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Control Engineering Practice 14 (2006) 1213–1229
CONTROL ENGINEERING PRACTICE
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An urban traffic network model via coloured timed Petri nets

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Available online 20 March 2006

Abstract

This paper deals with modelling of traffic networks (TNs) for control purposes. A modular framework based on coloured timed Petri nets (CTPNs) is proposed to model the dynamics of signalized TN systems: places represent link cells and crossing sections, tokens are vehicles and token colours represent the routing of the corresponding vehicle. In addition, ordinary timed Petri nets model the signal timing plans of the traffic lights controlling the area. The proposed modelling framework is applied to a real intersection located in Bari, Italy. A discrete event simulation of the controlled intersection validates the model and tests the signal timing plan obtained by an optimization strategy presented in the related literature.

Keywords: Modelling; Urban systems; Petri nets; Discrete event systems; Traffic control; Validation

1. Introduction

1.1. Problem statement

Traffic management systems address the objective of reducing congestion, vehicle delay time, fuel consumption and pollution. The most common technique to regulate and manage urban traffic areas and surface street networks is traffic signal control. The currently available traffic control strategies may be grouped into two main classes (Diakaki, Papageorgiou, & Aboudolas, 2002; Papageorgiou, Diakaki, Dinopoulou, Kotsialos, & Wang, 2003; Patel & Ranganathan, 2001): (1) fixed-time systems and (2) traffic-responsive systems. In the first group, the control system determines fixed timing plans by using an off line optimization method performed by computer programs. The second class of traffic control strategies employs actuated signal timing plans and performs an on-line optimization and synchronization of traffic signals. In the real-time optimization control strategies, detectors located on the intersections' branches monitor traffic conditions and feed information on the actual system state to the real-time controller. In both control classes, the traffic network (TN) has to be appropriately modelled, either for simulation purposes or in order to determine on line some states of the transportation network that are not available due to detector absence or failures (Gabard, 1991).

1.2. Literature review

Urban TNs exhibit high degree of concurrency and are characterized by resource sharing and conflicts. Hence, appropriate models of these systems have to take into account such distinctive features, in order to result in efficient traffic management strategies. In particular, urban TNs can be viewed as event driven and asynchronous systems. Their dynamics depends on the complex interactions of the timing of various discrete events, such as arrivals or departures of vehicles at intersections and beginning or completion of the various phases in the signal timing plans of the traffic lights controlling junctions (Tzes, Kim, & McShane, 1996). Thanks to the well-known ability of Petri nets (PNs) to capture concurrency and asynchrony, PN based models may be suitably derived for urban traffic systems. More precisely, PNs can be employed both for describing traffic signals controlling urban signalized areas, as well as for modelling concurrent activities that are typical of TNs. While an example of coloured PNs modelling a traffic light was first proposed in (Jensen, 1986), the idea of applying PNs to model TNs can be dated back to the approach presented in Giua (1991). Later on,

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doi:10.1016/j.conengprac.2006.02.005

Tzes et al. (1996), Gallego, Farges, and Henry (1996) and Di Febbraro, Giglio, and Sacco (2002) have modelled traffic lights controlling TNs by PNs: these approaches share the idea to adjust signals controlling an intersection according to the distinct tokens deposited in a PN controller. More recently, List and Cetin (2004) discuss the use of PNs in modelling traffic signal controls and perform a structural analysis of the control PN model by P-invariants, demonstrating how such a model enforces the traffic operation safety rules. With particular reference to the use of PNs for modelling TNs, Di Febbraro et al. (2002) present a traffic model in a timed PN framework, where tokens are vehicles and places are parts of lanes and intersections. Since in PNs tokens cannot distinguish among different vehicles and their associated routes, colours are introduced in Di Cesare, Kulp, Gile, and List (1994), where a different number (colour) is assigned to each vehicle entering the system. The model is realized by defining appropriate subnets modelling links and intersections. However, the TN modelling methodology developed in Di Cesare et al. (1994) does not use the coloured PN paradigm to simulate the system that is substantially modelled with ordinary PNs.

Fluid stochastic PNs are proposed by Bouyekhf, Abbas-Turki, Grunder, and El Moudni (2003) and hybrid PNs by Di Febbraro, and Sacco (2004a, 2004b) to model urban networks of signalized intersections. In particular, hybrid PNs employed by Di Febbraro and Sacco (2004a) are a combination of a "classical" PN and a continuous PN in order to suitably represent the event-driven dynamics of the traffic light and the traffic flows as fluids. Even if the model is efficient and able to estimate the network performance, it needs some input parameters that have to be partially measured or estimated. Some of these parameters require an identification algorithm based on real traffic data.

1.3. The proposed approach

This paper presents a coloured timed Petri net (CTPN) model to describe the flow of vehicles in an urban TN. The token colour represents the path that a vehicle has to follow and places are cells accommodating one vehicle at a time. Moreover, the paper models the TN traffic lights by ordinary timed Petri nets (TPNs). The model presents three main advantages. First, the parameters required by the model dynamics are the number of vehicles entering each road lane and their average rate. These parameters can be directly measured and, respectively, evaluated at each input branch of every intersection. Second, the model is modular and easy to build on the basis of the TN topology. Third, the code to simulate real intersections can be directly generated in order to test both fixed signal timing plans or actuated control strategies. Comparing the presented approach with other models based on PNs (e.g., Gallego et al., 1996; Giua, 1991; Tzes et al., 1996) or Hybrid PNs (e.g., Di Febbraro & Sacco, 2004a), it may be enlightened that CTPNs provide an intermediate level of representation by increasing the descriptive power of PNs while retaining the ease of obtaining the necessary input parameters. In addition, the introduced model is validated by the simulation of a case study describing a real intersection located in the city of Bari, Italy: the traffic dynamics is simulated under different traffic scenarios.

The paper is organized as follows. Section 2 recalls the basics of TPNs and CTPNs. Moreover, Section 3 defines the CTPN modelling the TN and the TPN synthesizing the traffic light. Section 4 describes the case study and reports the results of some simulations performed under different traffic scenarios. Finally, Section 5 summarizes the conclusions.

2. Basics of ordinary timed Petri nets and coloured timed Petri nets

2.1. Ordinary timed Petri nets

An ordinary TPN is a bipartite digraph described by the five-tuple $TPN = (P, T, \boldsymbol{Pre}, \boldsymbol{Post}, \boldsymbol{FT})$ where P is a set of places, T is a set of transitions (Murata 1989), $\boldsymbol{Pre}$ and $\boldsymbol{Post}$ are the pre-incidence and the post-incidence matrices, respectively, of dimension $|P| \times |T|$. Note that symbol $|A|$ is used to denote the cardinality of the generic set A. More precisely, the element $Pre(p_i,t_j)$ is equal to 1 if an arc joining p_i and t_j exists and 0 otherwise; the element $Post(p_i,t_j)$ is equal to 1 if an arc joining t_j and p_i exists and 0 otherwise. Moreover, $\boldsymbol{FT}$ is the firing time vector, specifying the deterministic duration of the firing of each transition.

The state of a TPN is given by its current marking that is a mapping $\boldsymbol{M}$: $P \rightarrow N$ where N is the set of non-negative integers. $\boldsymbol{M}$ is described by a $|P|$-vector and the ith component of $\boldsymbol{M}$, indicated with $M(p_i)$, represents the number of tokens in the ith place $p_i \in P$.

Given a TPN and a transition $t \in T$, the following sets of places may be defined: $\bullet t = \{p \in P : Pre(p,t) > 0\}$, named pre-set of t; $t\bullet = \{p \in P : Post(p,t) > 0\}$, named post-set of t.

A transition $t_j \in T$ is enabled at a marking $\boldsymbol{M}$ if and only if for each $p_i \in \bullet t_j$, $M(p_i) > 0$. When fired, t_j gives a new marking $\boldsymbol{M}'$, where for each $p_i \in P$ it holds $M'(p_i) = M(p_i) + Post(p_i, t_j) - Pre(p_i, t_j)$.

The firing time of transition t_j is FT_j (i.e., the jth element of vector $\boldsymbol{FT}$) that specifies the duration of the firing of t_j. If $FT_j = 0$, t_j is said immediate transition. Finally, a TPN system is denoted by $\langle TPN, \boldsymbol{M_0} \rangle$ and is a TPN with initial marking $\boldsymbol{M_0}$.

2.2. Coloured timed Petri nets

A CTPN is defined as a bipartite directed graph represented by a seven-tuple $CTPN = (P, T, Co, H, \boldsymbol{Pre}, \boldsymbol{Post}, \boldsymbol{FT})$, where P is a set of places, T is a set of transitions, Co is a colour function that associates with each element in $P \cup T$ a non empty ordered set of colours in the set of possible colours Cl (Jensen, 1992). Co maps each place $p_i \in P$ to the set of possible token colours $Co(p_i) = \{a_{i,1}, a_{i,2}, \ldots, a_{i,ui}\} \subseteq Cl$, where $u_i = |Co(p_i)|$ is the number of possible colours of tokens in p_i. Analogously, Co maps each transition $t_j \in T$ to the set of possible occurrence colours $Co(t_j) = \{b_{j,1}, b_{j,2}, \ldots, b_{j,uj}\} \subseteq Cl$ with $u_j = |Co(t_j)|$. Moreover, to limit the number of coloured tokens in a place, an additional modelling construct is introduced (Desrochers & Al-Jaar, 1995) called inhibitor arc. Hence, a weight function H is defined for an inhibitor arc that connects a transition to a place. More precisely, an inhibitor arc labelled $a \in N$ between $p_i \in P$ and $t_j \in T$ (i.e., $H(p_i, t_j) = a$) implies that as soon as there are a tokens in p_i, the arc inhibits the firing of t_j.

A non-negative multiset α over the set D is defined as a mapping α: $D \rightarrow N$ (Jensen, 1992). Analogously to ordinary TPNs, matrices $\boldsymbol{Pre}$ and $\boldsymbol{Post}$ are the pre-incidence and the post-incidence matrices, respectively. In particular, each element $\boldsymbol{Pre}(p_i,t_j)$ is a mapping from the set of occurrence colours of $t_j \in T$ to the set of non-negative multisets $N(Co(p_i))$ over the set of colours of $p_i \in P$. More precisely, $\boldsymbol{Pre}(p_i,t_j)$: $Co(t_j) \rightarrow N(Co(p_i))$ denotes a matrix of $u_i \times u_j$ non-negative integers, whose generic element $Pre(p_i,t_j)(h,k)$ is equal to the weight of the arc from place p_i with respect to colour $a_{i,h}$ to transition t_j with respect to colour $b_{j,k}$.

