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CCP-WSI

a Collaborative Computational Project in Wave Structure Interaction

# Applying Machine Learnt Explicit Algebraic Stress and Scalar Flux Models to a Fundamental Trailing Edge Slot

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# References

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RESEARCH-ARTICLE

## Applying Machine Learnt Explicit Algebraic Stress and Scalar Flux Models to a Fundamental Trailing Edge Slot

R. D. Sandberg, R. Tan, J. Weatheritt, A. Ooi, A. Haghiri, V. Michelassi, G. Laskowski

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Machine learning was applied to large-eddy simulation (LES) data to develop nonlinear turbulence stress and heat flux closures with increased prediction accuracy for trailing-edge cooling slot cases. The LES data were generated for a thick and a thin trailing-edge slot and shown to agree well with experimental data, thus providing suitable training data for model development. A gene expression programming (GEP) based algorithm was used to symbolically regress novel nonlinear explicit algebraic stress models and heat-flux closures based on either the gradient diffusion or the generalized gradient diffusion approaches. Steady Reynolds-averaged Navier–Stokes (RANS) calculations were then conducted with the new explicit algebraic stress models. The best overall agreement with LES data was found when selecting the near wall region, where high levels of anisotropy exist, as training region, and using the mean squared error of the anisotropy tensor as cost function. For the thin lip geometry, the adiabatic wall effectiveness was predicted in good agreement with the LES and experimental data when combining the GEP-trained model with the standard eddy-diffusivity model. Crucially, the same model combination also produced significant improvement in the predictive accuracy of adiabatic wall effectiveness for different blowing ratios (BRs), despite not having seen those in the training process. For the thick lip case, the match with reference values deteriorated due to the presence of large-scale, relative to slot height, vortex shedding. A GEP-trained scalar flux model, in conjunction with a trained RANS model, was found to significantly improve the prediction of the adiabatic wall effectiveness.

**Issue Section:** Research Papers
**Topics:** Anisotropy, Heat flux, Large eddy simulation, Reynolds-averaged Navier–Stokes equations, Scalars, Stress, Tensors, Turbulence, Eddies (Fluid dynamics), Algebra, Machine learning, Diffusion (Physics), Flow (Dynamics), Errors, Machinery, Temperature, Geometry

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## A novel evolutionary algorithm applied to algebraic modifications of the RANS stress–strain relationship

Jack Weatheritt, Richard Sandberg

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### Abstract

This paper presents a novel and promising approach to turbulence model formulation, rather than putting forward a particular new model. Evolutionary computation has brought symbolic regression of scalar fields into the domain of algorithms and this paper describes a novel expansion of Gene Expression Programming for the purpose of tensor modeling. By utilizing high-fidelity data and uncertainty measures, mathematical models for tensors are created. The philosophy behind the framework is to give freedom to the algorithm to produce a constraint-free model; its own functional form that was not previously imposed. Turbulence modeling is the target application, specifically the improvement of separated flow prediction. Models are created by considering the anisotropy of the turbulent stress tensor and formulating non-linear constitutive stress–strain relationships. A previously unseen flow field is computed and compared to the baseline linear model and an established non-linear model of comparable complexity. The results are highly encouraging.

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# Genetic Algorithm – Gene Expression Programming

# GEP – How does it work?

Evolutionary Algorithm that creates computer programs or models. These computer programs are complex tree structures that learn and adapt by changing their sizes, shapes, and composition, much like a living organism.

