CS 161: Homework 7

Jonathan Woong

804205763

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Discussion 1A

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Problem 1. Prove:

a) Generalized product rule: $Pr(A, B|K) = Pr(A|B, K)Pr(B|K)$

$$
\begin{aligned}
Pr(A, B|K) &= \frac{Pr(A, B, K)}{Pr(B|K)} \\
&= \frac{Pr(A|B, K)Pr(B, K)}{Pr(K)} \\
&= \frac{Pr(A|B, K)Pr(B|K)Pr(K)}{Pr(K)} \\
&= Pr(A|B, K)Pr(B|K)
\end{aligned}
$$

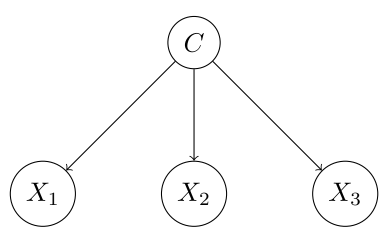
b) Generalized Bayes' rule: $Pr(A|B, K) = \frac{Pr(B|A,K)Pr(A|K)}{Pr(B|K)}$

$$
\begin{aligned}
Pr(A|B, K) &= \frac{Pr(A, B|K)}{Pr(B|K)} \\
&= \frac{Pr(A, B, K)}{\frac{Pr(K)}{Pr(B|K)}} \\
&= \frac{Pr(B|A, K)Pr(A, K)}{\frac{Pr(K)}{Pr(B|K)}} \\
&= \frac{Pr(B|A, K)Pr(A|K)}{Pr(B|K)}
\end{aligned}
$$

Problem 2. We have a bag of three biased coins a, b, and c with probabilities of coming up heads of 20%, 60%, and 80% respectively. One coin is drawn randomly from the bag (with equal likelyhood of drawing each of the three coins), and then the coin is flipped three times to generate the outcones X_1, X_2, and X_3. Draw the Bayesian network corresponding to this setup and define the necessary CPTs (Conditional Probability Table).

Let C be the random variable representing the coin that we drew:

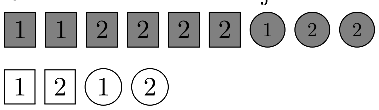
C	$Pr(C)$
a	$\frac{1}{3}$
b	$\frac{1}{3}$
c	$\frac{1}{3}$



The CPT for any X_i is given below:

C	X_i	$Pr(X_i \mid C)$
a	heads	0.2
a	tails	0.8
b	heads	0.6
b	tails	0.4
c	heads	0.8
c	tails	0.2

Problem 3. Consider the set of objects below.



Mr. Y picked up an object at random from the above set. We want to compute the probabilty of the following events:

α_1: the object is black;

α_2: the object is square;

α_3: if the object is one or black, then it is also square.

Construct the joint probability distribution of this problem. Use it to compute the above probabilities by explicitly identifying the worlds at which each α_i holds. Identify two sets of sentences α, β, γ such that α is independent of β given γ with respect to the constructed distribution.

world	$Black$	$Square$	One	count	probability
1	T	T	T	2	0.1538
2	T	T	F	4	0.3077
3	T	F	T	1	0.0769
4	T	F	F	2	0.1538
5	F	T	T	1	0.0769
6	F	T	F	1	0.0769
7	F	F	T	1	0.0769
8	F	F	F	1	0.0769

α_1: holds for worlds 1, 2, 3, 4

$$Pr(\alpha_1) = 0.1538 + 0.3077 + 0.0769 + 0.1538 = \boxed{0.6922}$$

α_2: holds for worlds 1, 2, 5, 6

$$Pr(\alpha_2) = 0.1538 + 0.3077 + 0.0769 + 0.7069 = \boxed{0.6153}$$

α_3: holds for worlds 1, 2, 5

$$Pr(\alpha_3) = 0.1538 + 0.3077 + 0.0769 = \boxed{0.5384}$$

1) $\boxed{\alpha = One, \beta = Square, \gamma = Black\text{: One is independent of Square given Black}}$

