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THE MEASUREMENT OF FARM-SPECIFIC TECHNICAL EFFICIENCY

K. P. KALIRAJAN and J. C. FLINN*

Measures of technical efficiency were estimated using a stochastic translog production frontier for a sample of rainfed rice farmers in Bicol, Philippines. These estimates were farm specific as opposed to being based on deviations from an average sample efficiency. A wide variation in the level of technical efficiencies (0.4 – 0.9) were observed. Differences between farms in technical efficiency were significantly related to method of crop establishment, fertilizer use and access to extension services.

I. Introduction

Technical efficiency is used as a measure of a firm's ability to produce maximum output from a given set of inputs and production technology. It is a relative concept insofar as the performance of each production unit is usually compared to a standard. This standard may be used on farm-specific estimates of best practice techniques [Herdt and Mandac (1981)] but more usually by relating farm output to population parameters based on production function analysis [Timmer (1971)]. Farrell (1957) points out that the frontier production function is appropriate for such analysis as it meets the theoretical definition of a production function.

Considerable progress has been made recently in the estimation of production frontiers consistent with Farrell's definition of production. The error structure of the frontier production function models of Aigner et al. (1977) avoid the statistical problems inherent in earlier formulations of this model. However, these models only allow the technical efficiency of a firm to be estimated in comparison with average population estimates. They do not provide a procedure for estimating farm-specific technical efficiency of individual sample observations.

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From an analytical and policy context farm-specific measures of technical efficiency provide greater insights than do average technical efficiency measures. The objectives of this paper, therefore, are: (1) to develop a procedure, based on a stochastic production frontier, to estimate farmer-specific technical efficiency; (2) to estimate farm-specific technical efficiency measures for individual sample participants; and (3) to identify the factors causing variations in technical efficiency among the sample farmers. The analysis relates to a sample of farmers in the Bicol region of the Philippines who grow modern varieties of rice under rainfed conditions [Mandac et al. (1982)].

The frontier production function is first described. Second the procedures used to empirically estimate the farm-specific technical efficiency from this model is discussed. The characteristics of rainfed rice production in Bicol are then briefly described. The empirical model and the results are then discussed. The final section provides a summary and conclusions of the study.

The Model

The theoretical notion of a frontier production function is that of the loci of most efficient technical transformation of inputs into outputs. Considering a location with n farms producing a homogeneous output y using a set of inputs x_i's, the production possibilities can be described as:

$$y_i = f_i(x_1, x_2, \ldots, x_m), \quad i = 1, 2, \ldots, n$$

where $y_i \in R_+$, $x_i \in X$, $X \subset R^n_+$

The best-practice or frontier production function is defined as:

$$y^* = \max_i f_i(x_1, x_2, \ldots, x_m) / T$$

where T refers to the level of technology known to the farms in the location. The understanding and use of technology at any point of time varies from farm to farm. The frontier function thus refers to those farms' production functions that yield maximum output from given quantities of a set of inputs. Consequently, any observed levels of production should lie either on or below the frontier production function. Therefore, the basic model of maximum production can be written as:

$$y = f(x) + u \tag{1}$$

where y refers to actual output; f(x) is the transformation between inputs x and output y; and u is a technical efficiency parameter. The parameter u takes the value zero ($y = y^*$) or less than zero ($y \leqslant y^*$) depending on whether the observation y lies on or below the frontier.

Equation (1) is deterministic and reveals that a farm using best practice techniques achieves the maximum output y and so u is zero. However, u will be negative for farms not using the best practice technique, and output is less than would be obtained if it used the best practice technique. The negative value of u will vary among farms depending on their technical efficiency according to how close they are to the best practice technique. Further the maximum output y may vary randomly across farms or over time for the same farm. Therefore, a random variable v is added to equation (1) to allow for this reality.

$$y = f(x) + u + v. \tag{2}$$

The presence of v in equation (2) also means that y is stochastic, and that v captures other random factors such as errors in measurements, weather, etc. The value of v in equation (2) may either be positive, negative or zero. The stochastic frontier production function defined by equation (2) is empirically estimated using maximum likelihood techniques, given a specified functional relationship between y and x, and density functions for u and v.

