Introducing Confidence Intervals

Confidence intervals measure how large or small a parameter value could be.
Example: Estimate a population mean $\mu$from a sample.
If we take many multiple samples, we will see slightly different estimates of the mean.
The different estimates are a natural result of randomization.
Confidence intervals tell us where those answers should be.

Samples From a Normally Distributed Population

Suppose 300 students each draw a random sample of 10 scores from a normally distributed population whose actual mean $\mu = 10$ and $\sigma = 1.5$.
Normal distribution theory says we should expect:
~68% of the drawn values will be within 1σ of μ
~95% of the drawn values will be within 2σ of μ
~99.7% of the drawn values will be within 3σ of μ
If we have appropriately obtained sample means, we can say:
95% of the sample mean ($\bar{X}$) values will be fairly close to the actual population mean (μ).
5% of the $\bar{X}$ values will be far away from μ.

What the Confidence Interval Tells Us

A 95% confidence interval means that, on average, we expect the endpoints of this particular interval to contain the population mean μin 95% of random samples.

Remember: The measure is not necessarily confidence in the interval itself but in the procedure that produces the interval.

In other words, if we draw repeated samples of size n from a normally distributed population, 95% of the sample means ($\bar{X}$) should lie within 1.96 standard errors of the actual μ.

The Confidence Interval Formula

Note that we use s instead of σ, where s is the sample standard deviation.

s is our estimate of σ.

This substitution introduces error.

So, confidence interval calculated with t-distribution instead of Normal distribution.

t-distribution allows for error in estimate of standard deviation.

t-distribution is like Normal distribution but with fatter tails that allow for error.

The t-distribution is the default with statistical software.

A Critical Value for the *t*-Distribution

The t_{crit} value depends on your chosen confidence interval (e.g., 90%, 95%, 99%, etc.).

Look up the t_{crit} values in a table if you're calculating by hand.

$t_{crit}$is a function of:

the degrees of freedom (defined as n – 1)

the confidence level (which you choose)

In our example, for a 95% confidence interval and n=120:

From a table, t_{crit} is 1.979.

Note that 1.979 is close to the 1.96 we'd get if we were using a Normal distribution.

The values are close because we have a large sample size.

Calculate the Confidence Interval

The confidence interval is:

$$\mathrm{CI} = 130 \pm 1.979\left(\frac{15}{\sqrt{120}}\right)$$

$$\mathrm{CI} = 130 \pm 2.71$$

$$\mathrm{CI}=127.29 \text{ to } 132.71$$

We could say:

"95% of the confidence intervals estimated with this procedure for random samples from the same population will contain the population mean μ."

"The other 5% of confidence intervals will not contain μ."

We should *not*say:

"There is a 95% chance that μ lies in this interval."

What we do say:

"With 95% confidence, the true mean μ is between 127.29 and 132.71, based on a sample n = 120 from this population."

Caveats

The value of the interval—whether it's really a 95% interval, and whether it's really a good estimate of the true population mean μ —depends on:

How good the random sampling procedure is

How representative the students are