   $Pr(One|Black) = \frac{1}{3}$

   $Pr(One|Black, Square) = \frac{2}{6} = \frac{1}{3}$

   $Pr(One|Black) \equiv Pr(One|Black, Square) \implies$ independence

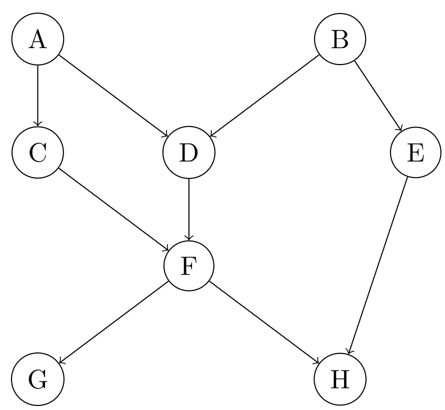
2) $\boxed{\alpha = One, \beta = Square, \gamma = \neg Black\text{: One is independent of Square given White}}$

   $Pr(One|\neg Black) = \frac{1}{2}$

   $Pr(One|\neg Black, Square) = \frac{1}{2}$

   $Pr(One|\neg Black) \equiv Pr(One|\neg Black, Square) \implies$ independence

Problem 4. Consider the DAG:



(a) List the Markovian assumptions asserted by the DAG.

   1) $I(A, \emptyset, \{B, E\})$
   2) $I(B, \emptyset, \{A, C\})$
   3) $I(C, A, \{B, D, E\})$
   4) $I(D, \{A, B\}, \{C, E\})$
   5) $I(E, B, \{A, C, D, F, G\})$
   6) $I(F, \{C, D\}, \{A, B, E\})$
   7) $I(G, F, \{A, B, C, D, E, H\})$
   8) $(H, \{E, F\}, \{A, B, C, D, G\})$

(b) True or false? Why?

- $d\_separated(A, BH, E)$: $\boxed{\text{False, there exists a path } A \rightarrow C \rightarrow F \rightarrow H \rightarrow E}$
- $d\_separated(G, D, E)$: $\boxed{\text{False, there exists a path } G \rightarrow F \rightarrow H \rightarrow E}$
- $d\_separated(AB, F, GH)$: $\boxed{\text{False, there exists a path } B \rightarrow E \rightarrow H}$

(c) Express $Pr(a, b, c, d, e, f, g, h)$ in factored form using the chain rule for Bayesian networks.

$$\begin{aligned} Pr(a, b, c, d, e, f, g, h) = Pr(a|b, c, d, e, f, g, h) \times \\ Pr(b|c, d, e, f, g, h) \times \\ Pr(c|d, e, f, g, h) \times \\ Pr(d|e, f, g, h) \times \\ Pr(e|f, g, h) \times \\ Pr(f|g, h) \times \\ Pr(g|h) \times \\ Pr(h) \end{aligned}$$

(d) Compute $Pr(A = 0, B = 0)$ and $Pr(E = 1|A = 1)$. Justify your answers.

$Pr(A = 0)$	$Pr(A = 1)$
0.8	0.2

$Pr(B = 0)$	$Pr(B = 1)$
0.3	0.7

	$Pr(E = 0\|B)$	$Pr(E = 1\|B)$
$B = 0$	0.1	0.9
$B = 1$	0.9	0.1

	$Pr(D = 0\|A, B)$	$Pr(D = 1\|A, B)$
$A = 0, B = 0$	0.2	0.8
$A = 0, B = 1$	0.9	0.1
$A = 1, B = 0$	0.4	0.6
$A = 1, B = 1$	0.5	0.5

1) $Pr(A = 0, B = 0)$

   Since A and B are independent:

$$\begin{aligned} Pr(A = 0, B = 0) &= Pr(A = 0)Pr(B = 0) \\ &= (0.8)(0.3) \\ &= \boxed{0.24} \end{aligned}$$

2) $Pr(E = 1|A = 1)$

   Since E and A are independent:

$$
\begin{aligned}
Pr(E=1|A=1) &= Pr(E=1) \\
&= Pr(E=1|B=0)Pr(B=0) + Pr(E=1|B=1)Pr(B=1) \\
&= (0.9)(0.3) + (0.1)(0.7) \qquad (1) \\
&= 0.07 + 0.27 \\
&= \boxed{0.34}
\end{aligned}
$$