Hypothesis Test

We use a formal hypothesis test to determine whether a sample supports a hypothesized parameter value.
Recall:
We used the central limit theorem to estimate a range of likely values for μ using confidence intervals.
We can also use the central limit theorem to estimate a sample distribution.
Example hypothesis test:
Is the normal human body temperature 98.6°F?
Can we support this hypothesis by checking a random sample of healthy individuals?

Example Test: Emotional Intelligence

Recall the emotional intelligence (EI) example from earlier.
What if we knew the μ for regular students (not gifted and talented)?
How do we determine whether the EI of gifted and talented students is different from the regular students?
We can do a two-sample test:
Sample the EI of gifted and talented students
Sample the EI of regular students
Is the difference 0, or is it larger than expected by chance?

Example Test: Sodium Content

Quality control at the potato chip factory
Is your sodium content as advertised?
To test:
Take a random sample from the assembly line.
Measure the sodium content.
Is the sample mean $\bar{X}$ close enough to the advertised target?
Is the difference small enough to be chance rather than a problem with the machine?

Significance Test Conditions

Correct significance level α.
α is typically 0.05.
α sets probability of **type I error** (when true H_0 is rejected).
Choose **directional** or **nondirectional**test (before looking at data).
Directional test: Is the sample value is greater than expected or less than expected?
Nondirectional test: Is the sample value different from expected?
"One-sided" and "two-sided" tests are alternative names for directional and nondirectional tests.

Significance Test Conditions (cont.)

Data must meet required assumptions:
Quantitative scores
Measured reliably
Nearly normally distributed
Outliers examined
Samples must be drawn randomly and/or representatively.
Perform one significance test at a time.

State the Hypothesis

Two types: the null hypothesis (H_0) and the alternative hypothesis (H_a)

A **hypothesis** is an assumption about the characteristics of one or more variables in one or more populations

Null hypothesis:the statement being tested

"The mean human body temperature is 98.6°F."

Alternative hypotheses:the statement we suspect is true instead of the null hypothesis

The true value is >98.6.
The true value is <98.6.
The true value is ≠ 98.6 (in either direction).
Note: The first two alternative hypotheses are one sided; the third is two sided.

The *p*-Value

***p*-value:** the chance of obtaining a particular sample result if the null hypothesis is true

We want a *p*-value that tells us—based on the condition of the test—whether to reject the H_0.

We always express our result in terms of whether or not we reject the H_0.

The *p*-value tells us whether random variation alone can account for the difference between between the null hypothesis and observations in the random sample.

Given the sample size and the standard deviation, is the *p*-value close enough to the expected value to support the null hypothesis?

Interpreting the *p*-Value

We calculate the *p*-value with the assumption that the H_0 is true.

The *p*-value is a probability between 0 and 1.

A small *p*-value implies random variation is *not*likely to account for the differences we see:

Evidence *against* the H_0

A large *p*-value suggests that the H_0 is valid.

How Small Is Small?

"Small" depends on the significance level you choose.
A typical significance level is 0.05.
If the p-value is much smaller than 0.05, you reject the H_0.
If the p-value is 0.05 or larger, you do not reject the H_0.
What if the p-value is very close to 0.05?
If the p-value is 0.049, do you reject the H_0?
What if the p-value is 0.051, which is not much larger than 0.05?
The p-value is a probability, not a hard cutoff.
State the p-value and let the reader decide whether your evidence is close enough to reject the H_0.

p-Value Example: Salt Content

We want the sodium content to be as advertised, therefore:
We want to support the H_0 (i.e., we want to see a large p-value).
$n = 40$
H_0: $\mu = 75$
H_a: $\mu \neq 75$ (nondirectional test)
$\bar{X} = 76.1$ and $s = 2.2878$
Is this consistent with our advertised claim (i.e., consistent with H_0)?
$t = 2.95$, which yields p-value = 0.0052.
0.5% chance (52 out of 10,000) we would see a value as large as 76.1 out of a sample of 40 if the H_0 were true.
We reject the null hypothesis (i.e., the p-value is too small to support H_0).
We need to fix the manufacturing process.

Reminder About Conditions

Conditions must be met.
The data must be a random sample.
The data must be close to normally distributed without too many outliers.