Analogously, $\boldsymbol{Post}(p_i,t_j)$: $Co(t_j) \rightarrow N(Co(p_i))$ for each $t_j \in T$ to each $p_i \in P$ corresponds to the set of directed arcs from T to P. Hence, $\boldsymbol{Post}(p_i,t_j)$ denotes a matrix of $u_i \times u_j$ non-negative integers and the scalar $Post(p_i,t_j)(h,k)$ is the weight of the arc from transition t_j with respect to colour $b_{j,k}$ to place p_i with respect to colour $a_{i,h}$.

For each place $p_i \in P$, the marking $\boldsymbol{m_i}$ of p_i is defined as a *non-negative multiset* over $Co(p_i)$. The mapping $\boldsymbol{m_i}$: $Co(p_i) \rightarrow N$ associates with each possible token colour in p_i a non-negative integer representing the number of tokens of that colour which is contained in p_i. In the following $\boldsymbol{m_i}$ denotes a $(u_i \times 1)$ vector of non-negative integers, whose hth component $m_i(h)$ is equal to the number of tokens of colour $a_{i,h}$ that are contained in p_i. The marking $\boldsymbol{M}$ of a CTPN is the column vector:

$$\boldsymbol{M} = \begin{bmatrix} \mathbf{m_1} \\ \cdots \\ \mathbf{m}_{|P|} \end{bmatrix}. \tag{1}$$

A transition $t_j \in T$ is *enabled* with respect to colour $b_{j,k}$ at a marking $\boldsymbol{M}$ if and only if the following conditions are verified:

(C1) for each $p_i \in \bullet t_j$ it holds $m_i(h) \geqslant Pre(p_i,t_j)(h,k)$ for $h = 1, \ldots, u_i$,
(C2) for each $p_i \in t_j \bullet$ such that $H(p_i,t_j) > 0$, it holds $\sum_{h=1}^{u_i} m_i(h) \leqslant H(p_i, t_j)$.

If an enabled transition $t_j \in T$ fires with respect to colour $b_{j,k}$, then one gets a new marking $\boldsymbol{M}'$, where for each $p_i \in P$ and for each $h = 1, \ldots, u_i$, it holds: $m_i'(h) = m_i(h) + Post(p_i, t_j)(h,k) - Pre(p_i, t_j)(h,k)$.

Considering that the temporization of a coloured PN can be achieved by attaching time either to places, to transitions (Desrochers & Al-Jaar, 1995) or to the expression functions of arcs (Jensen, 1992), here the second option is chosen and timed transitions and immediate transitions are considered. More precisely, $\boldsymbol{FT}$ denotes a timing vector and the firing time of each transition t_j is the positive number FT_j (i.e., the jth element of $\boldsymbol{FT}$) specifying the deterministic duration of the firing of t_j. In this method, each token has a time stamp attached to it, in addition to token colours. The time stamp is described by the function s: $Co \rightarrow \Re+$ where $s(c)$ indicates the earliest delay after which the token of colour $c \in Co$ becomes available and can be removed by an enabled transition. Hence, as soon as the c-colour token arrives to the place p_i enabling transition t_j, $s(c)$ is set equal to FT_j. Accordingly, after FT_j time instants, the enabled transition t_j becomes ready to fire with respect to colour c. If FT_j is equal to zero, the transition is said to be an immediate transition.

Finally, a coloured timed PN system $\langle CTPN, \boldsymbol{M_0} \rangle$ is a CTPN with initial marking $\boldsymbol{M_0}$.

3. Modelling a signalized traffic network

3.1. The traffic network description

In the proposed TN model, the following fundamental components are considered: signalized intersections, links, vehicles and traffic lights. More precisely, each link represents the space available between two adjacent intersections and may include one or several lanes. Hence, a generic signalized urban area comprises a number of junctions controlled by traffic lights pertaining to a common signal timing plan, including a set $L = \{L_i | i = 1, \ldots, I\}$ of I links. In addition, links can be classified as: (1) input links $L_i \in L_{in}$, that are controlled by a traffic light located at their end, (2) intermediate links $L_i \in L_{int}$, also equipped with a traffic light, and (3) output links $L_i \in L_{out}$, from which vehicles exit freely. A generic link L_i of length l_i with $i = 1, \ldots I$ has a finite capacity $C_i > 0$ denoting the number of passenger car units (PCUs) that the link can simultaneously accommodate. Hence, each link is usually divided in C_i cells with unit capacity. Moreover, it is necessary to take into account the physical space that a vehicle crossing the intersection occupies. Such a physical space, named intersection cell, can be occupied by only one vehicle and may coincide with the whole intersection area in a very simple intersection or with a part of the physical space in a multi-lane intersection.

Example 1. The TN depicted in Fig. 1 exhibits $I = 14$ links, defining set $L = \{L_i | i = 1, \ldots, 14\}$, partitioned in the input links set $L_{in} = \{L_i | i = 1, \ldots, 6\}$, the intermediate links set $L_{int} = \{L_i | i = 7, 8\}$ and the output links set $L_{out} = \{L_i | i = 9, \ldots, 14\}$.

In addition, each link in Fig. 1 includes only one lane and if it is assumed that it holds $l_i = 80\,\text{m}$ with

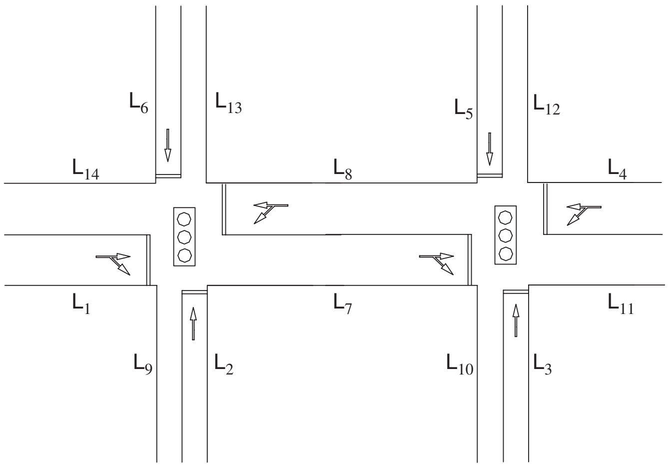

Fig. 1. Example of urban area comprising two intersections.

$i = 1, \ldots, 14$, the link capacities are $C_i = 16$ PCUs with $i = 1, \ldots, 14$, where it is supposed that one PCU is $l_0 = 5$ m long.

TN systems are modelled by using two nets: a CTPN describes roads and junctions of the TN, a TPN describes the traffic light pertaining to a common signal timing plan. Obviously, if some intersections have two or more independent signal timing plans, an independent TPN have to model the traffic lights of each timing plan. For the sake of clarity, CTPNs and TPNs were defined separately, however TPNs can be viewed as a particular case of the considered CTPNs, in which the set *Cl* contains only one colour. Consequently, one could consider the TPNs modelling the traffic lights as a subnet of the CTPN modelling the signalized TN.

3.2. The coloured timed Petri net modelling the urban area

In the proposed model the $CTPN = (P, T, Co, H, \boldsymbol{Pre}, \boldsymbol{Post}, \boldsymbol{FT})$ describes the urban area in a modular representation. A place $p_i \in P$ denotes a resource, i.e., a cell in a lane or in a crossing section that a vehicle can occupy. More precisely, two types of places are considered: places representing link cells and places corresponding to intersection cells.

In addition, transitions from T are timed transitions and model the flow of vehicles into the system between consecutive cells. Five types of transitions are considered:

(1) input transitions $t_{0,i} \in T_I$, modelling the arrival of a vehicle in a link from the outside of the considered intersection;
(2) output transitions $t_{i,0} \in T_O$, modelling the departure of a vehicle from a link to the outside of the considered intersection;
(3) flow transitions $t_j \in T_F$, modelling the flow of vehicles between two consecutive link cells;
(4) intersection transitions $t_j \in T_C$, modelling vehicles entering or leaving a crossing section. The intersection transitions that model a vehicle entering the crossing sections are regulated by a multi-phase traffic light;
(5) lane changing transitions $t_j \in T_L$, modelling vehicles that change lane in road.

The value of FT_j assigned to each $t_j \in T_O \cup T_F \cup T_C$ is equal to the average time interval in which a vehicle passes from a cell to the subsequent one or occupies an intersection cell and depends on the average vehicle speed in the considered traffic scenario. However, a longer time is assigned to transitions $t_j \in T_L$ modelling lane changing. In addition, the firing times of $t_{0,j} \in T_I$ are equal to the interval times in which vehicles can enter the system network: such values may change during the observation time in the same traffic scenario.

A coloured token in a place represents a vehicle. The colour of each token is the routing assigned to each vehicle indicating the different paths that a vehicle can follow starting from a particular position. For instance, with reference to Example 1 in Fig. 1, in link L_1 vehicle tokens have three different colours: a_1 if the vehicle has routing (L_1, L_7, L_{11}), a_2 if the vehicle route is (L_1, L_9) and a_3 if the vehicle route is (L_1, L_7, L_{10}). Finally, inhibitor arcs are introduced to model the finite capacities of each cell. More precisely, for each transition $t \in T_I \cup T_F \cup T_C \cup T_L$ and for each $p_i \in t\bullet$, there exists an inhibitor arc between p_i and t, i.e., $H(p_i, t) = 1$.

To show the modelling technique, Example 2 describes a real traffic intersection located in the urban area of Bari, Italy, depicted in Fig. 2. The example considers a single junction. However, the modularity of the proposed model allows one to represent several intersections and to connect two consecutive links simply considering the output transitions of the first link coincident with the input transitions of the next link.

Example 2. The real traffic intersection shown in Fig. 2 is composed by 6 links L_i ($i = 1, \ldots, 6$) with length $l_1 = 40$ m, $l_2 = l_3 = 45$ m and $l_4 = l_5 = l_6 = 60$ m, respectively. The capacities of the links are $C_1 = 16$ PCUs, $C_2 = C_3 = 9$ PCUs, $C_4 = C_6 = 24$ PCUs and $C_5 = 12$ PCUs, derived by assuming that one PCU is $l_0 = 5$ m long and taking into account the number of lanes in each link. Fig. 3 shows the CTPN modelling the intersection. Places p_i, p'_i, with $i = 1, \ldots, 9$ are a subset of places modelling cell links and places p_i, p'_i with $i = 10, 11, 12$ model the intersection crossing area composed of six cells. For the sake of simplicity, Fig. 3 shows just two places for each lane in an input link and one place for each lane in an output link. Moreover, considering the number of lanes in each link, L_1 is modelled by 16 link places, L_2 by 9 places, L_4 and L_6 by 24 places. In addition, links L_3 and L_4 are dedicated only to buses and, considering each bus of 3PCUs, links L_3 and L_5 are modelled by 3 and 4 places, respectively.