1. User decide on a set of mathematical basis function
2. Creates a population of Mathematical models (expression tree) through mathematical input bases defined by the users with (+, -, /, x, ^(), exp(),) - symbolic expression of mathematical models.
3. Load dataset for fitness evaluation (user decide on the loss/objective function)
4. Verify Stop condition
5. Select a subset of mathematical models
6. Replicate selected models to form the next population
7. Modify population of mathematical models using genetic operators (+, -, /, x, ^(), exp(),)
8. Go to Step 3

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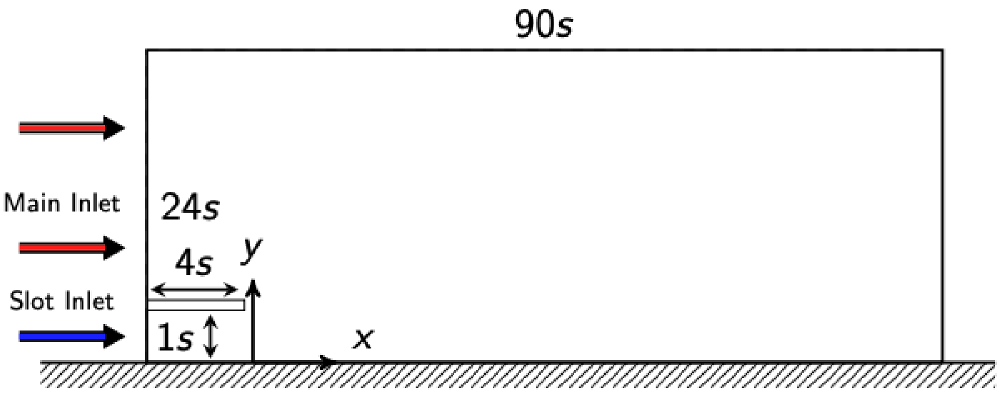
# Data Driven Turbulence Model Augmentation Framework

