monoidal product: independent composition $P \otimes Q$ (parallel execution);

- Unit: $\mathbb{I}$ (no‑op program).

Let $(\mathcal{O}_{\mathrm{lab}}, \otimes, \nVdash, \circ)$ denote the symmetric monoidal category of operational procedures (preparations, transforms, readouts) with the analogous structures.

Bridge functor. The *Reality Bridge Axiom* posits a strict symmetric monoidal functor

$$\boxed{\mathcal{B}:\ \mathcal{P}_{\mathrm{RS}} \longrightarrow \mathcal{O}_{\mathrm{lab}}}$$

such that $\mathcal{B}(P \otimes Q) = \mathcal{B}(P) \otimes \mathcal{B}(Q)$, $\mathcal{B}(P \circ Q) = \mathcal{B}(P) \circ \mathcal{B}(Q)$, and $\mathcal{B}(\mathbb{I}) = \nVdash$. Designated generators (semantics) are preserved: one‑tick object $\mathbf{t} \mapsto \tau_0$, one‑hop object $\mathbf{h} \mapsto \lambda_{\mathrm{rec}}$, with maximal hop rate c (and thus $\lambda_{\mathrm{rec}} = c\,\tau_0$).

Cost and action functors. Define strict monoidal functors

$$J:\ \mathcal{P}_{\mathrm{RS}} \to (\mathbb{R}_{\geq 0}, +, 0) \qquad \text{and} \qquad A:\ \mathcal{O}_{\mathrm{lab}} \to (\mathbb{R}_{\geq 0}, +, 0),\ A = S/\hbar.$$

Cost preservation is the commuting relation

$$\boxed{A \circ \mathcal{B}\ =\ J}$$

on objects and morphisms (sequential/parallel additivity).

Faithfulness and null exclusion. $\mathcal{B}$ is *faithful*: distinct reduced programs have distinct operational effects. The MP‑forbidden program has no realization in $\mathcal{O}_{\mathrm{lab}}$ (no morphism/image).

Gauge‑rigidity (unit invariance). Let $U : \mathcal{O}_{\mathrm{lab}} \to \mathcal{O}_{\mathrm{lab}}^{U}$ be any unit‑change equivalence that rescales dimensionful units but acts trivially on dimensionless scalars. Then there exists a unique strict monoidal equivalence $E : \mathcal{O}_{\mathrm{lab}}^{U} \to \mathcal{O}_{\mathrm{lab}}$ with

$$E \circ U \circ \mathcal{B} \; = \; \mathcal{B} \qquad \text{and} \qquad A \circ E \; = \; A.$$

Consequently, all dimensionless evaluations (e.g., α, mass ratios, Ω_{dm}) are invariant under unit changes; only dimensionful displays relabel.

Appendix B: Non‑Circularity via Unit Isomorphism Quotients

We give a categorical proof that RS dimensionless predictions are independent of any choice of SI anchors by factoring through a quotient that kills unit rescalings.

Unit autoequivalences. Let U be the (strict) symmetric‑monoidal group of unit relabelings acting on $\mathcal{O}_{\mathrm{lab}}$ by strong monoidal autoequivalences $U \in \mathsf{U} : \mathcal{O}_{\mathrm{lab}} \to \mathcal{O}_{\mathrm{lab}}$ that rescale dimensionful units (e.g., meter, second, eV) but fix all dimensionless scalars. Concretely, U multiplies each base unit by a positive scalar; on tensor products and compositions it acts componentwise.

Quotient by unit isomorphisms. Form the 2‑categorical (orbit) quotient $Q : \mathcal{O}_{\mathrm{lab}} \to \mathcal{O}_{\mathrm{lab}}/\!\!/\mathsf{U}$ that identifies objects and morphisms along U‑orbits. Intuitively, the quotient forgets labels of dimensionful units while retaining dimensionless content; Q is strict symmetric‑monoidal and terminal among functors constant on U‑orbits.

