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Estimation of Dynamic Origin Destination Matrix: A Genetic Algorithm Approach

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***Abstract*—Dynamic origin-destination (O-D) matrix estimation is one of the key components in the deployment of microscopic traffic simulation based real-time traffic predictions and estimations. Various theoretical methods have been proposed and tested via relatively small-scale networks. Very few practical studies have attempted to evaluate the performance of dynamic O-D matrix estimation methods for large-scale networks. This is because practical applications have not yet adopted dynamic O-D matrix estimation method, in part, due to the complexity and time requirements of advanced methods. This paper investigates the application of dynamic O-D matrix estimation methods for a large-scale network using a genetic algorithm (GA). The performance of GA-based method was compared with that of the QUEENSOD method using a microscopic traffic simulation program, PARAMICS. The evaluation results indicate that the GA-based method outperforms the QUEENSOD method.**

***Index Terms*—Microscopic Traffic Simulation, Dynamic Origin-Destination (O-D) Matrix Estimation, and ITS.**

I. INTRODUCTION

An estimation of accurate dynamic origin-destination (O-D) matrix is a crucial step for microscopic simulation-based evaluation studies. Microscopic traffic simulation models become practical tools for evaluating traffic operations, traffic management and Intelligent Transport Systems (ITS) application studies as they provide an inexpensive, fast, and risk-free evaluation environment [1]. The benefits of using microscopic traffic simulation models also include the capability to model ITS applications.

In general, there are two ways to obtain O-D matrix: a conventional household survey-based method and an estimation method based on link traffic counts. Since the former tends to be costly and labor intensive, the latter is of great interest as more traffic counts are becoming readily available through transportation management system deployments [2]. Over past several decades, researchers have proposed and tested a variety of techniques to estimate O-D matrices using link traffic counts.

Cascetta *et al*. [3] applied two generalized least squares (GLS) estimators and tested the performance of their methods on the Brescia-Verona-Vicenza-Padua motorway in Italy. They found that the accuracy of the O-D matrix obtained from their approach depends heavily on the number links with observed traffic counts. Hellianga and Van Aerde [4] compared a least square error (LSE) model and a least relative error (LRE) model at a 35-km section of Highway 401 in Toronto, Canada.

Van Aerde *et al.* [5] introduced a QUEENSOD method that generates dynamic synthetic O-D matrix. The QUEENSOD method was originally designed for practical needs as opposed to mathematical needs. The practical applicability of the QUEENSOD method was demonstrated at a 35-km section of Highway 401 in Toronto, Canada [5], Scottdale/Rural Rd. and Hayden Rd. network in Phoenix, Arizona [6], and Salt Lake metropolitan region [7].

Okutani [8] presented a Kalman filtering-based method that estimates unobserved link traffic counts from observed link traffic counts. Ashok and Ben-Akiva [9], and Ashok [10] formulated using Kalman filtering method for real-time dynamic O-D matrix estimation and prediction. To overcome the limitation of Okutani's [8] autoregressive specification for link traffic counts, they introduced the notion of deviations of O-D demands from historical estimations. The method was evaluated using various actual traffic data from Massachusetts Turnpike, Massachusetts, a stretch of I-880 near Hayward, California and a freeway encircling the city of Amsterdam, Netherlands [10]. The advantage of Kalman filtering-based method is in its ability of accommodation for on-line prediction of a dynamic O-D matrix, however it appears to be computationally costly for handling large-scale O-D matrix estimation problems [11].

A study on O-D matrix estimation from link traffic counts has been successfully conducted with GA. Kim *et al*. [12] introduced GA and compared it with a bi-level programming method to estimate a static O-D matrix from link traffic counts on a simple network.