# Problem Definition

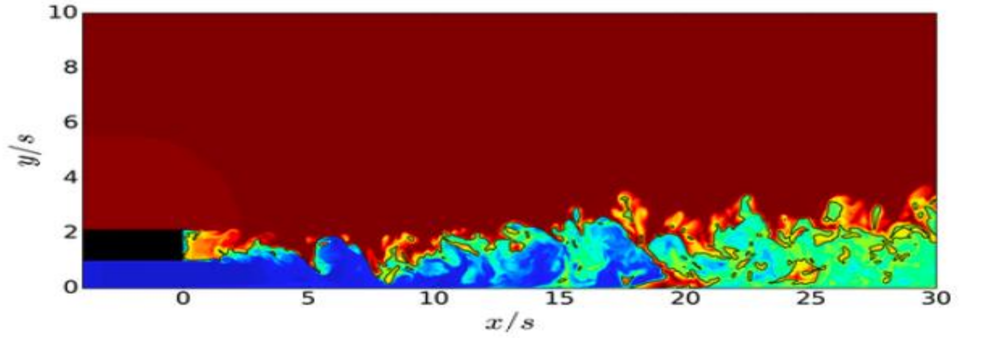
## Simulation Domain



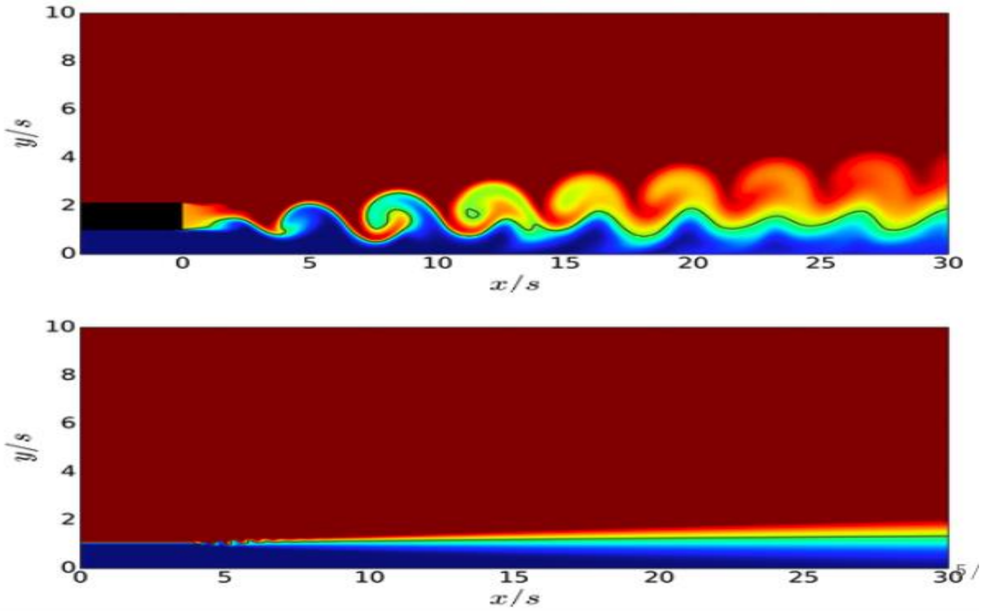

Computational Domain (not to scale).

| Simulation Parameters | Values |
|---|---|
| $U_{bulk}$ | 24.2 $m/s$ |
| $U_{fs}$ | 19.2 $m/s$ |
| $I_{slot}$ | 5 % |
| $I_{main}$ | 0.5 % |
| $l_t$ (turbulent length scale) | 10% of slot height |
| BR | 1.26 |
| $t/s$ | 1.14 & 0.126 |
| $Re_{slot}$ | $\approx$ 12000 |
| $T_{slot}$ | 273K |
| $T_{fs}$ | 323K |

Simulation Parameters




LES




RANS

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# Governing Equations – What are we trying to model?

RANS Equations - Momentum

$$\frac{d\overline{u_i}}{dt} + \overline{u_j}\frac{d\overline{u_i}}{dx_j} = -\frac{1}{\rho}\frac{d}{dx_j}(\overline{P} - \nu\frac{d\overline{u_i}}{dx_j} + \tau_{ij}).$$

$$\tau_{ij} = \frac{2}{3}k\delta_{ij} - 2\nu_t S_{ij} + a_{ij}^{mod}$$

Where

$$a_{ij}^{mod} = \sum_{\lambda=1}^{10} G^{\lambda} V_{ij}^{\lambda}.$$

RANS Equations – Scalar Flux

$$\frac{d\overline{T}}{dt} + \frac{d}{dx_j}(\overline{T}\overline{u_j}) = \frac{d}{dx_k}(\frac{\nu}{Pr}\frac{d\overline{T}}{dx_k}) + \frac{d}{dx_k}(\overline{u_k^{\prime}T^{\prime}})$$

$$\overline{u_i^{\prime}T^{\prime}} = -\frac{\nu_t}{Pr_t}\frac{d\overline{T}}{dx_i}.$$

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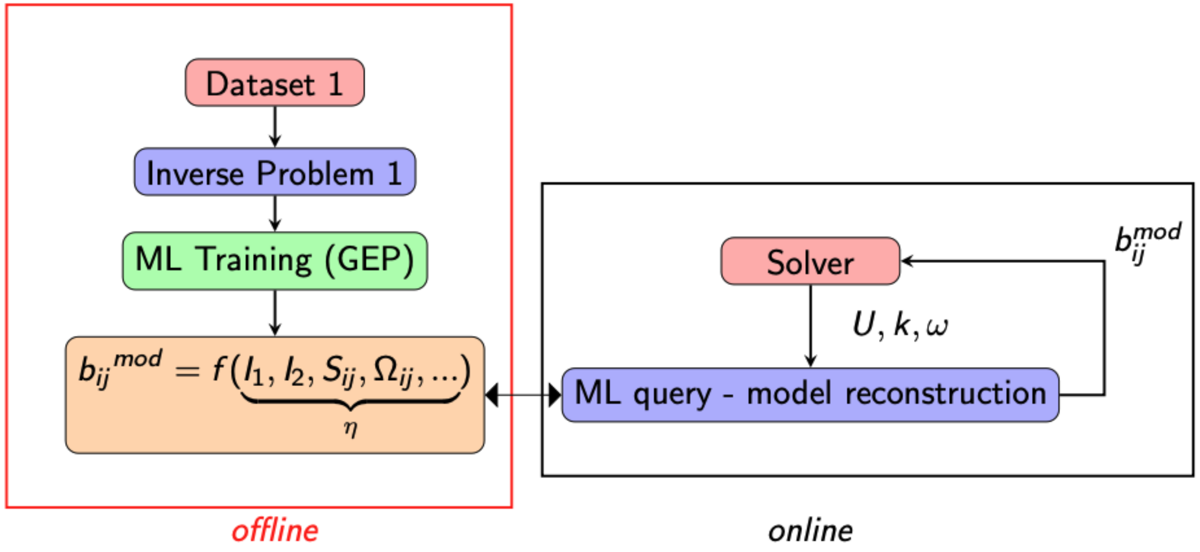
# Offline and Online Training