Aigner et al. (1977), Battese and Corra (1977), Meeusen and Broeck (1977) and Kalirajan (1982) estimated stochastic production frontiers of the form specified by equation (2). From these they estimated population average technical efficiency, but not estimates of the technical efficiency for individual observations in the sample. Further, the Cobb-Douglas form of the production process used by these authors imposes limitations on the generality of the result which can be deduced from the analysis.[1] Therefore, a flexible functional form, the translog, as used by Vinod (1973) was applied in this paper:

$$\ln y_i = \alpha_o + \Sigma\, \alpha_k \ln X_{ki} + \Sigma\Sigma\beta_{jk} \ln X_{ki} \ln X_{ji} + u_i + v_i \tag{3}$$

where x and y are as defined in equation (1); $u < 0$ and takes a half normal distribution and $v(\lesseqgtr 0)$ and takes a normal distribution.

[1] For example, the elasticities are constant, implying constant shares regardless of input level and the elasticity of substitution among input is unity. However, a statistical test is carried out to examine the suitability of the Cobb-Douglas model for the present data.

The density function of u and v can respectively be written as [Rao (1965)] :

$$f(u) = \frac{1}{\sqrt{\tfrac{1}{2}\pi}} \cdot \frac{1}{\sigma_u} \exp\left(-\frac{u_i^2}{2\sigma_u^2}\right) \qquad u \leqslant 0 \tag{4}$$

$$f(v) = \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma_v} \exp\left(-\frac{v_i^2}{2\sigma_v^2}\right) \qquad -\infty \leqslant v \leqslant \infty \tag{5}$$

The likelihood function of y is the product of density function of each y_i which is equal to the density function of $(u_i + v_i)$. By the convolution formula, the joint density function of $(u_i + v_i)$ can be written as [Rao (1965)] :

$$f(u_i + v_i) = \frac{1}{\sqrt{\tfrac{1}{2}\pi(\sigma_u^2 + \sigma_v^2)}} \exp\left(-\frac{(u_i + v_i)^2}{2(\sigma_u^2 + \sigma_v^2)}\right) \left[1 - F(u_i + v_i \cdot \frac{\sigma_u}{\sigma\sigma_v})\right] \tag{6}$$

where

(i) $F(\cdot)$ is the cumulative distribution function of the standard normal random variable,

(ii) $\sigma^2 = \sigma_u^2 + \sigma_v^2$,

iii) $\gamma = \dfrac{\sigma_u^2}{\sigma_u^2 + \sigma_v^2}$ where γ lies in the interval (0,1),

and

iv) $u_i + v_i = e_i$.

Using this notation the density function of y_i as defined in equation (6) may be written as:

$$f(y_i) = \frac{1}{\sigma\sqrt{\pi/2}} \exp\left(-\frac{1}{2}\frac{e_i^2}{\sigma^2}\right) \left[1 - F\left(\frac{e_i}{\sigma}\sqrt{\frac{\gamma}{1-\gamma}}\right)\right] \cdot \tag{7}$$

The likelihood function corresponding to (7) of the sample will thus be:

$$L^*(y;\, \theta) = \prod_{i=1}^{n} \left[\frac{1}{\sigma\sqrt{\pi/2}} \exp\left(-\frac{1}{2}\frac{e_i^2}{\sigma^2}\right)\left(1 - F\left(\frac{e_i}{\sigma}\sqrt{\frac{\gamma}{1-\gamma}}\right)\right)\right]. \tag{8}$$

where $e_i = \ln y - \alpha_o - \Sigma\alpha_i \ln x_i - \Sigma\Sigma\beta_{ij} \ln x_i \ln x_j$, and θ is the parameter to be estimated which contains the elements $(\alpha_o, \alpha', \beta', \sigma^2, \gamma)$.

The maximum likelihood (ML) estimators of θ maximizing the above likelihood function are obtained by setting its first order partial derivatives with respect to the elements of θ equal to zero, and solving them simultaneously.

The next issue is to estimate the farm-specific technical efficiency for each observation in the sample. Measurement of u for individual observations are derived from the conditional distribution of u, given (u + v) [Rao (1965)]. Given a normal distribution for v and a half-normal distribution for u, the conditional mean of u given (u + v) is:

$$E(u/u+v) = \int_{-\infty}^{o} u \cdot f(u/u+v),$$

where

$$f(u/u+v) = \frac{f(u, u + v)}{f(u + v)} .$$

The density function of u, given (u + v), using equations (4) and (6) is equivalent to:

$$f(u/u+v) = \frac{1}{\sqrt{2\pi}} \frac{\sigma}{\sigma_u^2 \sigma_v^2} \exp \left[- \frac{\sigma^2}{2\sigma_u^2 \sigma_v^2} (u + \frac{e\sigma_u^2}{\sigma^2})^2\right] \frac{1}{1-F(\cdot)} . \tag{9}$$