Recent research in dynamic O-D matrix estimation from link traffic counts has provided promising results on simple networks, long freeway sections and complex corridors [4-6], [8-10]. However, only few researchers have applied the dynamic O-D matrix estimation techniques to sizable large-scale networks [7]. The large-scale network in this paper refers to a network with more than 1,000 links as defined in

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Rakha *et al*. [7]. This paper focuses on the estimation of a dynamic O-D matrix from observed link traffic counts for a large-scale network, by using a GA-based method with an assign matrix obtained from a microscopic traffic simulation model. The objectives of this paper are (i) to develop a genetic algorithm (GA)-based dynamic O-D matrix estimation method applicable for a large-scale network and (ii) to evaluate its performance with practically well adopted the QUEENSOD method using a microscopic simulation model, PARAMICS.

II. Design of Dynamic O-D Matrix Estimation Methods

A. Model Formulation

O-D matrix estimation from available traffic information can be static or dynamic depending on the consideration of temporal variations [10]. In particular, dynamic estimations generally calculate O-D matrices based on the relationship between O-D distribution patterns [i.e., T_{ij}^{dt} in equation (1)] and resulting traffic counts [i.e., v_a^t in equation (1)] [2].

In other words, dynamic O-D matrix estimation searches the trips from zone *i* to zone *j* during time interval *dt,* T_{ij}^{dt} so as to match an estimated link traffic counts to the observed link traffic counts using spatial and temporal route choice of drivers, $p_{ij,dt}^{a,t}$. The route choice, $p_{ij,dt}^{a,t}$, is used to define the proportion of trips leaving from zone *i* to zone *j* during time interval *dt* and traveling through link *a* during time interval *t*. Thus, the observed link traffic counts, v_a^t, in a particular link *a* during time interval *t* is the summation of the temporal and spatial contributions of all trips between zones to that link. Mathematically, the relationship between traffic counts and O-D distribution patterns can be generally expressed as equation (1):

$$v_a^t = \sum_{dt} \sum_{ij} T_{ij}^{dt} p_{ij,dt}^{a,t}, \qquad 0 \le p_{ij,dt}^{a,t} \le 1 \tag{1}$$

where

v_a^t = observed traffic counts in link *a* during time interval *t,*
T_{ij}^{dt} = trips leaving zone *i* to zone *j* during time interval *dt,*
$p_{ij,dt}^{a,t}$ = proportion of trips leaving zone *i* to zone *j* during time interval *dt* and traveling through link *a* during time interval *t,*
i and j = origin and destination,
a = link identifier,
dt = time interval for departure time, and
t = time interval.

Thus, the objective of dynamic O-D matrix estimation is to find O-D matrices that minimize discrepancies between estimated and observed link traffic counts.

B. Use of Assignment Matrix

The most important input in the dynamic O-D matrix estimation is the estimate of spatial and temporal route choices of drivers. Unlike the static O-D matrix estimation, the dynamic O-D matrix estimation has to consider temporal and spatial variations of congestion. Hence, traditional assignment matrix is not adequate. In order to obtain spatial and temporal route choices, dynamic traffic assignment (DTA) models, microscopic traffic simulation models or a dynamic and multi-path assignment matrix are usually used [6], [9], [10], [13].

Dynamic and multi-path assignment matrix is commonly used to estimate link traffic counts as it requires less computational time than the other two methods in large-scale networks. The assignment matrix consists of the origin zone, destination zone, departure time, and time-varying link usage probabilities [5]. The assignment matrix considers multiple-path and temporal link usage. In this study, the assignment matrix is obtained from a microscopic traffic simulation model, PARAMICS.

C. Case Study and Link Traffic Counts

As a case study, this paper developed a microscopic traffic simulation network for the City of Hampton, Virginia, USA using a microscopic traffic simulation model, PARAMICS. The PARAMICS network consists of 50 zones including seven external zones, 3,364 links, 1,464 nodes, and 154 traffic signals. Since PARAMICS does not provide a built-in or external actuated signal control module, an application programming interface (API) for the coordinated and isolated actuated signal control logics was developed for this study. Default values of network wide calibration parameters were used. PARAMICS simulation was conducted with 60 minutes of network initialization, 60 minutes of simulation time (5 pm ~ 6 pm), time step of 0.5 seconds and dynamic feedback of 120 seconds.