With reference to Fig. 3, transitions $t_{0,1}$, $t_{0,2}$, $t_{0,3}$, $t_{0,4}$ and $t_{0,5}$ are input transitions and transitions $t_{1,0}$, $t_{2,0}$, $t_{3,0}$ and $t_{4,0}$ are output transitions. Transitions t_j with $j = 12, \ldots, 21$

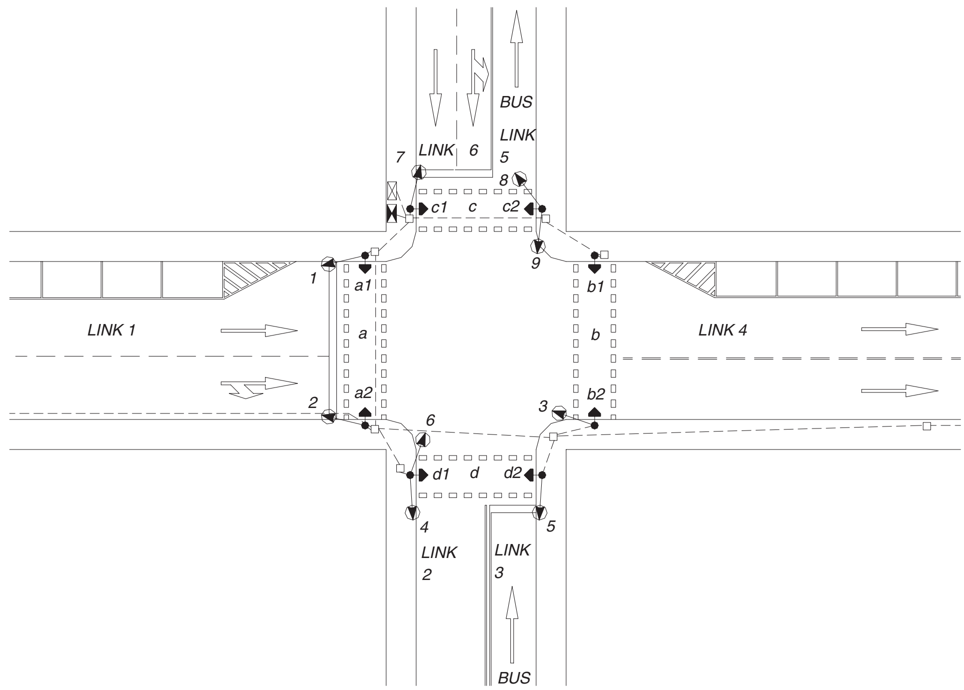

Fig. 2. Layout of a real traffic intersection located in the urban area of Bari, Italy.

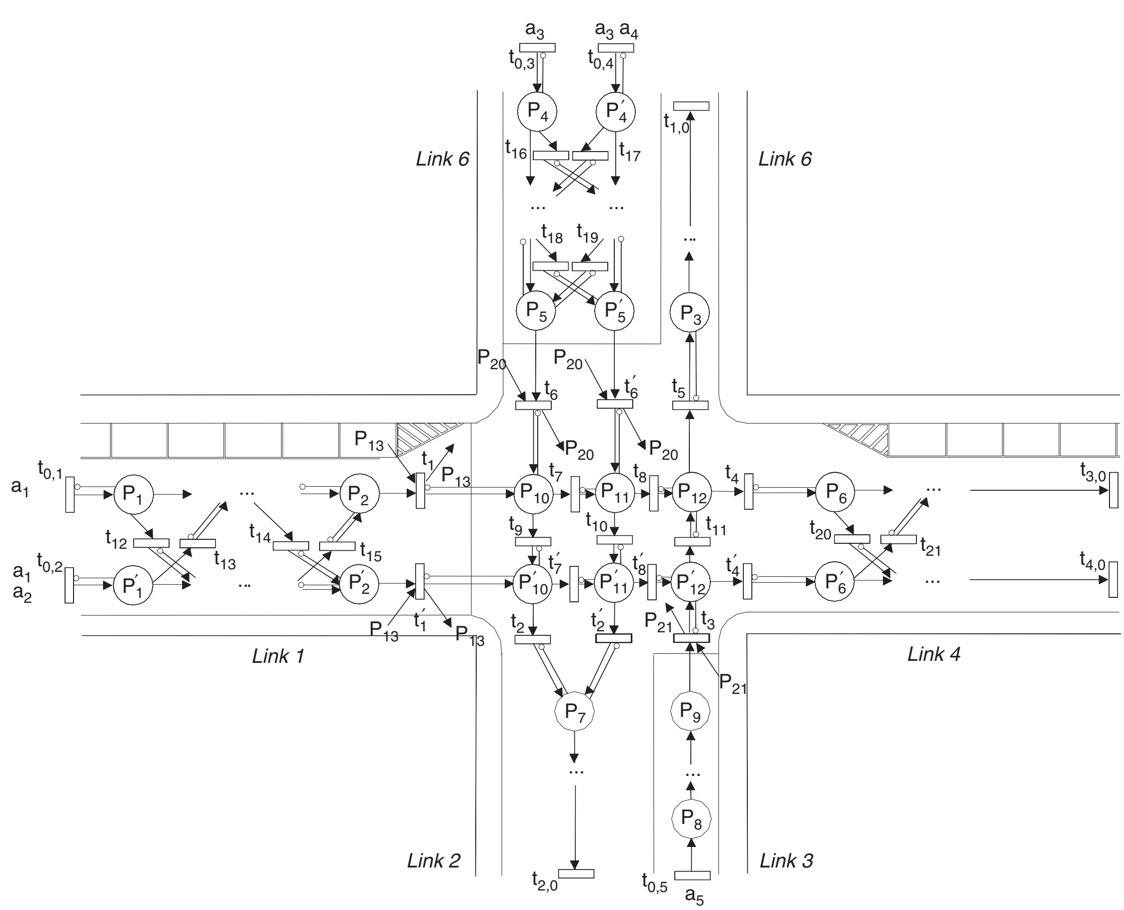

Fig. 3. The CTPN modelling the intersection of Example 2.

model lane changing in the two-lanes intersection links (i.e., links L_1, L_4 and L_6), transitions t_1, t'_1, t_3, t_6, t'_6 model vehicles entering a crossing section and are controlled by the traffic light TPN. Vehicles travelling in the intersection are tokens of 5 colours. More precisely, colours a_1 and a_2 are associated with vehicles following the routings (L_1, L_4) and (L_1, L_2), respectively. Moreover, colours a_3, a_4 refer to vehicles following the routings (L_6, L_2) and (L_6, L_4), respectively. Finally, colour a_5 represents vehicles following the routing (L_3, L_5). Referring to the definitions of matrices $\boldsymbol{Post}(p_i,t_j)$ and $\boldsymbol{Pre}(p_i,t_j)$, as an example the following matrix entries are shown, obviously the other ones are determined similarly:

$Pre(p'_{10},t'_7)(1,1) = 1$, $Pre(p'_{10},t'_7)(h,k) = 0$ for $h,k = 2,\ldots,5$,
$Post(p'_{11},t'_7)(1,1) = 1$, $Post(p'_{11},t'_2)(h,k) = 0$ for $h,k = 2,\ldots,5$,
$Post(p_7,t_2)(2,2) = 1$, $Post(p_7,t_2)(3,3) = 1$,
$Post(p_7,t_2)(h,k) = 0$ for $h,k = 1,4,5$,
$Pre(p'_{10},t_2)(2,2) = 1$, $Pre(p'_{10},t_2)(3,3) = 1$,
$Pre(p'_{10},t_2)(h,k) = 0$ for $h,k = 1,4,5$,
$Pre(p_2,t_{13})(1,1) = 1$, $Pre(p_2,t_{13})(h,k) = 0$ for $h,k = 2,\ldots,5$,
$Post(p'_2,t_{13})(1,1) = 1$ $Post(p'_2,t_{13})(h,k) = 0$ for $h,k = 2,\ldots,5$.

3.3. The timed Petri net modelling the traffic lights

The traffic lights of a generic TN are defined according to a signal timing plan, including green, red and amber signals that in most American and European cities correspond respectively to clear way, stop and caution signal after green and before red. In addition, the lost (or intergreen) times are taken into account, i.e., short duration phases in which all traffic lights in one intersection are red, in order to let vehicles, previously allowed to occupy the crossing area and late due to congestion, clear the junction.

Among the main decision variables in a timing plan, cycle time and green splits are here recalled (Diakaki et al., 2002). Cycle time is defined as the duration of time from the centre of the red phase to the centre of the next red phase. Green split for a signal in a given direction of movement is defined as the fraction of cycle time when the light is green in that direction. Moreover, a phase is the time interval during which a given combination of traffic signals in the area is unchanged. Finally, during each phase different streams may be allowed to proceed, where a stream of vehicles in a junction is a portion of traffic formed by all vehicles that cross the intersection from the same departure link and are directed to the same arrival link in the considered phase (Diakaki et al., 2002).

The traffic light controller pertaining to a common signal timing plan is modelled by the definition of the $TPN = (P, T, \boldsymbol{Pre}, \boldsymbol{Post}, \boldsymbol{FT})$ (Di Febbraro et al., 2002, Giua, 1991). In particular, places from P represent phases and transitions from T model the succession of red, yellow and green phases. In order to clearly explain the method to model a generic signal timing plan, the TPN representing the traffic light of the signalized intersection of Example 2 is described. For this intersection, the streams allowed to proceed during the phases of the signal timing plan are depicted in Fig. 4 and labelled with numbers from 1 to 5. Moreover, amber and intergreen times are taken into account, so that the considered fixed timing plan comprises 8 phases, including 3 amber phases (i.e., phases 2, 4 and 7) and 2 lost times (i.e., phases 5 and 8). Fig. 5 shows the phases of the signal timing plan for Example 2 and the duration of the phases in seconds is indicated by τ_i with $i = 1,\ldots,8$, while the cycle time is CT. For the sake of simplicity, in Fig. 4 phases 5 and 8 are omitted, being lost times (see Fig. 5): indeed, no stream is allowed to proceed during such phases. Note that each stream is modelled by a token colour of the CTPN in Fig. 3, ruled by the described signal timing plan.