- $\tau_{ij}{}^{RANS} = -2\nu_t S_{ij} + \frac{2}{3} k \delta_{ij} + a_{ij}^{mod}$
- Model $b_{ij} = \frac{\tau_{ij}}{2k} - \frac{1}{3}\delta_{ij}$ from high fidelity dataset

$$J = \frac{1}{N} \sum_{n=1}^{N} \sum_{i=1}^{3} \sum_{j=1}^{3} w_{ij} \left( b_{ij}^{mod} - b_{ij}^{LES} \right)^2$$

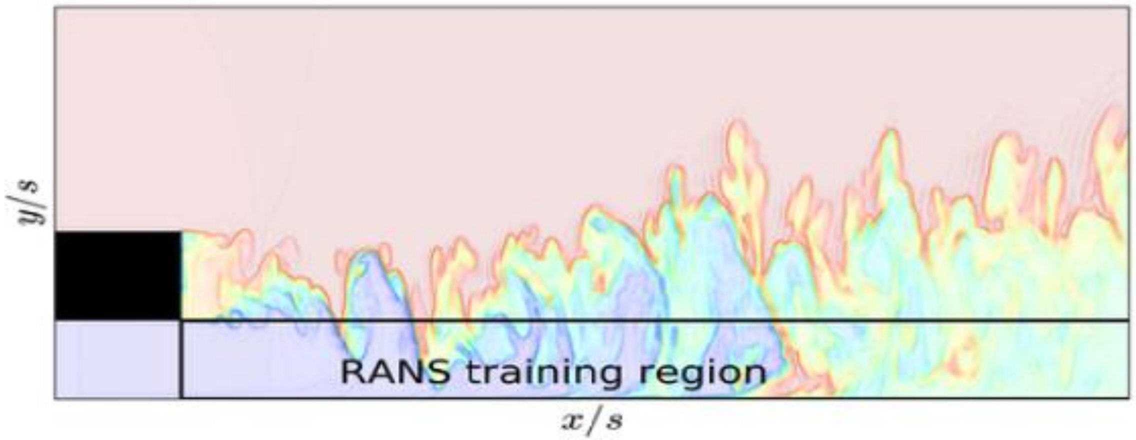
Tensor regression Problem!

$$b_{ij}^{mod} = \sum_{\lambda=1}^{10} G^{\lambda}(I_1^{LES}, I_2^{LES}, I_3^{LES}, \ldots) T_{ij}^{\lambda} \qquad T_{ij}^{\lambda} = f(S_{ij}^{LES}, \Omega_{ij}^{LES})$$

strain rate and vorticity tensor

$$\begin{aligned} b_{ij}^{mod1} = {} & V_{ij}^1(-2I^1 I^2 - 1.0 I^2 - 0.43) \\ & + V_{ij}^2(1.0 I^1 I^2 + I^1 + (I^2)^2 - 3.0 I^2 - 3.903) \\ & + V_{ij}^3(4 I^2) \end{aligned}$$

$$\begin{aligned} b_{ij}^{mod2} = {} & V_{ij}^1(I^2 - 0.3) + V_{ij}^2(I^1 - 3.73) \\ & + V_{ij}^3(4 I^1 (I^2)^2 + 4(I^2)^3 + 8.0 I^2) \end{aligned}$$