Observed link traffic counts for the dynamic O-D matrix estimation were available from the City of Hampton and the Virginia Department of Transportation (VDOT) Database. These observed link traffic counts data cover about 10% of entire links on the network.

D. QUEENSOD Method

The QUEENSOD method, introduced by Van Aerde *et al*. [5], generates static and dynamic synthetic O-D matrices from observed link traffic counts. The QUEENSOD method was specifically developed to satisfy practical traffic engineering needs, as opposed to mathematical needs [5]. The QUEENSOD method initiates the first iteration using a seed O-D matrix that may be either a uniform or historical O-D matrix. The seed matrix is utilized to generate estimates of link traffic counts based on drivers' expected route choices in the form of either an assignment matrix (used in this study) or a microscopic traffic simulation model. The seed O-D matrix is systematically modified to produce a new matrix that minimizes discrepancies between observed and estimated link traffic counts.

In this study, the QUEENSOD method starts from a uniform O-D matrix, which actually consists of four 15-minute dynamic O-D matrices. Especially, the total 15-minute O-D demands produced and attracted by the seven external zones are adjusted by matching them to the observed directional link volumes (15-minute interval) after estimating a new O-D matrix in each iteration. Similar adjustment is applied to the GA-based method. In other words, the network used in the case study has seven external zones that connect the City of Hampton to other adjacent areas and each external zone is connected to the City of Hampton by a single link. Thus, observed traffic counts (15-minute inbound and outbound link traffic counts) on the link connected to each of seven external zones were used as total trips (i.e., 15-minute productions and attractions) generated by each of the external zones.

As it is not uncommon assumption that either the target (or true) O-D matrix or historical dynamic O-D matrix are unknown, this study starts with a uniform O-D matrix to generate an assignment matrix. Given that an initial assignment matrix obtained from a uniform O-D matrix using the PARAMICS simulation does not properly represent drivers' route choice behavior, the initial uniform assignment matrix is updated through a multi-step approach. Note that the first step should be considered as an initialization step. The next step is the estimation of a better assignment matrix using an O-D matrix found in Step 1. Of course, if a historical O-D matrix is available the first step can be skipped and an assignment matrix can be generated from the historical O-D matrix. The procedure of QUEENSOD method is shown below:

Step 1: Finding a better assignment matrix

(1) Prepare a uniform O-D matrix as a seed O-D matrix,
(2) Create an assignment matrix based on the uniform O-D from the PARAMICS simulation output,
(3) Run QUEENSOD method,
(4) Evaluate the estimated O-D matrix from the QUEENSOD method using the PARAMICS simulation, and
(5) Determine the final estimate of the O-D matrix.

Step 2: Determining the best estimate of O-D matrix

(1) Create a new assignment matrix from the final O-D matrix in Step 1,
(2) Run QUEENSOD method,
(3) Evaluate the estimated O-D matrix from the QUEENSOD iteration using the PARAMICS simulation, and
(4) Find the best O-D matrix estimate.

E. GA-based Method

The genetic algorithm (GA) was developed by John Holland in the early 1970s at the University of Michigan [14]. In this study, the GA-based method is designed to generate the best O-D matrix for a given total O-D demand, and to overcome some problems found in the implementation of the QUEENSOD method, which will be explained later.

Through intensive preliminary tests, a binary GA model with simple mutation (probability of 0.05), simple crossover (probability of 0.5) and the normalized geometric ranking method [15] was selected. An elitist selection method was also used to keep the best individual from generation to generation [16]. More detailed information can be found in [17].

In GA, each individual solution is systemically represented by a chromosome-like data structure. The solution representation scheme determines how the problem and its solution are structured. More efficient and natural solution representation schemes produce faster convergence and better solutions [15]. The major concern on the design of solution representation scheme for this paper is two-fold: (i) naturally structured solution representation of dynamic O-D matrix, and (ii) the utilization of observed 15-minute link volumes on the links connected to external zones as the total trips generated by each of the external zones as explained in the QUEENSOD method.

