CS161: FUNDAMENTALS OF ARTIFICIAL INTELLIGENCE

Fall 2017

Assignment 7. Due Sunday, November 26, 2017, 11:55pm

Please submit your solutions on CCLE. The submitted file should be plain text or a for-matted PDF file (no scans or pictures). Text files should have lines no longer than 100 characters and should be well-aligned when viewed with a monospace font.

1. Prove:

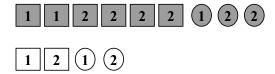
Generalized product rule: $Pr(A, B \mid K) = Pr(A \mid B, K)Pr(B \mid K)$.

Generalized Bayes' rule: $Pr(A \mid B, K) = Pr(B \mid A, K)Pr(A \mid K)/Pr(B \mid K)$.

2. We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 60%, and 80%, respectively. One coin is drawn randomly from the bag (with equal likelihood of drawing each of the three coins), and then the coin is flipped three times to generate the outcomes X_1 , X_2 , and X_3 .

Draw the Bayesian network corresponding to this setup and define the necessary CPTs (Conditional Probability Table).

3. (From Bayesian Networks by R. Neapolitan) Consider the set of objects below.



Mr. Y picked up an object at random from the above set. We want to compute the probabilities of the following events:

 α_1 : the object is black;

 α_2 : the object is square;

 α_3 : if the object is one or black, then it is also square.

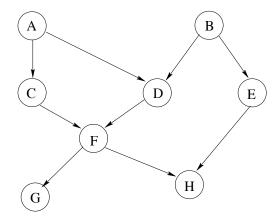


Figure 1: The DAG of a Bayesian network.

Construct the joint probability distribution of this problem. Use it to compute the above probabilities by explicitly identifying the worlds at which each α_i holds. Identify two sets of sentences α, β, γ such that α is independent of β given γ with respect to the constructed distribution.

4. Consider the DAG in Figure 1:

- (a) List the Markovian assumptions asserted by the DAG.
- (b) True or false? Why?
 - $d_separated(A, BH, E)$
 - d-separated(G, D, E)
 - d-separated (AB, F, GH)
- (c) Express Pr(a, b, c, d, e, f, g, h) in factored form using the chain rule for Bayesian networks.
- (d) Compute Pr(A = 0, B = 0) and $Pr(E = 1 \mid A = 1)$. Justify your answers.

Pr(A=0)	Pr(A=1)	Pr(B=0)	Pr(B=1)
.8	.2	.3	.7

	$Pr(E = 0 \mid B)$	$Pr(E=1 \mid B)$
B = 0	.1	.9
B=1	.9	.1

	$\Pr(D = 0 \mid A, B)$	$Pr(D=1 \mid A,B)$
A = 0, B = 0	.2	.8
A = 0, B = 1	.9	.1
A = 1, B = 0	.4	.6
A = 1, B = 1	.5	.5