bij - Scaled Reynolds stress anisotropy tensor

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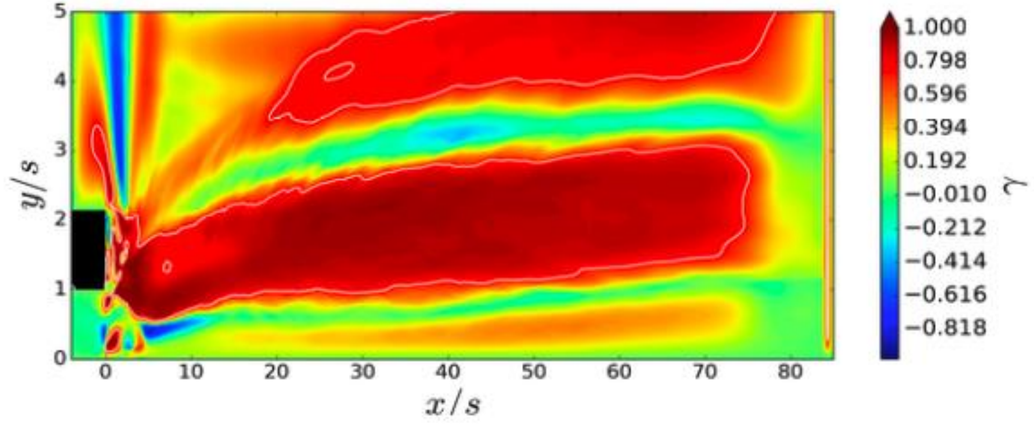
# Results

- Compare the alignment of the linear and trained models:

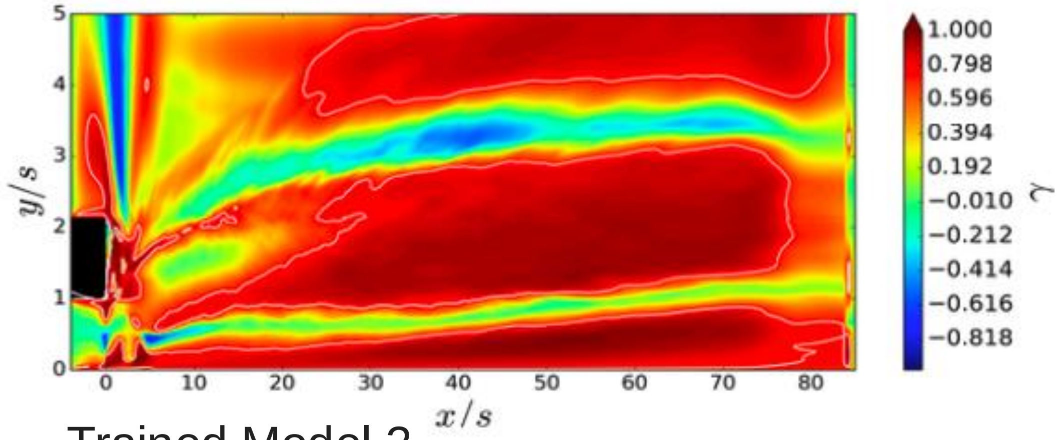
$$\gamma = -\frac{a_{ij}^{LES} S_{ij}}{\sqrt{a_{mn}^{LES} a_{nm}^{LES} S_{pq} S_{qp}}}, \quad -1 < \gamma < 1.$$

$$\gamma = \frac{a_{ij}^{LES} a_{ij}^{mod}}{\sqrt{a_{mn}^{LES} a_{nm}^{LES} S_{pq} S_{qp}}}, \quad -1 < \gamma < 1.$$