The signal timing plan, described by Figs. 4 and 5, is realized by three traffic lights, each one with three phases modelled by three places representing the red, yellow and green phases, respectively. Hence, to model the phases of the cycle, nine places are necessary. In particular, a neutral colour token in place p_{13} enables transitions t_1 and t'_1 to rule the vehicles in L_1. Moreover, places p_{14}, p_{15} and p_{16} describe the state of green, yellow and red, respectively. Hence, in order for the TPN in Fig. 6 to describe the signal timing plan in Fig. 5, it is selected $FT_{22} = \tau_6$ (the duration of the green signal controlling L_1), $FT_{23} = \tau_7$ (the length of the amber phase ruling L_1) and $FT_{24} = \tau_8$ (the duration of

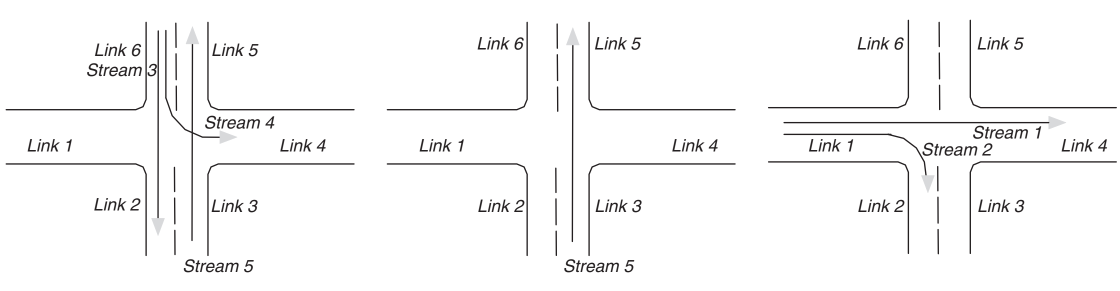

Fig. 4. The streams of the signal timing plan controlling the intersection in Fig. 2.

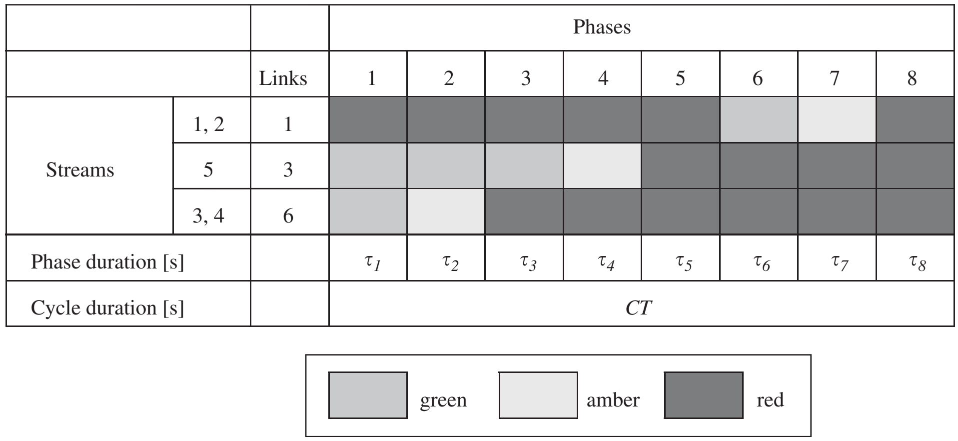

Fig. 5. The signal timing plan of the intersection in Fig. 2.

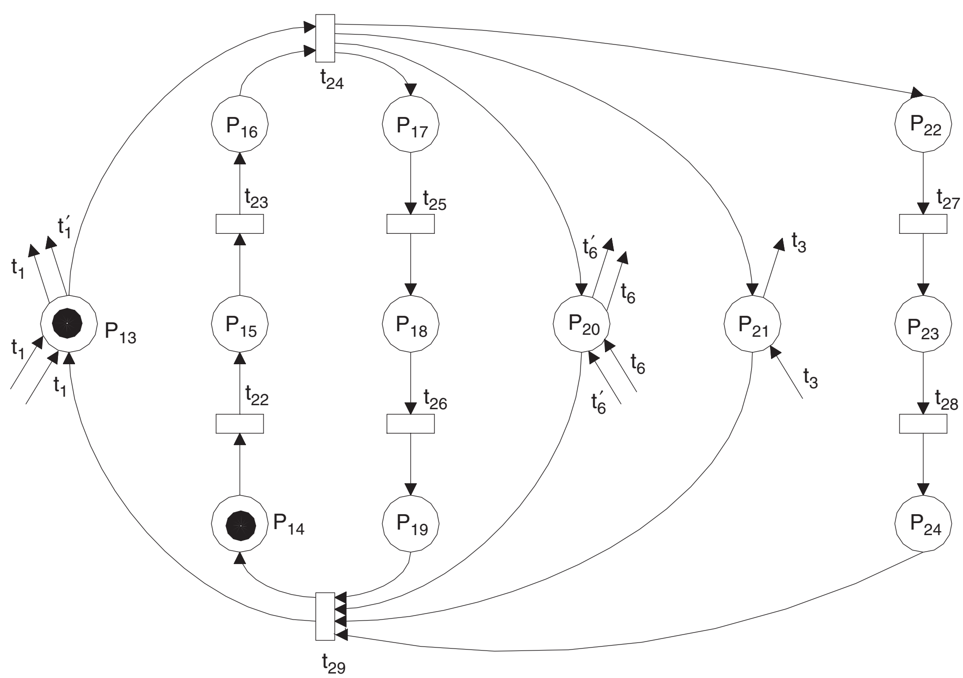

Fig. 6. The TPN modelling the traffic light of the intersection in Fig. 2.

the subsequent lost time). In a similar way, the traffic light ruling L_6 and modelled by places p_{17}, p_{18}, p_{19} and p_{20} is described with $FT_{25} = \tau_1$ (the duration of the green signal for L_6) and $FT_{26} = \tau_2$ (the duration of the amber phase for L_6). Finally, the traffic light controlling L_3 is modelled by places p_{21}, p_{22}, p_{23} and p_{24} with $FT_{27} = \tau_1 + \tau_2 + \tau_3$ (the duration of the green signal for L_3), $FT_{28} = \tau_4$ (the duration of the amber phase for L_3). In addition, transition t_{29} models the duration of the intergreen signal subsequent to the allowed movement of the stream originating by L_3 and hence it holds $FT_{29} = \tau_5$. The start of the signal timing plan cycle is with a token in p_{14} and in p_{13} (green for L_1). When a token is in p_{16}, the red phase begins for L_1. Hence, after the lost time FT_{24} transition t_{24} fires and the green phases start for links L_6 and L_3 (tokens in p_{17}, p_{20}, p_{21} and p_{22}). If there is a token in p_{20}, p_{21}, p_{24} and p_{19}, then t_{29} can fire when it is ready (that is after the lost time FT_{29}) and the green phase starts for L_1, so that the cycle begins again.

3.4. The model parameters

This section enlightens that the peculiarity of the model is the low number of parameters necessary to describe the structure and the behaviour of the TN. More precisely, the physical characteristics of the TN are collected to build the CTPN structure and the traffic signal timing plan is used to realize the traffic light TPN. Moreover, the parameters necessary to describe the TN behaviour are the values of FT_j assigned to each $t_j \in T_I \cup T_O \cup T_F \cup T_C \cup T_L$ and the colours assigned to each token arriving to the TN. In particular, the value of the interarrival times FT_j assigned to each $t_j \in T_I$ are measured by the detectors positioned at each input link. In addition, the average values of FT_j assigned to each $t_j \in T_O \cup T_F \cup T_C \cup T_L$ can be easily evaluated. Finally, the rate of vehicles travelling from two subsequent links and necessary to assign the colour at each token, can be obtained off-line by a suited algorithm (e.g. see Willumsen, 1991).

4. The case study

4.1. The system specification

In this section the CTPN model of the signalized intersection described in Example 2 and shown in Fig. 2 is validated on the basis of real traffic data. To this aim, some simulation results referred to the TN behaviour of the system modelled by the CTPN of Fig. 3 and the TPN of Fig. 6 are presented. Moreover, to determine the deterministic firing times of the input transitions $t_{0,j} \in T_I$, the time instants of the vehicle arrivals are obtained for two scenarios from the direct measures at the input links registered at the considered time of the day, during the red and green-amber signals for 20 cycles. Hence, the interarrival time of the vehicles during green and amber phases and during the red phase of each cycle are approximated as shown in Table 1 for scenario 1 (weekday around 3 p.m.) and in Table 2 for scenario 2 (weekday around 6 p.m.). Being the cycle time $CT = 70$ s, each registration has been performed for 1400 s.

Tables 1 and 2 show that the second scenario features a more significant congestion than the first case with regard to link L_6 (with increased arrivals per cycle on average), while link L_3 is always under-saturated, corresponding to the fact that such a link is dedicated to buses, that run with fixed and non-overlapping schedules. In addition, link L_1 displays similar traffic conditions under both scenarios.

On the basis of experimental evidence, $v = 40\,\mathrm{km\,h^{-1}}$ is assumed to be the vehicle average speed. Hence, the firing time of the transitions $t_j \in T_O \cup T_F \cup T_C$ is equal to $FT_j = l_0/v = 0.45$ s in each scenario. However, if transitions $t_j \in T_O \cup T_F$ belong to links L_3 and L_5, then it is assumed $FT_j = 1.35$ s to model the bus movements. For the sake of simplicity, lane changing is not considered in the model of the real case study. Indeed, such an assumption is not restrictive and it is supported by experimental observations, showing that lane changing occurrences are rare in the case study.

In the considered intersection, traffic is currently ruled by a fixed time control strategy with a given signal timing plan. The fixed durations of the traffic light phases applied to the real intersection independently from the traffic scenarios are listed in Table 3 (second column). In particular, all amber phases last 4 s and lost times are 2 s long. Moreover, as described in Section 3.3, the firing times FT_i with $i = 22, \ldots, 29$ referring to the corresponding transitions of the TPN modelling the traffic light are obtained from the phases of the signal timing plan.

