One-Sided and Two-Sided Tests

A **one-tail** or **one-sided**test of a population mean has these null and alternative hypotheses:

H_0: μ = [a specific number] H_a: μ < [a specific number]   OR

H_0: μ = [a specific number] H_a: μ > [a specific number]

A **two-tail** or **two-sided**test of a population mean has these null and alternative hypotheses:

H_0: μ = [a specific number] H_a: $\mu \neq$ [a specific number]

Calculating *p*-Value

We compare the *p*-value with a level that we regard as decisive called the **significance level**, α. Usually, α = 0.05.

Everything is calculated on the basis of the *t*-distribution.

The ***p*-value** is the probability, if H_0 is true, of randomly drawing a sample like the one obtained or more extreme, in the direction of H_α.

The ***p*-value** is calculated as the corresponding area under the curve, one-tailed or two-tailed depending on H_α.

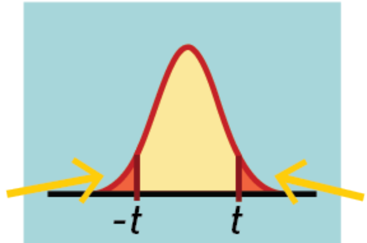
I.e., the areas to the right and the left of the *t*-statistic

Recall the *t*-statistic formula (sodium example):

$$t = \frac{(x - \mu_0)}{(s / \sqrt{n})} = \frac{(76.1 - 75)}{(2.287 / \sqrt{40})} = 3.04$$

Two-Sided *p*-Value

Two sided
(two tailed)

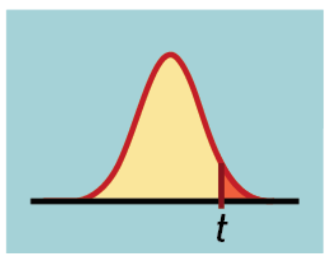


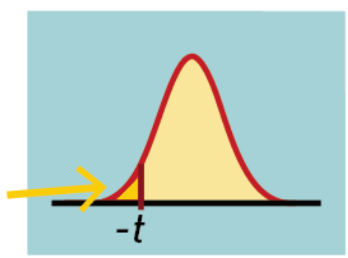
The *t*-values are marked on the *x*-axis (in our case, 2.95 on either side of the center).
The H_a probability corresponds to the area under both tails.
The software tells us the total area under both right and left tails = 0.0052.
The area under either single tail is 0.0052/2 because the function is symmetric.
Rephrased, we want the areas to the left of −2.95 and to the right of +2.95.
These two areas total 0.0052, which is our *p*-value.

One-Sided *p*-Value

One sided
(one tailed)

$$t = \frac{(\bar{x} - \mu_0)}{(s/\sqrt{n})}$$





If your H_a is that the true mean is >75 mg, the *p*-value is represented by the shaded area right of *t*in the upper graph.
This *p*-value is 0.0026 (the two-sided *p*-value, 0.0052, divided by 2).
If your H_a is that the true mean is < 75 mg, the *p*-value is represented by the shaded area left of *t*in the bottom graph.
$p = 1 - 0.0026 = 0.9974$
We would *not* reject the H_0 in that case.
But here the area left of the *t* is actually very small.

Keep Track of Signs

For a one-sided hypothesis, pay attention to the sign of the numerator.

A negative value indicates that $\bar{X}$ is smaller than μ_0.

If H_a: $\mu < \mu_0$, you want to see a *negative* value.

If H_a: $\mu > \mu_0$, you want to see a *positive* value.

It's always the sample mean minus the population mean in the numerator.

Summary

Remember: Software gives the two-sided *p*-value.

For the one-sided *p*-value, divide by two.

Remember: The difference between the sample mean and hypothesized mean keeps with the direction of the alternative hypothesis.