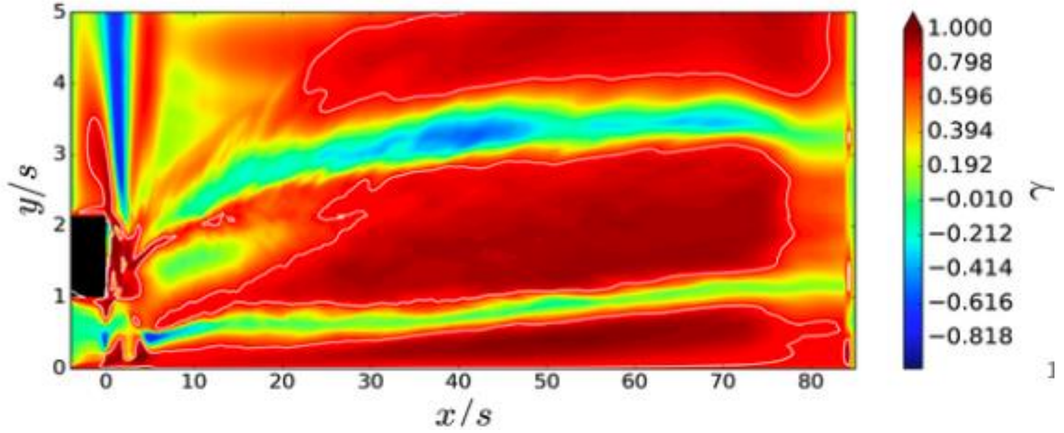



Linear Model

Trained Model 1




Trained Model 2




Conclusion: Better alignment close to the wall !!

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# Scalar Flux Model Development

Seek a functional form for the thermal diffusivity which can be a scalar or a tensor (generalized gradient diffusion Hypothesis) depending on how the Reynolds averaged heat flux is modelled

$$\overline{u_i'T'} = -\alpha_i^{LES}\frac{\partial \overline{T}}{\partial x_i}$$

$$\alpha_i^{LES} = -\frac{\overline{u_i'T'}\,\frac{\partial \overline{T}}{\partial x_i}}{\frac{\partial \overline{T}}{\partial x_i}\,\frac{\partial \overline{T}}{\partial x_i}}.$$

$$\overline{u_i'T'} = -f(I_1, IT_1, \ldots)\frac{1}{\omega}\overline{u_i u_j}\frac{\partial \overline{T}}{\partial x_j}$$

$$f(I_1, IT_1, \ldots) = -\frac{\omega\,\overline{u_i T'}\,\frac{\partial \overline{T}}{\partial x_i}}{\overline{u_i u_j}\,\frac{\partial \overline{T}}{\partial x_j}\,\frac{\partial \overline{T}}{\partial x_i}}.$$