The proposed GA-based method is designed to search for an optimal O-D matrix under a given total O-D demand. Thus, the proposed solution representation uses proportional allocation approach. One solution representing a dynamic O-D matrix consists of four 15-minute interval O-D matrices. In order to estimate the 15-minute O-D matrix, the parameters in the first group of one solution divide a given total O-D demand (1-hour interval) into four total O-D demands (i.e., four 15-minute intervals). The parameters in the second group determine the total trips that are generated from each internal zone during each 15-minute interval. Note that only total trips starting from internal zones are determined in this stage. In the case of external zones, the observed 15-minute volumes of the links connected to the external zones are directly used as explained in the QUEENSOD. Based on the 15-minute total trips, parameters in the third group determine 15-minute O-D demands that start from each zone to other zones during the 15-minute interval. Fifty solutions, each determined by 9,843 parameters, are randomly generated to make up an initial population.

As with the QUEENSOD method, the first step is an initialization to obtain a realistic assignment matrix. The main O-D estimation begins from Step 2. Step 2 determines total O-D demand by evaluating a range of total O-D demands, while Step 3 determines the best O-D matrix using a new assignment matrix obtained from the O-D matrix in Step 2. The proposed GA-based dynamic O-D estimation involves the following three steps:

Step 1: Determining a better assignment matrix
Step 2: Determining an appropriate total O-D demand
Step 3: Determining the best O-D matrix

Step 1 follows the same process as that in the QUEENSOD method, except running GA instead of the QUEENSOD method. Steps 2 and 3 also follow Step 2 in the QUEENSOD method.

III. Implementation of Dynamic O-D Matrix Estimation

A. QUEENSOD Method

In Step 1, an initial assignment matrix is obtained from a PARAMICS simulation run with a uniform O-D matrix, and the same uniform O-D matrix is used as a seed O-D matrix in the QUEENSOD method. In the convergence of the QUEENSOD method, it was observed that the average sum of square error (SSE) value dramatically decreases during the first two iterations and then gradually decreases up to the 25th iterations with the increase in the total O-D demand over the iterations. Note that in the QUEENSOD method, the adjustment to the O-D demand between an O-D pair is conducted independently such that the total O-D demand is not constrained during iterations. In this case study, the QUEENSOD method produced better SSE measure with larger total O-D demand. An investigation with PARAMICS animations revealed that the estimated O-D demand from QUEENSOD was too high to be accepted. This is further discussed later.

The O-D matrix estimated in Step 1 is then validated via PARAMICS evaluation. PARAMICS evaluation simulates each O-D matrix five times with different random number seeds. Finally, the O-D matrix obtained at the 8th iteration was selected as the O-D matrix in Step 1.

In Step 2, a new assignment matrix is first developed from the final O-D matrix in Step 1 using the PARAMICS simulation. Also, the final O-D matrix is used as a seed O-D matrix in this step. The QUEENSOD method is implemented with the new assignment matrix and the seed O-D matrix. Interestingly, calculated SSE increased for the first five iterations and then decreased. However, the changes in the total O-D demand exhibited similar patterns when compared to that of Step 1. Again, it was found that the O-D matrix at the 8th iteration resulted in the best estimation.

B. GA-based Method

In Step 1, five different total O-D demands, ranging from 40,000 to 60,000 vehicles per hour with 5,000 vehicles per hour interval, are used to find a better assignment matrix. It is noted that the assignment matrix used here is the same matrix used in the Step 1 of the QUEENSOD method. The estimated O-D matrices are validated through the PARAMICS evaluations. PARAMICS evaluation simulates each O-D matrix five times with different random number seeds. In the PARAMICS evaluation, the total O-D demand of 45,000 vehicles per hour provided the best result. However, the SSE value from GA runs using the assignment matrix and the SSE value from PARAMICS evaluation showed, as expected, significant discrepancy. This is because the assignment matrix used in this step was obtained from a PARAMICS simulation run using a uniform O-D matrix using.