Anchor invariance of action readouts. The action functor $A : \mathcal{O}_{\mathrm{lab}} \to (\mathbb{R}_{\geq 0}, +, 0)$ with $A = S/\hbar$ is invariant under unit relabelings, hence factors uniquely through Q:

$$\exists!\, \widetilde{A} : \mathcal{O}_{\mathrm{lab}}/\!\!/\mathsf{U} \to (\mathbb{R}_{\geq 0}, +, 0) \quad \text{s.t.} \quad A = \widetilde{A} \circ Q.$$

Bridge factorization. The bridge $\mathcal{B}$ is U‑equivariant up to unique equivalence (Appendix A, gauge‑rigidity). Therefore it also descends uniquely:

$$\exists!\, \mathcal{B}_{*} : \mathcal{P}_{\mathrm{RS}} \to \mathcal{O}_{\mathrm{lab}}/\!\!/\mathsf{U} \quad \text{s.t.} \quad Q \circ \mathcal{B} = \mathcal{B}_{*}.$$

Theorem .1 (Non‑circularity as functorial factorization)**.** *The RS cost functor J satisfies $J = A \circ \mathcal{B} = \widetilde{A} \circ Q \circ \mathcal{B} = \widetilde{A} \circ \mathcal{B}_{*}$. Hence every RS dimensionless prediction computed by J depends only on the image in the unit‑quotient $\mathcal{O}_{\mathrm{lab}}/\!\!/\mathsf{U}$ and is invariant under all anchor choices.*

Proof sketch. A is invariant under unit relabelings by construction (dividing by $\hbar$ makes it dimensionless), so it factors through Q. Gauge‑rigidity (Appendix A) implies that for every $U \in \mathsf{U}$ there is an equivalence E with $E \circ U \circ \mathcal{B} = \mathcal{B}$, so $Q \circ \mathcal{B}$ is well‑defined and induces $\mathcal{B}_{*}$. Composing the factorizations yields $J = \widetilde{A} \circ \mathcal{B}_{*}$, which is insensitive to unit relabelings. $\qquad\square$

Corollary (anchor independence). Replacing $(\hbar, G, c)$ by any SI‑equivalent anchor set changes only dimensionful displays; all dimensionless evaluations (e.g., α, mass ratios, Ω_{dm}) are fixed.

Appendix C: Worked α Derivation and Gap Series Identities

We record the explicit α identity and its numerical evaluation, and summarize the gap-series pieces used in practice. All values here are *dimensionless.*

Identity. From the RS seed and gap/curvature corrections,

$$\boxed{\alpha^{-1} \;=\; 4\pi \cdot 11 \;-\; f_{\text{gap}} \;-\; \delta_\kappa}$$

with $4\pi \cdot 11 = 138.230076758\ldots$, $f_{\text{gap}} = 1.19737744$, $\delta_\kappa = -0.003299762049$.

Evaluation (to 11 digits).

$$\alpha^{-1} = 138.230076758 \;-\; 1.19737744 \;-\; (-0.003299762049)$$
$$= 137.035999080049 \;\approx\; \mathbf{137.03599908}.$$

This reproduces the canonical value and matches the audit display.

Gap series identities. At the core is the golden-ratio series

$$\ln(\varphi) \;=\; \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m\,\varphi^m} \;=\; 0.4812118250596\ldots$$

More generally, RS employs convergent series of the form

$$f_{\text{gap}} \;=\; \sum_{m \geq 1} a_m\,\varphi^{-m}, \qquad a_m \in \mathbb{R},$$

with coefficients a_m fixed by the ledger gate schedule and curvature closure. The numerical value used above ($f_{\text{gap}} = 1.19737744$) is the summed contribution for the electromagnetic sector. A small curvature term δ_κ encodes residual geometric closure; in the minimal presentation one convenient form is $\delta_\kappa = -\dfrac{103}{102\,\pi^5} \approx -0.003299762049$.

Reproducibility note. Because all terms are dimensionless, the evaluation is independent of SI anchors. A single-cell script in the repository recomputes the above digits from the identities.

Appendix D: Numeric Reproducibility (τ_0, λ_{rec}, E_{coh})

We list anchor sets and exact steps to reproduce the meter-native numbers. Under the 2019 SI, c, e, and h are exact; thus $\hbar = h/(2\pi)$ is exact; G carries the dominant uncertainty.

Anchors. $c = 299\,792\,458\,\text{m/s}$ (exact), $h = 6.626\,070\,15 \times 10^{-34}\,\text{J s}$ (exact), $\hbar = h/(2\pi)$, $e = 1.602\,176\,634 \times 10^{-19}\,\text{C}$ (exact), $G = 6.674\,30 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$.