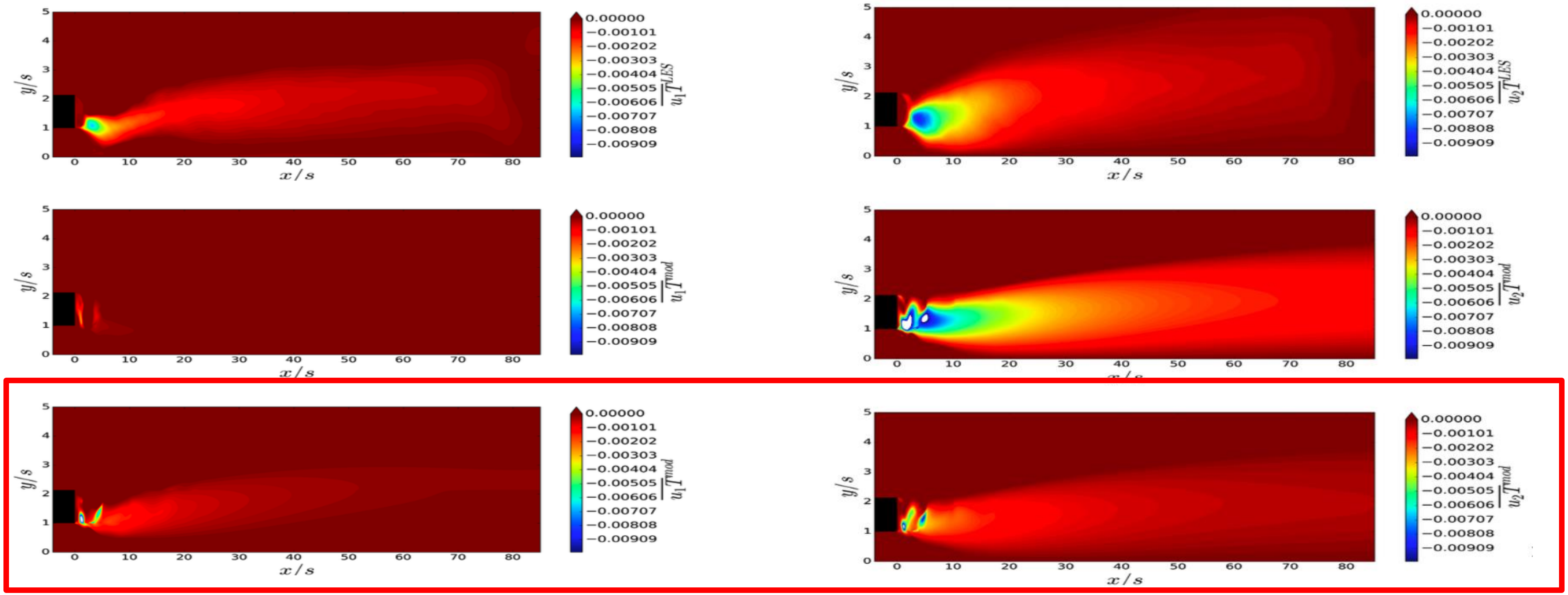
$x^{LES}$ $x^{LES}$

$$J = \frac{\sum_{n=1}^{N}(x^{mod} - x^{LES})^2}{\sum_{n=1}^{N}(x^{LES})^2}.$$

| | | |
|---|---|---|
| $I_1 = tr(S^2)$ | $I_2 = tr(\Omega^2)$ | $I_3 = tr(S^3)$ |
| $I_4 = tr(\Omega^2 S)$ | $IT_1 = T_i^T T_i$ | $IT_2 = T_i^T S T_i$ |
| $IT_3 = T_i^T S^2 T_i$ | $IT_4 = T_i^T \Omega^2 T_i$ | $IT_5 = T_i^T \Omega S T_i$ |
| $IT_6 = T_i^T \Omega S^2 T_i$ | $IT_7 = T_i^T S\Omega S^2 T_i$ | $IT_8 = T_i^T \Omega^2 S T_i$ |

**TABLE 2**: Definitions of Scalar Invariants [11]

# Comparison between LES and GEP Trained Heat flux Model




Conclusion: Tensorial Heat Flux Model performs better for both $\overline{u_1T}$ and $\overline{u_2T}$