4.2. The simulation results

The CTPN model is implemented and simulated in the Matlab environment. Indeed, the simplicity and modularity of the model suggest to use an efficient software such as Matlab, that allows to model systems with a large number of places and transitions. Moreover, such a matrix-based software appears particularly appropriate for simulating

Table 1
Deterministic firing times of the input transitions for Scenario 1

Cycle	Green and amber phases $FT_{0,1} = FT_{0,2}$	Red phase $FT_{0,1} = FT_{0,2}$	Green and amber phases $FT_{0,3} = FT_{0,4}$	Red phase $FT_{0,3} = FT_{0,4}$	Green and amber phases $FT_{0,5}$	Red phase $FT_{0,5}$
1	8.80	88.00	20.00	15.00	20.67	23.33
2	8.80	17.60	13.33	30.00	20.67	70.00
3	7.33	14.67	13.33	30.00	15.50	70.00
4	6.29	17.60	20.00	30.00	20.67	35.00
5	11.00	17.60	10.00	30.00	12.40	35.00
6	6.29	17.60	40.00	30.00	12.40	35.00
7	3.67	12.57	40.00	30.00	15.50	35.00
8	5.50	22.00	20.00	30.00	15.50	35.00
9	3.67	14.67	40.00	30.00	12.40	14.00
10	4.40	44.00	40.00	30.00	12.40	35.00
11	4.89	12.57	20.00	30.00	8.86	17.50
12	8.80	29.33	40.00	30.00	15.50	23.33
13	3.14	29.33	40.00	30.00	15.50	35.00
14	4.40	11.00	13.33	30.00	12.40	23.33
15	7.33	12.57	10.00	30.00	31.00	17.50
16	4.40	14.67	40.00	30.00	7.75	35.00
17	6.29	17.60	40.00	30.00	15.50	35.00
18	7.33	9.78	40.00	30.00	20.67	23.33
19	8.80	8.80	13.33	30.00	31.00	23.33
20	8.80	1.00	40.00	30.00	20.67	23.33

Table 2
Deterministic firing times of the input transitions for Scenario 2

Cycle	Green and amber phases $FT_{0,1} = FT_{0,2}$	Red phase $FT_{0,1} = FT_{0,2}$	Green and amber phases $FT_{0,3} = FT_{0,4}$	Red phase $FT_{0,3} = FT_{0,4}$	Green and amber phases $FT_{0,5}$	Red phase $FT_{0,5}$
1	14.67	11.00	20.00	30.00	15.50	23.33
2	8.80	17.60	40.00	30.00	10.33	14.00
3	8.80	8.00	20.00	15.00	12.40	23.33
4	7.33	17.60	20.00	30.00	7.75	17.50
5	6.29	9.78	20.00	30.00	12.40	23.33
6	44.00	14.67	40.00	30.00	12.40	8.75
7	11.00	11.00	40.00	30.00	31.00	10.00
8	14.67	12.57	40.00	30.00	31.00	17.50
9	7.33	11.00	40.00	30.00	8.86	17.50
10	14.67	14.67	40.00	30.00	10.33	14.00
11	11.00	11.00	40.00	30.00	10.33	14.00
12	8.80	14.67	20.00	30.00	10.33	70.00
13	8.80	29.33	20.00	30.00	6.89	14.00
14	5.50	11.00	13.33	30.00	31.00	70.00
15	14.67	17.60	40.00	30.00	7.75	14.00
16	5.50	17.60	13.33	30.00	31.00	70.00
17	4.89	22.00	20.00	30.00	20.67	14.00
18	6.29	14.67	20.00	30.00	6.89	14.00
19	14.67	17.60	20.00	30.00	8.86	23.33
20	6.29	1.33	40.00	30.00	15.50	35.00

Table 3
The signal timing plan phases

τ_j [s]	Fixed	Scenario 1		Scenario 2	
		Dynamic ($K = 20$)	Dynamic ($K = 5$)	Dynamic ($K = 20$)	Dynamic ($K = 5$)
τ_1	31	18	15	21	19
τ_2	4	4	4	4	4
τ_3	1	1	2.5	1	1
τ_4	4	4	4	4	4
τ_5	2	2	2	2	2
τ_6	22	35	36.5	32	34
τ_7	4	4	4	4	4
τ_8	2	2	2	2	2

the dynamics of CTPNs and TPNs based on the matrix formulation of the marking update.

Obviously, the CTPN simulation could be carried out in a generic commercial or freeware tool for discrete events systems. Indeed, the same results determined with Matlab are obtained by using ARENA, that is ideally suited when dealing with large-scale and modular systems, like the one of interest here. In particular, places are represented with unit capacity resources, transitions with delay travel times, tokens with entities and colours with attributes. Moreover, the ARENA software environment makes it possible to associate to an entity, modelling a moving vehicle, other attributes, such as arrival and departure time instants, that are useful to determine the system performance.

Both the chosen software programs are able to integrate modelling and simulation of event-driven systems (e.g., the TN dynamics) with the execution of generic control algorithms (e.g., developing the traffic signal control strategy and the resulting signal timing plan), while keeping track of time by way of a software clock. Therefore, such environments have been preferred to the many available specific CTPN simulators. Indeed, the resulting simulation model is compact, simple to implement and keeps the CTPN modularity feature while reproducing its structure.

Considering the two described traffic scenarios, two simulations are performed for 20 timing plan cycles, for a total run time $T = 1400$ s. Starting from the initial state describing the CTPN in scenario 1 (scenario 2), with 4 (8) vehicles present in link L_1, 0 (0) vehicles in link L_3 and 0 (4) vehicles in link L_6, the discrete event simulation is performed in the fixed run time. The simulation model generates as outputs for each input link L_i with $i = 1, 3, 6$, the number of vehicles $n_i(k)$ with $i = 1, 3, 6$ at the beginning of the kth cycle, and the number of vehicles $w_i(k)$ with

$i = 1, 3, 6$ that enter the junction at the kth cycle and have to wait the $k+1$th cycle to cross the intersection.

Variables $n_i(k)$ for links L_i with $i = 1, 3, 6$ and $k = 1, \ldots, 20$ are, respectively, reported in Figs. 7–9 for scenarios 1 and in Figs. 10–12 for scenario 2. In particular, the figures show that the simulation results are consistent with real traffic data. The slight differences are due to accidental events that are not modelled in the proposed CTPN model, such as parking and temporary stops at the sides of the roads (that are more frequent in scenario 2 because of the increased traffic). Moreover, a comparison of Figs. 7 and 10 shows that under both scenarios link L_1 is similarly congested and under-saturated (its capacity being $C_1 = 16$ PCUs). As regards link L_3, since such a link is dedicated to buses, it is extremely under-saturated and most buses entering the junction during a cycle are able to cross it in the same cycle, as shown by Figs. 8 and 11, depicting an empty link at the beginning of most cycles. In addition, the comparison of Figs. 9 and 12 shows that link L_6 is always under-saturated but it is more congested in the second scenario, with longer queues at the beginning of the cycles in its lanes.

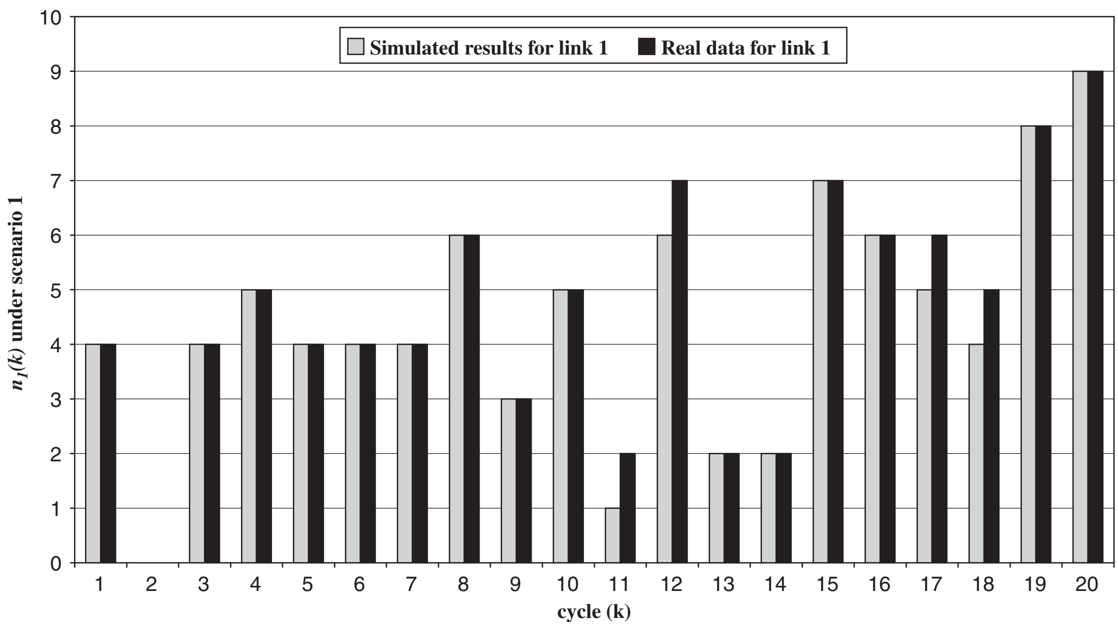

Fig. 7. Simulated and measured number of vehicles in link L_1 under scenario 1.

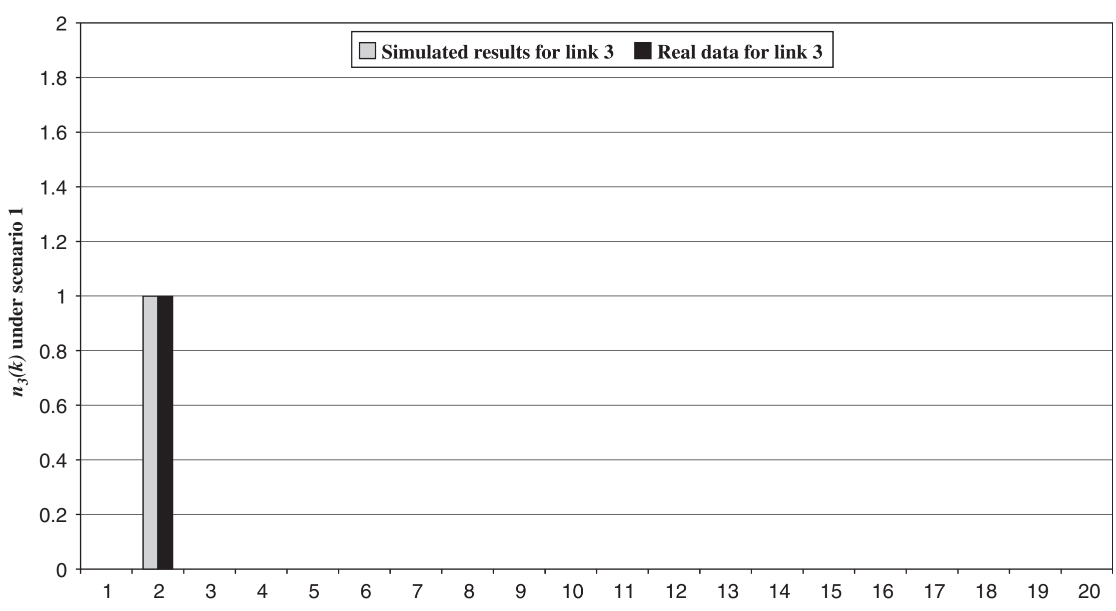

Fig. 8. Simulated and measured number of vehicles in link L_3 under scenario 1.