In Step 2, a new assignment matrix is developed again from the PARAMICS simulation run using an estimated O-D matrix at the total O-D demand of 45,000 vehicles per hour. Based on the new assignment matrix, the GA-based method was implemented with the same total O-D demands as used in Step 1. Interestingly, the O-D matrix estimated at the total O-D demand of 45,000 vehicles per hour produced the best SEE value. Even though the discrepancy in the SSE value between GA-based method and PARAMICS evaluation is reduced from that of Step 1, it is still somewhat substantial. Thus, the Step 3 is conducted.

During Step 3, a new assignment matrix is again as it was done in Step 2. In this step, two additional total O-D demands are used along with the 45,000 vehicles per hour estimate to improve the accuracy in total O-D demand. The two additional total O-D demands are 42,500 and 47,500 vehicles per hour. The PARAMICS evaluation results indicate that the O-D matrix at the total O-D demand of 45,000 vehicles per hour produces the best SSE value. The discrepancy in the SSE values between the GA-based method and PARAMICS evaluation is significantly reduced.

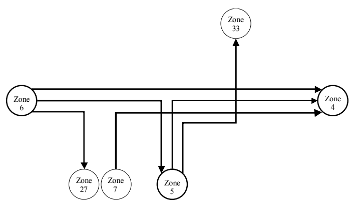
C. Comparison of GA-based and QUEENSOD Methods

In the QUEENSOD method, some individual estimates of four 15-minute interval O-D pairs change dramatically over iterations. However, the GA-based method is designed to avoid this phenomenon by restricting the variation of individual 15-minute interval O-D matrices within certain possible ranges. The ranges were found from the variation of 15-minute link flows.

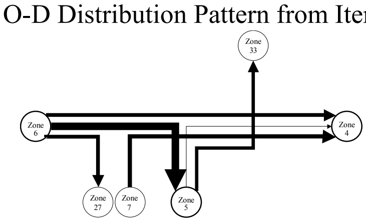
The comparison of SSE values from the QUEENSOD method and the PARAMICS evaluation shows sizeable discrepancies. There are two reasons for this: (i) the use of inadequate assignment matrix, and (ii) the way that the QUEENSOD method adjusts dynamic O-D demands between zone pairs.

In the case of former, since the assignment matrix used in the first steps of both methods was initially developed from the PARAMICS simulation using a uniform O-D matrix, the assignment matrix is unlikely to replicate realistic drivers' route choice behaviors. However, it was corrected subsequently through the multi-step approaches.

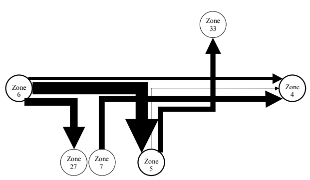
The latter factor is relevant only for the QUEENSOD method. This is because the QUEENSOD method does not constraint total O-D demand such that O-D demands between O-D pairs can be freely adjusted. For example, heavy O-D demands resulting in high link traffic counts on I-64 come from three external zones (i.e., zones 4, 5 and 6 in Fig. 1), as those are the main entry points to the City of Hampton. It was observed that the QUEENSOD method not only increases O-D demands among these three zones but also increases O-D demands among other zones adjacent to I-64. The high traffic demands from zone 6 (I-64, west city limit) and zone 4 (I-64, east city limit) can easily be understood. However, as the number of iterations increases, the demands from zone 7 to zone 4 and zone 5 to zone 33 are increased to match the high link traffic counts on I-64 instead of the increasing the demands from zone 6 to zone 4 as shown in Fig. 1. Animations of PARAMICS simulation show that the estimated O-D matrix at a large number of iterations causes a great deal of congestion in simulations due to heavy turning traffic exceeding the ramp capacity. This characteristic of the QUEENSOD method resulted in huge turning demands, particularly on major roads.