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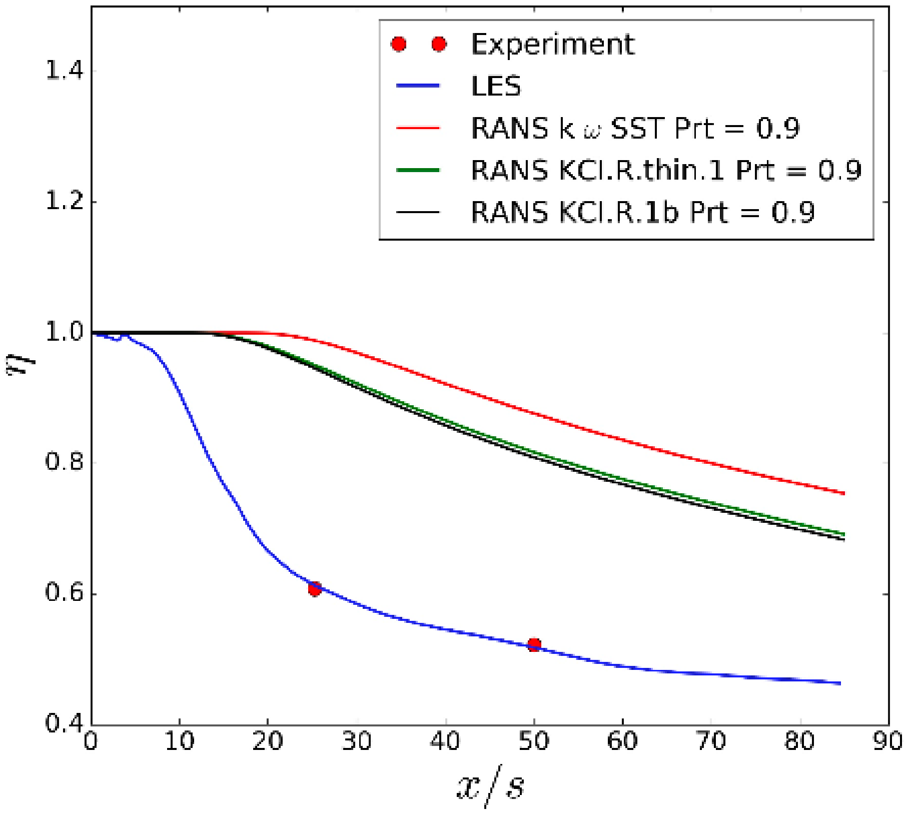
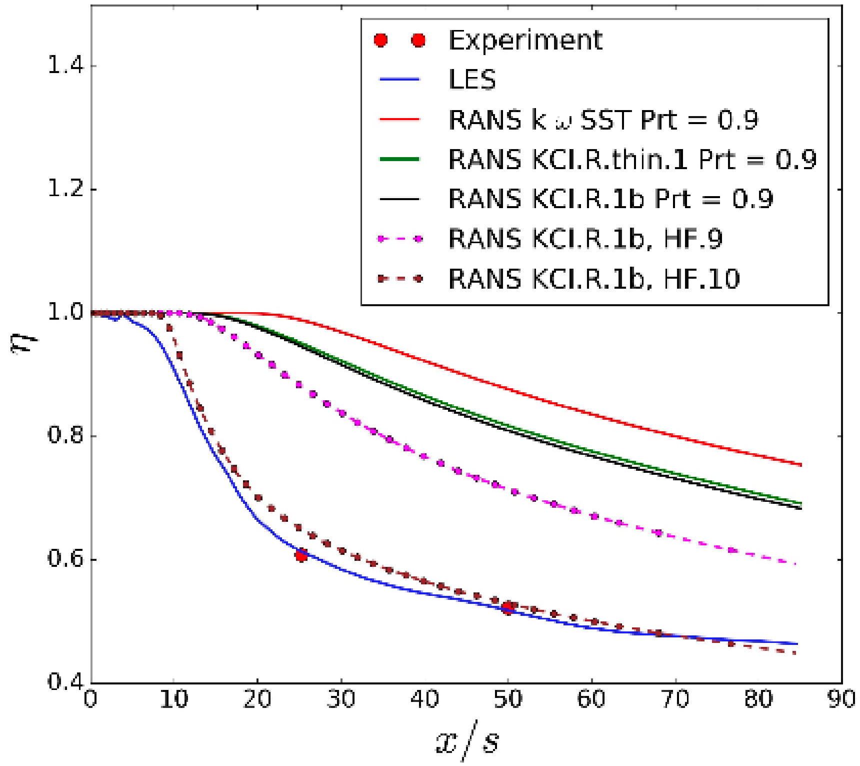
# Results




RANS model augmentation




RANS + Heat flux model augmentation

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# Conclusions – What can we learn?

- Velocity field from the RANS solver is incorrect although augmented model is developed based on 'correct' velocity field – How should model training process account for this incorrect velocity field?
- Turbulence model augmentation should be developed based on generic/fundament flow setup and not specific cases for generality
- Physical constraints can be embedded in the training algorithm through the loss function (Physical basis of the Reynolds Stress tensor as well as boundary conditions that should be adhered) -> Data and physics driven!

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# Questions?

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# Thank you

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