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An iterative scheme for signal settings and network flows

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Abstract

An area traffic control optimization is considered where the signal setting variables and link flows are simultaneously determined. The optimal signal settings and network flows (OPSF) can be formulated as a mathematical program with equilibrium constraints (MPEC) where the user equilibrium is expressed as a variational inequality problem. Due to the non-differentiability of the perturbed solutions in equilibrium constraints, a non-smooth model is established. A Quasi–Newton subgradient projection is presented to effectively solve the OPSF with global convergence. Numerical calculations are conducted on an example network where promising results are obtained.

Keywords: Area traffic control; Non-smooth model; Subgradients; Sensitivity analysis

1. Introduction

Optimal signal settings and network flows for road traffic networks are considered where the sum of total travel times for all users is minimized. Consider the OPSF problem, in past decades a lot of rich work have been done from the literature [1,14,9,2,16,7,15] and associated references therein. Recently, [4] proposed a genetic algorithm (GA) to deal with area traffic control problem. Also, [3] discussed models and algorithms for the optimization of urban signal settings with stochastic traffic assignments where numerical tests are reported on a small-scale real network. However, these developed heuristics have not been tested on general networks for comparative analysis.

In this paper, we formulate the OPSF as a generalized mathematical program with equilibrium constraints (MPEC) where the response of users behaviour is taken into account in the decision-making process. The OPSF problem can be formulated as an MPEC, among which the coordinated signal timing plan is defined by the common cycle time, the start and duration of greens. The performance index (PI) is defined as the sum of a weighted linear combination of rate of delay and number of stops per unit time for all traffic streams. Due to the non-differentiability of the perturbed solutions in the equilibrium constraints with respect to the decision variables, ways in dealing with the optimisation problem are adopted from a non-smooth analysis approach. Following the pioneer work in non-smooth optimization [11], a non-smooth model for the OPSF

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is firstly proposed in this paper. The first-order sensitivity analysis is conducted by solving an affine variational inequality problem in which the directional derivatives and associated generalized gradient of variable of interests can be found. A Quasi–Newton subgradient projection (QSP) approach is firstly presented for which the accumulation points of the OPSF problem can be obtained. Global convergence analysis is delivered. Numerical calculations are carried out on an example road network. The computational results have shown the potential of the proposed approach in solving the OPSF by consistently yielding closer value to the system optimum.

The organization of this paper is as follows. In the next section, an MPEC program for OPSF is given. The first-order sensitivity analysis for obtaining the generalized gradient [6] and the associated directional derivatives are conducted. In the Section 2, a new solution scheme is presented for solving a non-smooth OPSF problem. In the Section 3 numerical calculations are conducted on an example road network. Good computational results are obtained when compared to earlier results. Conclusions and further work are summarized in the Section 4.

2. Problem formulations

In this section, the OPSF problem can be formulated as an MPEC programme. The first-order sensitivity analysis is conducted for which the generalized gradient and directional derivatives of variable of interests are derived. Provided that the objective function of the MPEC problem is semi-smooth, the directional derivatives can be found and a single level MPEC is presented. Notation used throughout this paper is given below.

2.1. Notation

$G(N,L)$ a directed road network, where N is the set of nodes and L is the set of links
$\Psi = (\zeta, \theta, \phi)$ the set of signal setting variables, respectively for the reciprocal of cycle time, start and duration of greens
Λ_a the duration of effective green for link a
s_a the saturation flow on link a
g_{jm} the minimum green for signal group j at junction m
$\bar{c}_{jlm}$ the clearance time between the end of green for group j and the start of green for incompatible group l at junction m
$\Omega_m(j,l)$ a collection of numbers 0 and 1 for each pair of incompatible signal groups at junction m; where $\Omega_m(j,l) = 0$ if the start of green for signal group j proceeds that of l, and $\Omega_m(j,l) = 1$, otherwise
D_a the rate of delay on link a
S_a the number of stops per unit time on link a
W a set of OD pairs
T the OD trip rates
R the set of paths between OD pairs
f the path flow vector
q the link flow vector
Λ the link-path incidence matrix
Γ the OD-path incidence matrix
c the link flow travel time
C the path flow travel time
π the minimum travel time

2.2. The traffic assignment problem

A traffic assignment problem can be expressed as a variational inequality. To find values $q \in K$ such that

$$c^t(q)(z - q) \geqslant 0 \tag{1}$$

for all $z \in K = \{q : q = \Lambda f, \Gamma f = T, f \geqslant 0\}$ where the superscript t denotes matrix transpose operation.

2.3. A parametric variational inequality

For a traffic assignment problem subject to signal settings, a parametric variational inequality for (1) can be expressed as follows:

$$c^t(q,\Psi)(z-q(\Psi)) \geqslant 0 \tag{2}$$

for all $z \in K(\Psi) = \{q(\Psi) : q(\Psi) = \Lambda f(\Psi), \Gamma f(\Psi) = T, f(\Psi) \geqslant 0\}$.

