CS 181: Homework 2

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Discussion 1A

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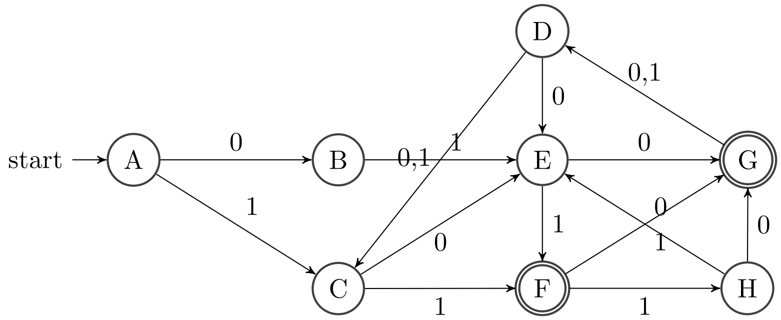
Problem 1. Minimize:



State	**0**	**1**
A	B	C
B	E	E
C	E	F
D	E	C
E	G	F
F	G	H
G	D	D
H	G	E

TFA	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
A								
B	X							
C	X	X						
D	X	X	X					
E	X	X	X	X				
F	X	X	X	X	X			
G	X	X	X	X	X	X		
H	X	X	X	X	X	X	X	

This DFA cannot be minimized.

Problem 2. L $= \{a^n b^m c^{2(n+m)}; n \geq 0, m \geq 0\}$

Find if L is:

a) a regular language

Assume that L is regular.

Let p = pumping length and $w = a^n b^p c^{2(n+p)} = xyz$.

Suppose $n = 0$ and $p = 3$, then $w = b^3c^6 = bbbcccccc = xyz$.

1) Let $x = b, y = bb, c = cccccc$. By pumping lemma, we expect $xy^kz \in L\ \forall k \geq 0$.

   Suppose $k = 2$, then $w = xy^2z = bbbbbcccccc = b^5c^6 \notin L$.

2) Let $x = bb, y = bc, z = ccccc$. By pumping lemma, we expect $xy^kz \in L\ \forall k \geq 0$.

   Suppose $k = 2$, then $w = xy^2z = bbbcbcccccc \notin L$.

3) Let $x = bbb, y = cc, z = cccc$. By pumping lemma, we expect $xy^kz \in L\ \forall k \geq 0$.

   Suppose $k = 2$, then $w = xy^2z = bbbcccccccc = b^3c^8 \notin L$.

$\therefore$ By contradiction, L is not a regular language. We cannot build a finite automata for a non regular language.

b) a context-free language

Assume that L is context-free.

Let p = pumping length and $w = a^n b^p c^{2(n+p)} = uvxyz$.

Suppose $n = 2$ and $p = 2$, then $w = a^2b^2c^8 = aabbcccccccc$.

1) Let $u = a, v = a, x = bb, y = c, z = cccccc$. By pumping lemma, we expect $uv^kxy^kz \in L \; \forall k \geq 0$.

Suppose $k = 2$, then $w = uv^2xy^2z = aaabbccccccccc = a^3b^2c^9 \notin L$.

2) Let $u = a, v = ab, x = b, y = c, z = cccccc$. By pumping lemma, we expect $uv^kxy^kz \in L \; \forall k \geq 0$.

Suppose $k = 2$, then $w = uv^2xy^2z = aababbcccccccc \notin L$.

$\therefore$ By contradiction, L is not a context-free language. We cannot build a pushdown automata for a non context-free language.

Problem 3. Is any finite language regular? Prove your answer is correct.

Yes. A finite language L must have a finite set of strings that are accepted by L. Let these strings be denoted as $w_1, w_2, \ldots, w_n$. We can express L as $w_1 \cup w_2 \cup \cdots \cup w_n = \cup_{i=1}^{n} w_i$, which is a regular expression. Since any language which can be defined using a regular expression is defined to be a regular language, then by definition any finite language is regular.