Therefore

$$E(u/u+v) = \frac{\sigma}{\sigma_u \sigma_v} \left[\frac{f(\cdot)}{1 - F(\cdot)} - e \frac{\gamma}{1 - \gamma}\right] \tag{10}$$

where $f(\cdot)$ is the standard normal density function. The value of u for each observation is then obtained by substituting the values of σ, σ_u and γ from the ML estimate of equation (8), along with e – the residual specific for each observation into equation (10).

The Data

The data used in the analysis was derived from a random sample of 79 rainfed rice farmers included in a farm management survey conducted by the International Rice Research Institute during crop year 1980-81 [Mandac et al. (1982)]. Though the sample was divided equally between tenant and owner operators input use and yields did not differ significantly by tenure group. Therefore, for the purpose of this analysis the two tenurial groups are combined.

Modal farm size of respondents was 0.6 hectare (ha); only 4 farmers cultivated more than 2 ha of rice, 75 per cent cultivated less than 1 ha (Table 1). All farmers grew modern rice varieties, mainly IR36 and IR42.

TABLE 1

Typical rain-fed rice production system, Bicol, Philippines[1]

Characteristics	Units	Mean Values
Operational holding	ha	0.73
Rice Output	tons/ha	2.38
Labour	man days	68.00
Pesticides	Peso/ha	45.00
Mechanized land preparation*	days/ha	3.20
Sample size	number	79.00

[1]US$ = 7.50 Pesos approximately in 1980-81.

*Animal power used was converted to mechanical power days at a rate of 1:3 as suggested by Barker et al. (1969) and used by Orcino and Duff (1974).

Only 8 per cent of sampled farmers applied fertilizer, and of those using fertilizer, rates (measured in terms of total nutrients) were lower than recommended for rainfed conditions in the Philippines [PCARRD (1977)]. Over 90 per cent of farmers used herbicides and over 80 per cent applied insecticides, the mean rice yield was 2.4 tons/ha.

In the analysis which follows the cost of pesticides (i.e., insecticides plus herbicides) was included as a composite variable. Fertilizer was excluded because of its infrequent use, and also adding an arbitrarily constant to zero fertilizer values to allow logarithmic transformation in the model specification, results in a biased estimate [Johnson and Rausser, (1971)].

The Empirical Model

The empirically estimated production frontier for the rainfed rice production process was as follows:

$$\ln y_i = \alpha_o + \Sigma\, \alpha_k \ln X_{ki} + \Sigma\Sigma\beta_{jk} \ln X_{ki} \ln X_{ji} + u_i + v_i \qquad (11)$$

where

y = output of rice, per farm, in kgs;

x_1 = number of pre-harvest days of labour used;

x_2 = the cost of pesticides (insecticides plus herbicides in Pesos);

x_3 = number of days of mechanical (hand tractor) power used in land preparation;

x_4 = rice area in ha;

v = a disturbance terms with 'normal' properties;

u = farm-specific error term as defined earlier and follows a half normal distribution.

The estimation of such a single equation production frontier yields biased estimates of the true parameters because it assumes instantaneous adjustment of output with respect to changes in the level of inputs. This is known as simultaneous equation bias. This limitation can be overcome by assuming that uncertainty surrounds future output only, with all prices being known at the beginning of the period, and the farmer maximises expected profit rather than the actual one [Zellner et al. (1966)].

The method used to obtain the ML estimates was that suggested by Fletcher and Powell (1963), and programmed in ALGOL by Wells (1967) which incorporates the constraint that γ lies in the interval (0,1). The ordinary least squares estimates of α, β and σ^2 were used as the starting values for the Fletcher-Powell method.

II. Results and Discussion

Estimate of Production Frontier

The MLE estimates of the production frontier (11) are given in Table 2, along with the OLS estimates. Both sets of estimates are similar with the exception of the intercept term. The estimate of the intercept term obtained by OLS is smaller than that obtained by MLE.