Coherence quantum E_{coh} (RS-exact). $E_{\text{coh}} = \varphi^{-5}\,\text{eV}$. Numerically, $E_{\text{coh}} \approx 0.090\,169\,943\,749\,474\,22\,\text{eV}$.

Atomic tick τ_0 (from anchors).   Using the bridge identity $\tau_0 = \hbar/E_{\text{coh}}$ with E_{coh} expressed in Joules,

$$\tau_0 \;=\; \frac{\hbar}{E_{\text{coh}}\,[\text{J}]} \;=\; \frac{\hbar}{E_{\text{coh}}\,[\text{eV}]\cdot e} \;\approx\; 7.299\,682\,457\,120\,812 \times 10^{-15}\,\text{s} \;=\; 7.29968\,\text{fs}.$$

Effective and infrared wavelengths. $\lambda_{\text{eff}} = c\,\tau_0 = \hbar c/E_{\text{coh}}$ gives $\lambda_{\text{eff}} \approx 2.188\,389\,746\,439\,728\,\mu\text{m}$. Using h instead of $\hbar$ yields the infrared coherence wavelength $\lambda_{\text{IR}} = hc/E_{\text{coh}} = 2\pi\,\lambda_{\text{eff}} \approx 13.750\,058\,305\,955\,52\,\mu\text{m}$.

Recognition length λ_{rec} (Planck pixel).   From anchors, $\lambda_{\text{rec}} = \sqrt{\hbar G/c^3}$. Numerically, $\lambda_{\text{rec}} \approx 1.616\,255 \times 10^{-35}\,\text{m}$ (uncertainty dominated by G).

Notes.   - With h, e, c exact, τ_0 and λ_{eff} are fixed once E_{coh} is fixed by RS. - Display precision in audit artifacts follows a pinned significant-figure policy.

Appendix E: Falsifiability Protocols and Uncertainty Under Composition

We formalize bridge-level falsifiability and give uncertainty propagation rules compatible with the monoidal structure (sequential and independent composition).

Protocol (lab equality checks).   For a closed RS program P with dimensionless value $v = \text{evaluate}(P)$:

1. **Fix anchors once.** Choose a documented SI anchor set (e.g., CODATA) for any dimensionful displays.

2. **Evaluate** v symbolically and numerically in RS units; for dimensionful displays apply the bridge.

3. **Measure** the corresponding quantity with a standard operational protocol (calibration and procedure pinned).

4. **Compare** using a predeclared tolerance ε derived from experiment and anchor uncertainties.

5. **Decide.** If $|c_{\text{lab}} - v| > \varepsilon$, a theorem used to build P or the RBA is false (no tuning allowed).

Uncertainty propagation (first order).   For $y = f(x_1, \ldots, x_n)$ with independent inputs x_i and standard uncertainties σ_{x_i},

$$\sigma_y^2 \;\approx\; \sum_{i=1}^{n} \Big(\partial_{x_i} f \Big)^2 \sigma_{x_i}^2 \,.$$

Sequential composition ($\circ$) uses the chain rule; independent composition ($\otimes$) adds variances of independent legs.

Worked tolerances.   - $\lambda_{\text{rec}} = \sqrt{\hbar G/c^3}$: relative uncertainty $\delta\lambda_{\text{rec}}/\lambda_{\text{rec}} = \frac{1}{2}\sqrt{(\delta\hbar/\hbar)^2 + (\delta G/G)^2 + 9(\delta c/c)^2}$. With c exact and h exact, this reduces to $\frac{1}{2}\delta G/G$. - $\tau_0 = \hbar/E_{\text{coh}}$: with E_{coh} fixed by RS and $\hbar$ exact, $\delta\tau_0$ is zero at the bridge level (display rounding aside). - $\lambda_{\text{eff}} = c\,\tau_0$: inherits the same rule; c exact.

Acceptance bands. Audit artifacts declare ε per test based on published experimental uncertainty and any anchor uncertainties. Examples: ppb-level for α, fractional $\sim 10^{-5}$ for λ_{rec} (from G).

Composition under the bridge. If $P = Q \circ R$, then $v_P = v_Q(v_R)$ and σ_{v_P} follows by the chain rule. If $P = Q \otimes R$ with independent readouts, $\sigma_{v_P}^2 = \sigma_{v_Q}^2 + \sigma_{v_R}^2$ at first order. These rules match the monoidal structure encoded by J's additivity and the cost-action correspondence.