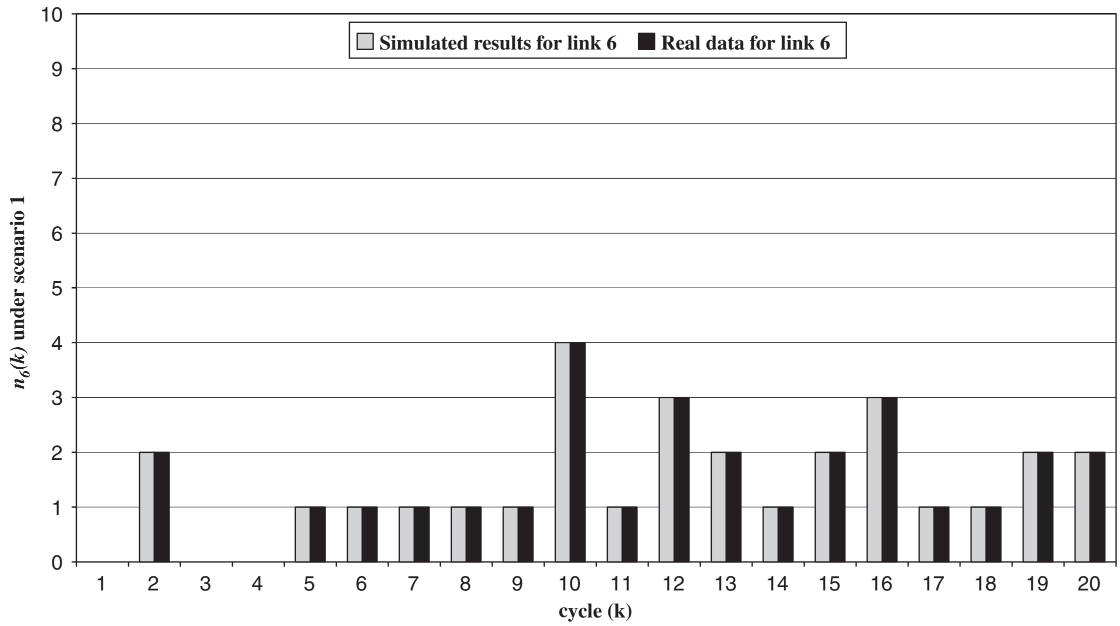

Fig. 9. Simulated and measured number of vehicles in link L_6 under scenario 1.

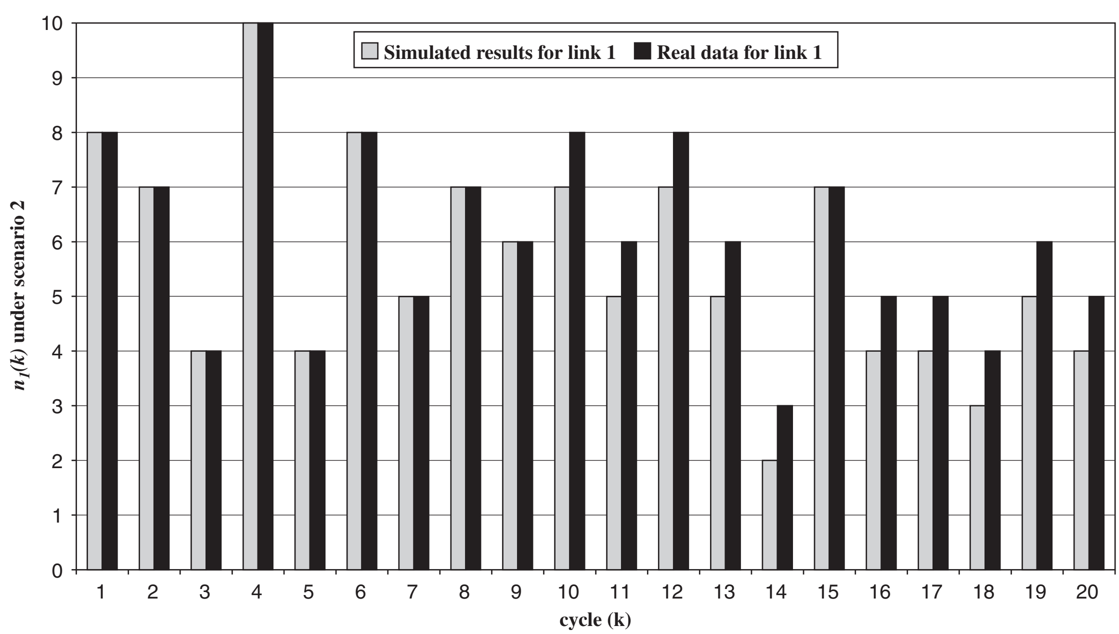

Fig. 10. Simulated and measured number of vehicles in link L_1 under scenario 2.

For the considered real scenarios 1 and 2 the case study is under-saturated and, due to the small dimension of the considered example TN, most vehicles are able to cross the intersection in the same cycle in which they enter the junction, i.e. $w_i(k) = 0$ or $w_i(k) = 1$ for $i = 1, 3, 6$ and $k = 1, \ldots, 20$. Hence, detailing such results is neglected for the sake of brevity.

A third simulation scenario is considered in order to show that the proposed model is able to describe the behaviour of a TN even under saturated conditions. In particular, scenario 3 considers interarrival times equal to half the values reported in Table 2, so that traffic is doubled in the intersection with respect to scenario 2. In addition, in accordance with such a congested scenario, $v = 25\,\mathrm{km\,h^{-1}}$ is selected as the vehicle average speed and the firing times of the transitions are accordingly modified. Results for links L_1 and L_6 are reported in Figs. 13 and 14, showing that the queues in the links at the beginning of each cycle are doubled, with the exception of link L_1 at the beginning of cycles 4 and 7, when the link reaches its capacity $C_1 = 16$ PCU (see Fig. 13). The values of $n_3(k)$ for $k = 1, \ldots, 20$ are omitted for the sake of brevity, since link

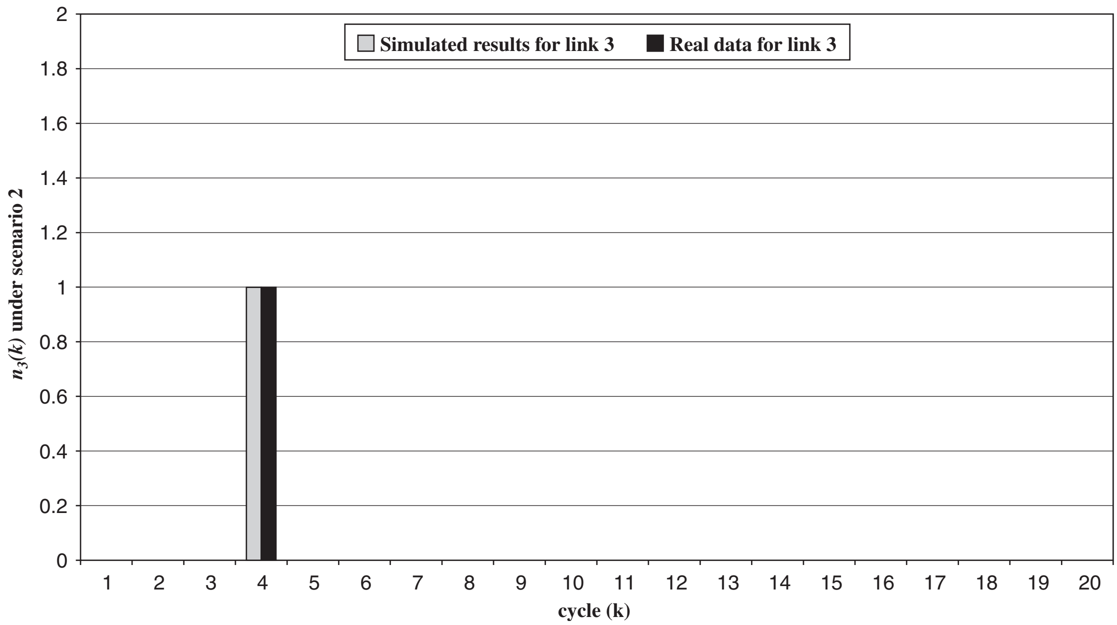

Fig. 11. Simulated and measured number of vehicles in link L_3 under scenario 2.

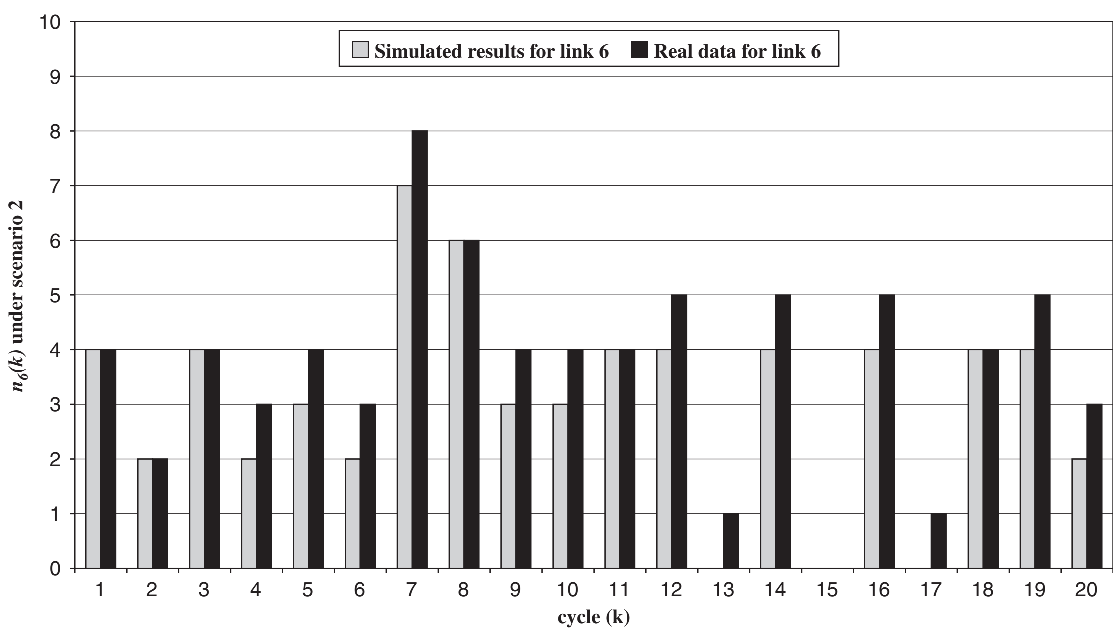

Fig. 12. Simulated and measured number of vehicles in link L_6 under scenario 2.

L_3 remains under-saturated also in scenario 3. Moreover, Figs. 15 and 16, respectively, report the values of $w_1(k)$ and $w_6(k)$ for $k = 1, \ldots, 20$. Since scenario 3 corresponds to saturated conditions, the results show that some vehicles cannot cross the junction in the same cycle in which they enter the intersection. Such a circumstance is particularly apparent for link L_6, since during its green phase vehicles of stream 4 turning from L_6 to L_4 have to give right of way to the simultaneous stream 5 of vehicles going from L_3 to L_5 (see Fig. 4).