First, a statistical test has been carried out to test whether the functional form of the production frontier is non-homothetic (Trans-log) or homothetic (Cobb-Douglas). For the non-homothetic production frontier given in equation (11) to be Cobb-Douglas, it is necessary that all the interaction terms

TABLE 2

MLE and OLS estimates of stochastic translog production frontier for sample farmers

Parameters	Estimates	
	MLE	OLS
α_0	4.4438	3.5621
	(1.0213)	(1.2100)
α_1	−0.2601	−0.2552
	(0.0803)	(0.8560)
α_2	0.5203	0.5163
	(0.0401)	(0.0523)
α_3	0.3931	0.4025
	(0.1233)	(0.1425)
α_4	−0.6342	−0.6204
	(0.2018)	(0.2201)
β_{11}	0.1565	0.1472
	(0.0702)	(0.0811)
β_{12}	−0.1223	−0.1303
	(0.0562)	(0.0621)
β_{13}	−0.0935	−0.1026
	(0.0622)	(0.0693)
β_{14}	0.4201	0.4021
	(0.1983)	(0.2006)
β_{22}	−0.0030	−0.0039
	(0.0121)	(0.0158)
β_{23}	0.0876	0.0821
	(0.0321)	(0.0363)
β_{24}	−0.1235	0.1189
	(0.0337)	(0.0421)
β_{33}	−0.0086	−0.0093
	(0.0041)	(0.0053)
β_{34}	0.0582	0.0602
	(0.0339)	(0.0382)
β_{44}	0.0682	0.0713
	(0.0248)	(0.0318)
Log likelihood function	−21.8316	−78.2103
σ^2	0.4818	0.2512
σ_u^2	0.3927	
γ	0.8151	
	(0.1825)	

Note: Figures in parentheses are asymptotic standard errors.

(β_{ij}'s) in equation (11) are zero. The conventional F-test is done to test the null hypothesis that all $\beta_{ij} = 0$. The computed F value (3.63) is greater than the tabulated F value (2.61) for (10,64) degrees of freedom at the 1 per cent level of significance. This implies that the null hypothesis cannot be accepted. Thus, the non-homothetic functional form fits the data better than the homothetic form for this particular type of specification.

Production elasticities with respect to individual inputs are calculated using the frontier estimates reported in Table 2. They are calculated at the mean level of inputs reported in Table 1. The calculated production elasticities are presented in Table 3 along with the formula. All production coefficients have theoretically acceptable signs and are statistically significant.

TABLE 3

Frontier production elasticities for sample participants

Input	Output elasticity
Labour	0.2737
Pesticides	0.1544
Mechanical Power	0.1703
Land (operational area)	0.5146

Note: 1. Output elasticities are calculated at the mean level of input applications.
2. Mean level of inputs are taken from Table 1.
3. The following formula is used to calculate the frontier production elasticities:

$$\frac{\partial \ln y}{\partial \ln x_i} = \alpha_i + 2\beta_{ii} \ln x_i + \sum_{\substack{j=1 \\ j \neq j}}^{4} \beta_{ij} \ln x_j$$

$$i = 1, 2, 3, 4.$$

4. α and β's are the MLE estimates reported in Table 2.

An important result for the present study from Table 2 is that the variance ratio parameter γ is comparatively large, given the interval within which it lies, and differs significantly from zero. This implies that about 82 per cent of the difference between the observed output and the maximum production frontier output is caused by differences in farmers' levels of technical efficiency as opposed to the conventional random variability.

Farm Specific Technical Efficiency

The technical efficiency relative to the stochastic production frontier is derived as the rate of observed to expected y. That is:

$$e^{u_i} = \frac{y_i}{f(x_i)e^{v_i}}$$

and $$u_i = \ln y_i - (\alpha_o + \Sigma\alpha_i \ln x_i + \Sigma\Sigma\beta_{ij} \ln x_i \ln x_j + v_i).$$

The mean technical efficiency as reported in earlier studies is calculated as:

$$E(u_i) = -\sigma_u \sqrt{2/\pi}\,.$$

Substituting the value of σ_u from Table 2 provides an estimate of a 50 per cent mean technical efficiency for the study area.

The farm-specific technical efficiency for individual sample farmers was estimated using equation (10) as described. They are reported in Table 4. These results reveal wide variation in the level of technical efficiencies across sample farms, ranging from 0.38 to 0.91. Less than 20 per cent of the sample participants obtained output in the neighbourhood of (i.e., within 20 per cent) the maximum output estimated through the frontier.

