CS 161: Homework 5

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Discussion 1A

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Problem 1. Use truth tables (worlds) to show that the following pairs of sentences are equivalent:

- $P \implies \neg Q$, $Q \implies \neg P$

  $P \implies \neg Q \rightarrow \neg P \vee \neg Q$

  $Q \implies \neg P \rightarrow \neg Q \vee \neg P$

P	Q	$\neg P \vee \neg Q$	$\neg Q \vee \neg P$
F	F	T	T
F	T	T	T
T	F	T	T
T	T	F	F

- $P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

  $P \Leftrightarrow \neg Q \equiv (P \implies \neg Q) \wedge (\neg Q \implies P)$

  $P \implies \neg Q \equiv \neg P \vee \neg Q$

  $\neg Q \implies P \equiv Q \vee P$

P	Q	$\neg P \vee \neg Q$	$Q \vee P$	$((P \vee \neg Q) \wedge (Q \vee P))$	$P \wedge \neg Q$	$\neg P \wedge Q$	$((P \wedge \neg Q) \vee (\neg P \wedge Q))$
F	F	T	F	F	F	F	F
F	T	T	T	T	F	T	T
T	F	T	T	T	T	F	T
T	T	F	T	F	F	F	F

Problem 2. Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither:

- $(\text{Smoke} \implies \text{Fire}) \implies (\neg \text{Smoke} \implies \neg \text{Fire}) \equiv (\neg \text{Smoke} \vee \text{Fire}) \implies (\text{Smoke} \vee \neg \text{Fire})$

Smoke	Fire	$\neg$ Smoke $\vee$ Fire	Smoke $\vee \neg$ Fire
F	F	T	T
F	T	T	F
T	F	F	T
T	T	T	T

This is not valid and not unsatisfiable, but it is satisfiable.

- $(\text{Smoke} \implies \text{Fire}) \implies ((\text{Smoke} \vee \text{Heat}) \implies \text{Fire}) \equiv (\neg \text{Smoke} \vee \text{Fire}) \implies ((\neg \text{Smoke} \wedge \neg \text{Heat}) \vee \text{Fire})$

Smoke	Fire	Heat	$\neg$ Smoke $\vee$ Fire	$\neg$ Smoke $\wedge \neg$ Heat	($\neg$ Smoke $\wedge \neg$ Heat) $\vee$ Fire
F	F	F	T	T	T
F	T	T	T	F	T
T	F	F	F	F	F
T	T	T	T	F	T

This is valid.

- $((\text{Smoke} \wedge \text{Heat} \implies \text{Fire}) \Leftrightarrow ((\text{Smoke} \implies \text{Fire}) \vee (\text{Heat} \implies \text{Fire})))$

$\equiv ((\neg \text{Smoke} \vee \neg \text{Heat} \vee \text{Fire}) \Leftrightarrow ((\neg \text{Smoke} \vee \text{Fire}) \vee (\neg \text{Heat} \vee \text{Fire})))$

$\equiv (\neg \text{Smoke} \vee \neg \text{Heat} \vee \text{Fire}) \implies ((\neg \text{Smoke} \vee \text{Fire}) \vee (\neg \text{Heat} \vee \text{Fire})))$

$\wedge ((\neg \text{Smoke} \vee \text{Fire}) \vee (\neg \text{Heat} \vee \text{Fire}))) \implies (\neg \text{Smoke} \vee \neg \text{Heat} \vee \text{Fire})$

Smoke	Fire	Heat	$(\neg$ Smoke $\vee \neg$ Heat $\vee$ Fire)	$\neg$ Smoke $\vee$ Fire	$\neg$ Heat $\vee$ Fire	$((\neg$ Smoke $\vee$ Fire$) \vee (\neg$ Heat $\vee$ Fire$))$
F	F	F	T	T	T	T
F	F	T	T	T	F	T
F	T	F	T	T	T	T
F	T	T	T	T	T	T
T	F	F	T	F	T	T
T	F	T	F	F	F	F
T	T	F	T	T	T	T
T	T	T	T	T	T	T

This is valid.

Problem 3. Consider the following:

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

(a) Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).

MYTHICAL = unicorn is mythical

IMMORTAL = unicorn is immortal

MAMMAL = unicorn is mammal

MAGICAL = unicorn is magical

HORNED = unicorn is horned

I. MYTHICAL $\implies$ IMMORTAL

II. $\neg$ MYTHICAL $\implies \neg$ IMMORTAL $\wedge$ MAMMAL

III. (IMMORTAL $\vee$ MAMMAL) $\implies$ HORNED

IV. HORNED $\implies$ MAGICAL

(b) Convert the knowledge base into CNF.

I. $\neg$ MYTHICAL $\vee$ IMMORTAL

II. MYTHICAL $\vee$ ($\neg$ IMMORTAL $\wedge$ MAMMAL)

III. ($\neg$IMMORTAL $\wedge$ $\neg$MAMMAL) $\vee$ HORNED $\equiv$ ($\neg$IMMORTAL $\vee$ HORNED) $\wedge$ ($\neg$MAMMAL $\vee$ HORNED)

IIIa. $\neg$IMMORTAL $\vee$ HORNED

IIIb. $\neg$MAMMAL $\vee$ HORNED

IV. ¬HORNED ∨ MAGICAL

(c) Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned?

1) Resolve I and II: IMMORTAL ∨ (¬ IMMORTAL ∧ MAMMAL)
2) Expand 1: (IMMORTAL ∨¬IMMORTAL) ∧ (IMMORTAL ∨ MAMMAL)
3) Simplify 2: (IMMORTAL ∨ MAMMAL)
4) Resolve 3 and IIIa: HORNED ∨ MAMMAL
5) Resolve 4 and IIIb: (HORNED ∨ HORNED)
6) Simplify 5: HORNED
7) Resolve 6 and IV: MAGICAL

Step 1 shows that the unicorn is mythical, step 6 shows that the unicorn is horned, and step 7 shows that the unicorn is magical.