In order to show an application of the proposed model, a different signal timing plan, obtained by the optimization method proposed in Barisone, Giglio, Minciardi, and Poggi, (2002), Dotoli, Fanti, and Meloni (2004), is realized for the case study and scenarios 1 and 2. More precisely, while keeping the cycle time constant, the green phases of the traffic light are determined dynamically, i.e., on the basis of traffic data, in order to minimize congestion. In particular, for each input link L_i $(i = 1, 3, 6)$, the mean number of vehicles over K

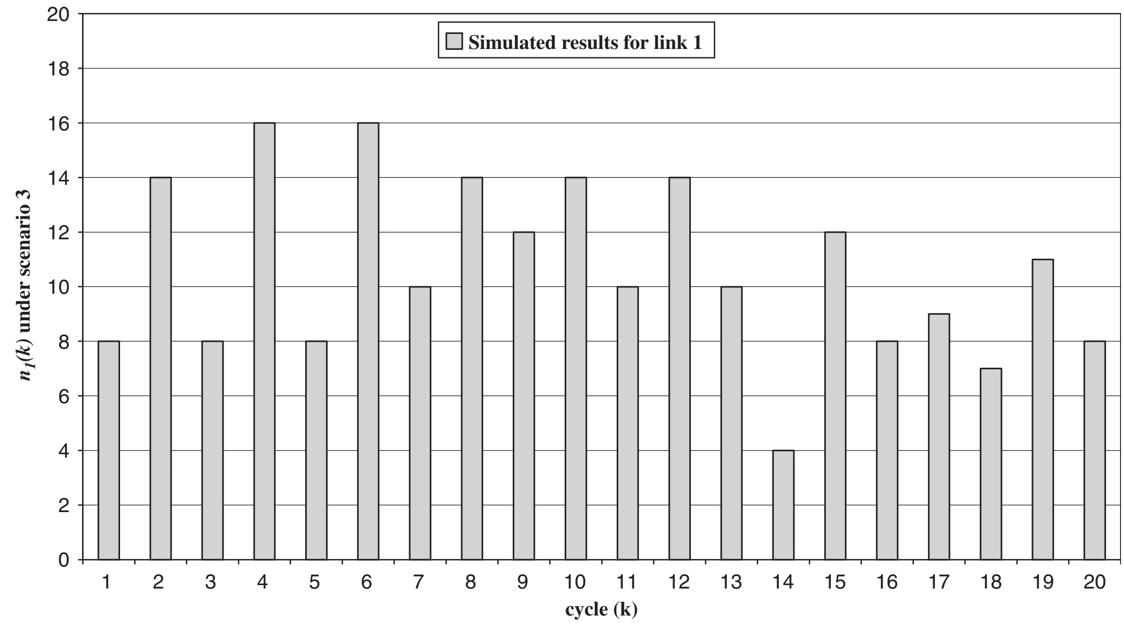

Fig. 13. Simulated number of vehicles in link L_1 under scenario 3.

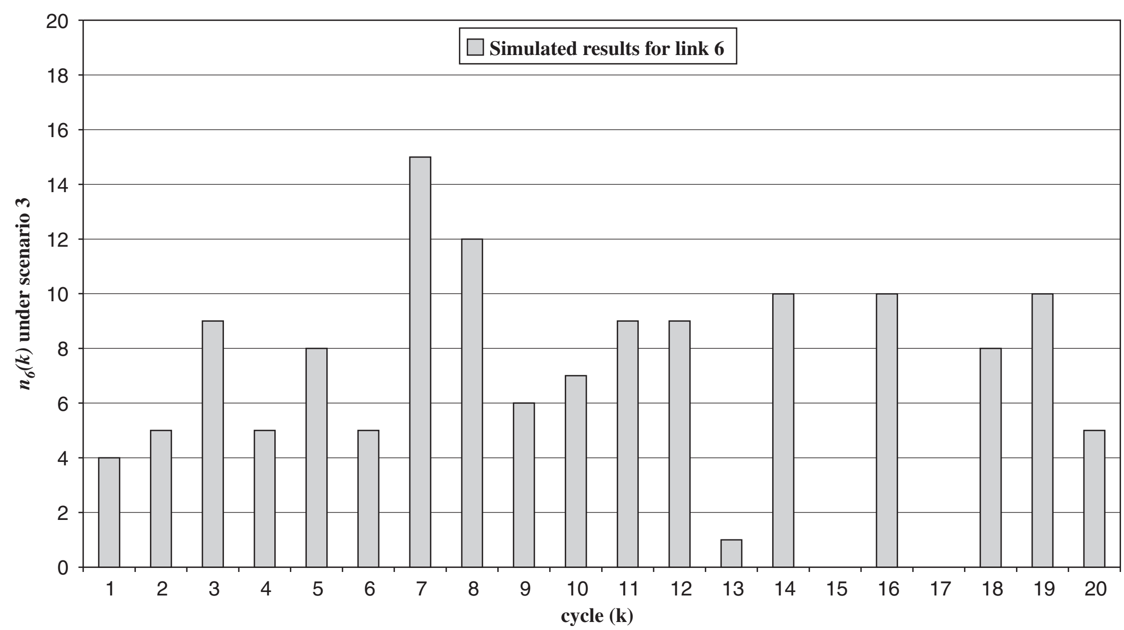

Fig. 14. Simulated number of vehicles in link L_6 under scenario 3.

cycles in PCU is defined as follows:

$$OF_i(K) = \frac{1}{K}\left[\sum_{k=1}^{K} n_i(k)\right] \quad \text{for } i = 1, 3, 6, \tag{2}$$

where $n_i(k)$ denotes the number of vehicles at the beginning of the kth cycle in the input link L_i with $i = 1, 3, 6$. The control objective is to minimize the number of vehicles in the TN in the optimization horizon of K cycles and the objective function is defined as follows:

$$OF(K) = \sum_{i=\{1,3,6\}} OF_i(K). \tag{3}$$

Table 3 reports the values of the timing plan phases obtained by the optimization method for scenarios 1 and 2 with $K = 20$ and 5, in turn. In particular, the optimized signal timing plan differs from the original plan in the green phases allowing flow from L_1 to L_2 and L_4, from L_3

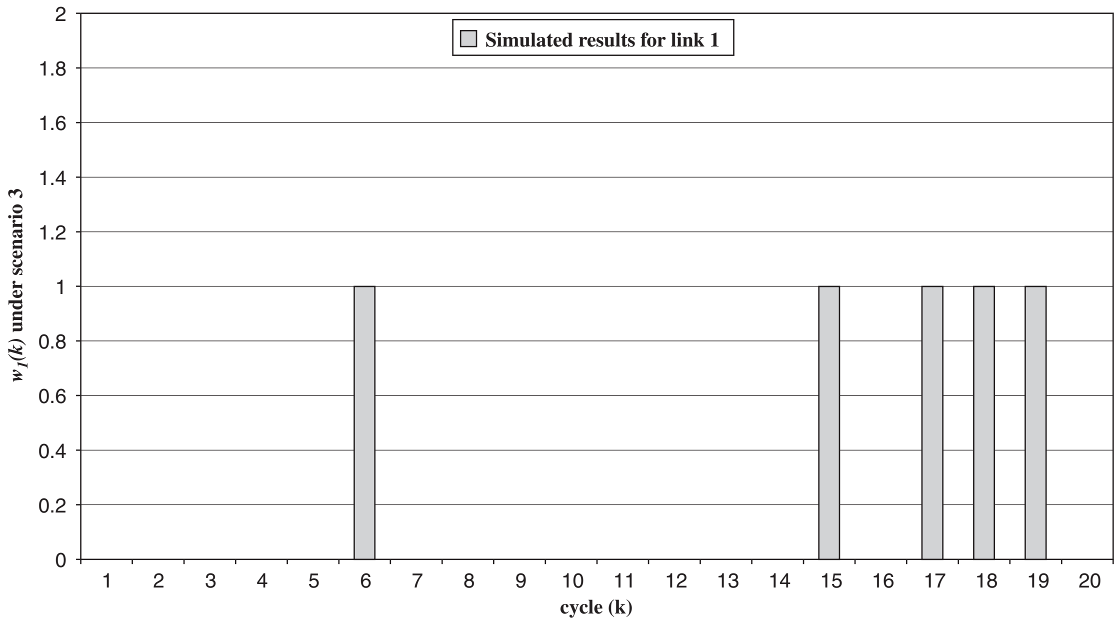

Fig. 15. Simulated number of vehicles in link L_1 that have to wait the next cycle to cross the intersection under scenario 3.

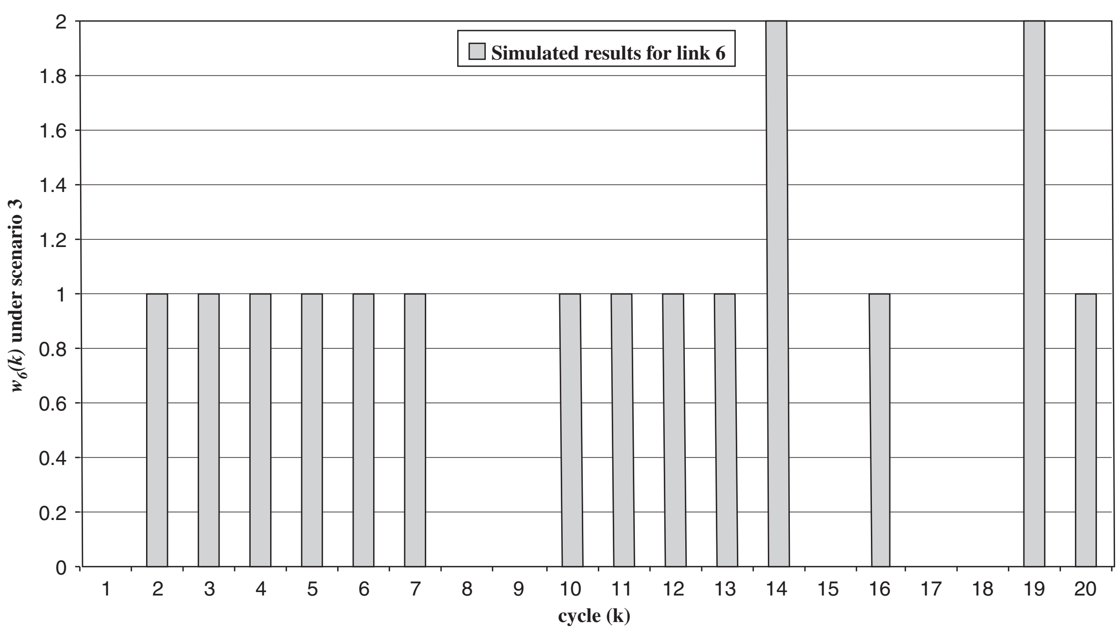

Fig. 16. Simulated number of vehicles in link L_6 that have to wait the next cycle to cross the intersection under scenario 3.

to L_4, from L_6 to L_2 and L_4, respectively. Figs. 17 and 19 show that the simulation gives a lower number of vehicles at the beginning of each cycle in input link L_1 under both scenarios if the intersection is controlled with the dynamically optimized timing plans obtained for $K = 20$ or for $K = 5$ cycles. Analogously, Figs. 18 and 20 compare the results before and after optimization in link L_6. Again, simulation results for link L_3 are omitted, since such a link, that displays short queues, is virtually unaffected by the change in the signal timing plan. Although link L_6 becomes slightly more congested with the optimized timing plan, the net result is that the performance index $OF(20)$ is reduced after optimization in both scenarios (see Table 4). Hence, the results show the benefits in applying an actuated traffic control strategy that is able to modify the signal timing plan with traffic congestion. On the other hand, optimizing the signal timing plan over $K = 5$ or $K = 20$ cycles leads to similar results in the considered cases (see Figs. 17–20), since the traffic congestion modelled by the input transitions of the CTPN is assumed identical over all the cycles.