Factors Associated with Differences in Technical Efficiency

Farmer specific factors were examined to identify those associated with between farm differences in technical efficiency. Two groups of factors – those biological in nature (e.g., soil characteristics, method of crop establishment) and socio-economic in nature (e.g., education, farming experience) were examined for association with farmer's level of technical efficiency. Due to data constraints, only the method of crop establishment (direct seeded or transplanted) and fertilizer application were included as biological factors. Farmer's age, farming experience, number of years of formal schooling, and contact with agricultural extension officials were the socio-economic factors included in the analysis.

The linear function estimated to relate farmer specific technical efficiency and these independent variables was:

$$\hat{u}_i = a_o + \sum_{i=1}^{6} a_i s_i + w_i \qquad (12)$$

TABLE 4

Frequency distribution of farm-specific technical efficiency for sample farmers

Efficiency levels	Number of farms	Percentage
0.35 – 0.40	8	10
0.41 – 0.45	12	15
0.46 – 0.50	18	23
0.51 – 0.55	5	6
0.56 – 0.60	3	4
0.61 – 0.65	5	6
0.66 – 0.70	6	8
0.71 – 0.75	5	6
0.76 – 0.80	3	4
0.81 – 0.85	8	10
0.86 – 0.91	6	8
Total	79	100

Derived via equation (10).

where $\hat{u}_i$ is the estimated technical efficiency reported in Table 3, s_i's are the biological and socio-economic factors described above and w_i is a disturbance term with 'normal' properties.

The OLS estimates of equation (12) are given in Table 5. All coefficients are statistically significant except the age of the farmer and the number of years of formal schooling. The model explains about 46 per cent of the variation in the farmer specific technical efficiency across the sample.

The results indicate that technical efficiency is significantly higher on transplanted farms than on direct seeded farms. Results of field experiments

TABLE 5

OLS estimates of the determinants of variation in the technical efficiency across sample farms*

Variables	Units	Estimates	Marginal R^2
Intercept		0.3282 (0.1346)	
Crop establishment (Transplanting = 1; Direct seeded = 0)	Dummy 0.1	0.2023 (0.0562)	0.15
Fertilizer application (Applied = 1; Not applied = 0)	Dummy 0.1	0.1031 (0.0318)	0.12
Age	Years	0.0004 (0.0013)	0.02
Education	Years of schooling	0.0056 (0.0135)	0.01
Experience	Years of farming	0.0037 (0.0013)	0.07
Extension contact	Number of visits	0.0245 (0.0138)	0.13
$\bar{R}^2 = 0.4582$			

*Figures in brackets are standard errors of estimates.

conducted in the same location showed that transplanted rice yielded more output than the direct seeded rice [Mandac, et al. (1982)]. This crop establishment variable is confounded with the moisture status of the fields, as it was observed that the incidence of direct seeding was highest on plots short of water at planting time. The finding that technical efficiency increases with the increase in the number of extension contacts, is in conformity with the earlier findings of Herdt and Mandac (1981). They concluded that a lack of effective extension service contributed to lower rice output in the Philippines.

The relative importance of each factor in explaining variation in the levels of farm-specific technical efficiency are determined by looking at the marginal R^2 given in Table 5. The marginal R^2 for the method of crop establishment' (0.15) indicates that it causes about 33 per cent of the explained variability. Extension officials' contact and the fertilizer use variable both explain a further 26 per cent of variability. These three factors, thus, appear to be important determinants of between-farm technical efficiency variability.

III. Summary and Conclusion

Technical efficiency may be measured in a number of ways. Among these, measuring technical efficiency relative to a stochastic production frontier offers potential to overcome the shortcomings of other methods such as deterministic frontier and probabilistic frontier models. Earlier studies, however, have estimated only population average technical efficiency relative to a stochastic production frontier. From the policy perspective, what is necessary is a measure of technical efficiency for each observation in a sample. This paper provides such a measure for a sample of Filipino rice farmers. These individual technical efficiency measures were used to identify the factors associated with variation in the level of efficiency across the sample.

The empirical results showed that the level of technical efficiency among sample farmers varied from 0.40 to 0.91 with an average value of 0.50. The method of crop establishment, extension officials' contact and fertilizer application were identified as important factors causing variations in the level of technical efficiency among sample farmers.

From a policy point of view comparison between best practice (frontier production function) production and individual production levels give a valuable description of the prevailing production technology in a particular area. Identification of the factors causing differences between best practice and individual outputs, could help policy-makers to formulate appropriate programmes to reduce such gaps.

National University of Singapore
International Rice Research Institute, Phillipines

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