(a) O-D Distribution Pattern from Iteration 1



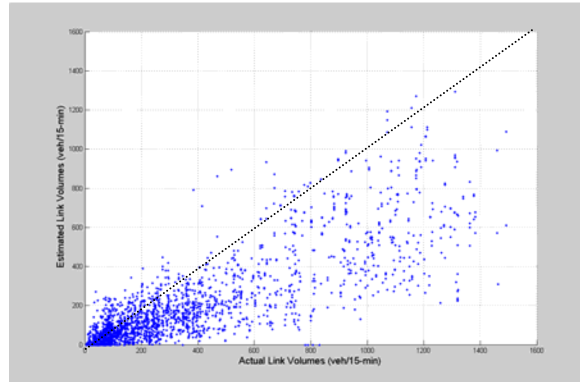
(b) O-D Distribution Pattern from Iteration 8



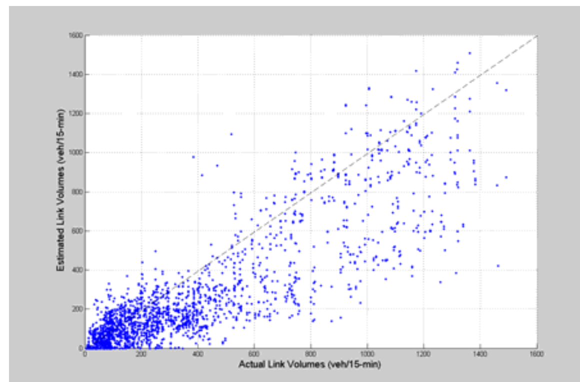
(c) O-D Distribution Pattern from Iteration 25

Fig. 1. O-D Demand Adjustments by QUEENSOD Method (Note: The widths of arrows are proportional to the values of O-D demands)

Conclusively, the SSE values from the PARAMICS evaluations indicate that the GA-based method outperforms the QUEENSOD method. It can be observed in Fig. 2 that illustrates the difference between the observed link flows and the estimated link flows from the QUEENSOD and GA-based methods using the PARAMICS evaluations.



(a) QUEENSOD Method (SSE = 4.57E+07)



(b) GA-based Method (SSE = 2.75E+07)

Fig. 2. Comparison of Estimated Link Flows vs. Observed Link Flows (Note: the estimated link flows were obtained from the optimal O-D matrices of both methods using PARAMICS simulation)

IV. CONCLUSIONS AND FUTURE RESEARCH

This paper presented a GA-based dynamic O-D matrix estimation method and compared it with the QUEENSOD method using a large-scale traffic simulation network. A microscopic traffic simulation program, PARAMICS, was used to generate an assignment matrix and to evaluate estimated dynamic O-D matrix.

Both the QUEENSOD method and the proposed GA-based method were implemented in the City of Hampton network using PARAMICS. The results showed that the GA-based method has potential in estimating a dynamic O-D matrix for large-scale networks, whereas the QUEENSOD method showed a tendency to be trapped at a local optimal solution.

The following key factors are emphasized for the development and implementation of dynamic O-D matrix estimation methods in large-scale networks.

1) Importance of updating the assignment matrix in the dynamic O-D matrix estimation: Both methods used in this paper utilize multi-step approaches to update the assignment matrix. Since these methods are assumed to start from a uniform O-D matrix, the initial assignment matrix obtained from the PARAMICS simulation using

a uniform O-D matrix does not represent proper drivers' route choice behavior. Through the case study, the use of an improved assignment matrix, which is updated through the first step, showed better results in dynamic O-D matrix estimations.

2) Importance of turning movement counts: Both methods use only link traffic counts in their criteria for the dynamic O-D matrix estimation. Using only link traffic counts has a limitation as shown in the implementation of the QUEENSOD method. This is because the O-D matrix estimation from only link traffic counts could cause unrealistic turning movement counts. A method that can accommodate turning movement counts as well as link traffic counts should be used for dynamic O-D matrix estimation.

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