Table 4
The performance indices in the simulations

Performance index [PCU]	Scenario 1			Scenario 2		
	Fixed	Dynamic ($K = 20$)	Dynamic ($K = 5$)	Fixed	Dynamic ($K = 20$)	Dynamic ($K = 5$)
$OF(20)$	6.05	5.45	5.50	8.85	8.60	8.60
$OF_1(20)$	4.45	3.35	3.05	5.60	4.65	4.30
$OF_3(20)$	0.15	0.15	0.15	0.15	0.15	0.15
$OF_6(20)$	1.45	1.95	2.30	3.10	3.80	4.15

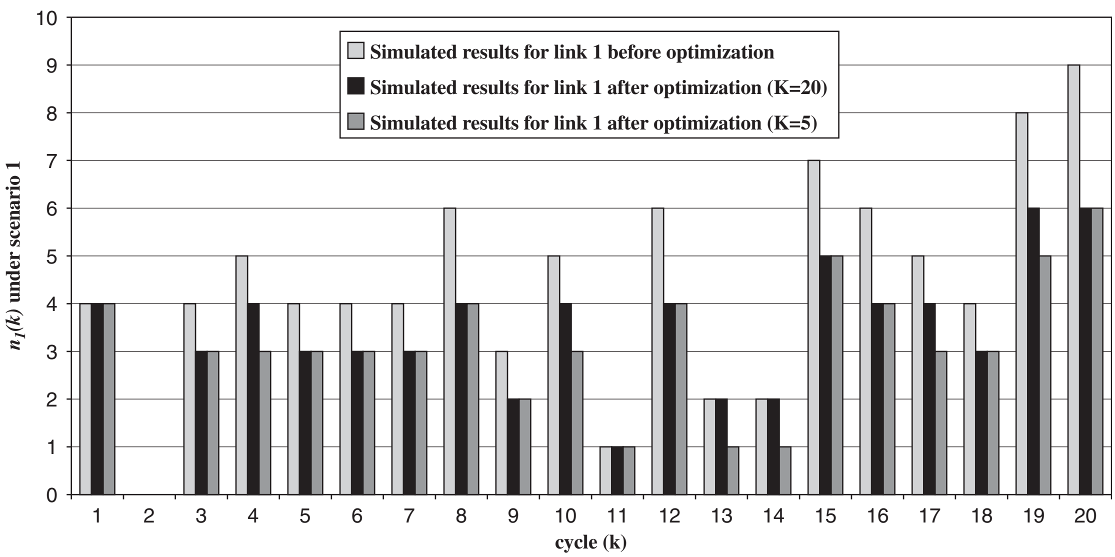

Fig. 17. Simulated number of vehicles in link L_1 under scenario 1 before and after optimization.

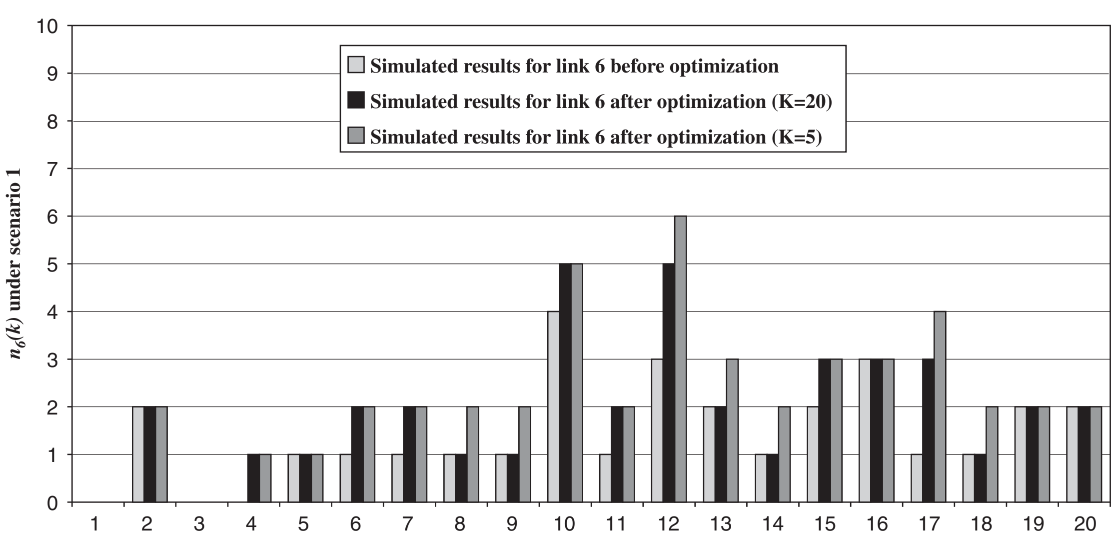

Fig. 18. Simulated number of vehicles in link L_6 under scenario 1 before and after optimization.

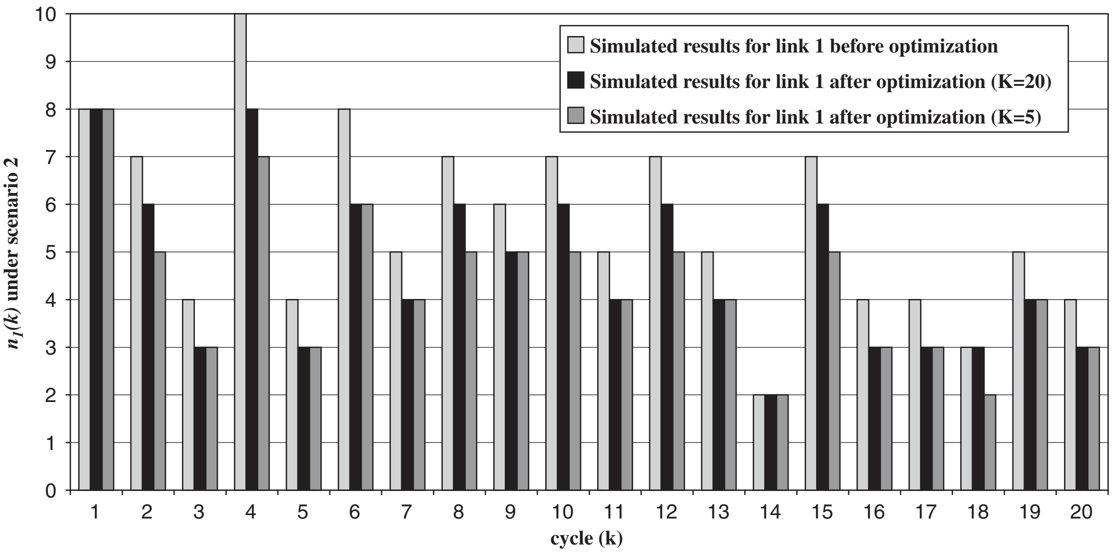

Fig. 19. Simulated number of vehicles in link L_1 under scenario 2 before and after optimizsation.

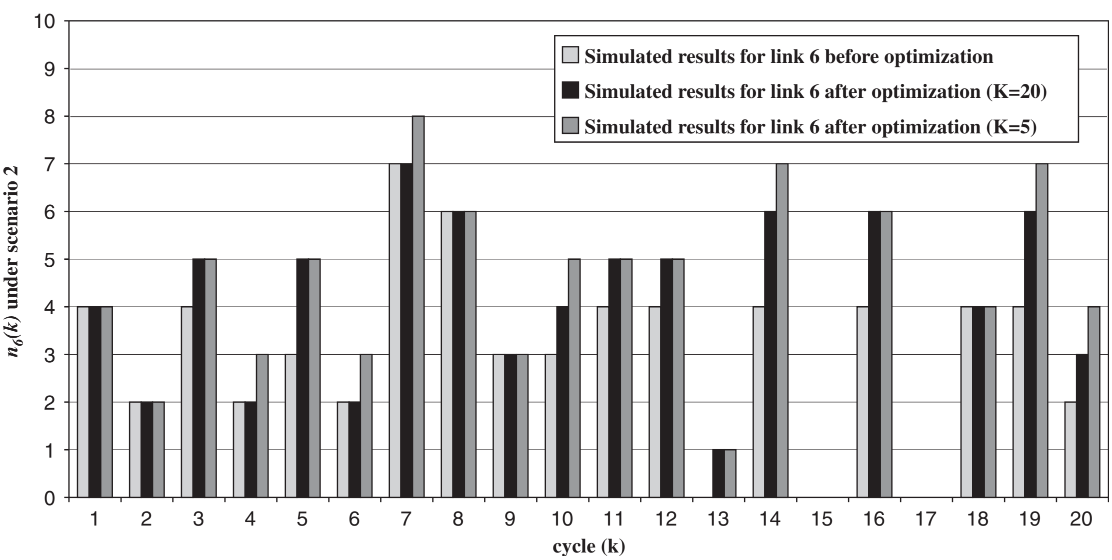

Fig. 20. Simulated number of vehicles in link L_6 under scenario 2 before and after optimization.

Finally, note that the change in the traffic control strategy simply results in modifying the firing times of the transitions in the TPN modelling the traffic light, while the CTPN model of the TN is unaffected.

5. Conclusions

This paper introduces a modelling technique to describe the behaviour of urban TN systems. CTPNs model links, intersections and vehicles of the urban area and timed PNs synthesize the traffic lights. The obtained model is able to provide a sufficiently accurate and valid representation of the TN system using data that are collected by detectors, positioned at the input links of the traffic area. Moreover, the model can be easily translated in a simulation software not costly to develop.

In order to show the model efficacy, the paper presents a case study describing a real intersection located in the city of Bari, Italy. The flexibility of the model is enlightened by the description of a link where buses only are admitted. Moreover, simulation results give a confirmation of the model capability to correctly predict traffic performance measures and to test different signal timing plans.

The proposed modelling framework is employed to test a signal timing plan optimization method proposed in the related literature. Further research may study in more depth the variables involved in real time traffic responsive control strategies by way of the presented modelling technique. In addition, the introduction of more complex modelling features such as the presence of pedestrians, motorcycles and accidental events will be the subject of future research.

Acknowledgements

The authors wish to thank Mr. Giorgio Iacobellis for his support in the collection of the experimental and simulation data.

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