2.4. An MPEC programme

An OPSF problem can be formulated as an MPEC program as follows:

$$\begin{aligned} &\min_{\Psi,q} && \Theta = P_0(\Psi,q) \\ &\text{subject to} && \Psi \in \Omega \quad \text{and} \quad q \in S(\Psi), \end{aligned} \tag{3}$$

where Ω defines the constraints of decision variables and $S(\cdot)$ denotes the solution set for equilibrium flow as given in (2).

2.5. Sensitivity analysis by directional derivatives

Suppose the solution set $S(\cdot)$ is locally Lipschitz. Following the results developed by [13, Theorem 3.2.1], the first-order sensitivity analysis of (2) can be established in the following way. Given an arbitrary path flow f^* together with q^* and π^* at Ψ^*, let the changes in link or path flow with respect to the changes in Ψ^* denoted by $g_{q(\Psi^*)}$ or $g_{f(\Psi^*)}$ and the corresponding changes in path flow travel time denoted by g_{C_f}. The directional derivatives g_q of (2) can be obtained by solving the following affine variational inequality. For all $\bar{q} \in g_{K(\Psi)}$,

$$(\nabla_\Psi c(\Psi^*,q^*)g_\Psi + \nabla_q c(\Psi^*,q^*)g_q)^t(\bar{q}-g_q) \geqslant 0, \tag{4}$$

where $\nabla_\Psi c$ are gradients evaluated at (Ψ^*,q^*) when changes in signal settings g_Ψ are specified, and

$$g_{K(\Psi)} = \{g_q : \exists g_{f(\Psi)} \text{ such that } g_{q(\Psi)} = \Lambda g_{f(\Psi)}, \Gamma g_{f(\Psi)} = 0, \text{ and } g_{f(\Psi)} \in g_{K_0(\Psi)}\}, \tag{5}$$

where

$$g_{K_0} = \left\{ g_{f(\Psi^*)} : \begin{array}{lllll} \text{(i)} & g_{f_u(\Psi^*)} \text{ free}, & \text{if} & f_u^* > 0, & \\ \text{(ii)} & g_{f_u(\Psi^*)} \geqslant 0, & \text{if} & f_u^* = 0, & \text{and} \quad C_u^* = \pi_w^* \\ \text{(iii)} & g_{f_u(\Psi^*)} = 0, & \text{if} & f_u^* = 0, & \text{and} \quad C_u^* > \pi_w^* \quad \forall u \in R_w, w \in W \end{array} \right\}.$$

According to Rademacher's theorem [6], the solution set $S(\cdot)$ is differentiable almost everywhere. Thus, the generalized gradient for $S(\cdot)$ can be denoted as

$$\partial S(\Psi^*) = \text{conv}\left\{ g_{q(\Psi^*)} = \lim_{k\to\infty} \nabla q(\Psi^k) : \Psi^k \to \Psi^*, \nabla q(\Psi^k) \text{exists} \right\}, \tag{6}$$

where *conv* denotes the convex hull.

2.6. A single-level problem

Consider the OPSF problem given in (3), the set Ω defines the constraints of signal settings Ψ, which can be expressed as system of linear inequalities for minimum green, maximum cycle time and capacity constraints, following the generalized gradient given in (6) a single-level problem can be expressed. Suppose the solution set $S(\cdot)$ is locally Lipschitz, a one level problem of (3) is to

$$\begin{aligned} &\min_{\Psi} && \Theta(\Psi) = P_0(\Psi, S(\Psi)) \\ &\text{subject to} && \Psi \in \Omega. \end{aligned} \tag{7}$$

Let W^1_{aD} and W^1_{aS} be respectively link-specific weighting factors for the rate of delay and the number of stops per unit time, the detail of P_0 is

$$\sum_{a\in L} D_a W^1_{aD} + S_a W^1_{aS}. \tag{8}$$

Also the constraints of signal settings are

$$\zeta_{\min} \leqslant \zeta \leqslant \zeta_{\max}, \tag{9}$$

$$g_{jm}\zeta \leqslant \phi_{jm} \leqslant 1 \quad \forall j, m, \tag{10}$$

$$q_a \leqslant s_a \Lambda_a \quad \forall a \in L, \tag{11}$$

$$\theta_{jm} + \phi_{jm} + \bar{c}_{jlm}\zeta \leqslant \theta_{lm} + \Omega_m(j,l), \quad j \neq l\ \forall j, l, m. \tag{12}$$

The constraint (9) is for the common cycle time and for each junction m the constraints (10)–(12) are for the green phase, link capacity and clearance time. As it seen obviously from literature [8], $\Theta(\cdot)$ function is a non-smooth and non-convex function with respect to the decision variable because the solution set $S(\cdot)$ may not be explicitly expressed as a closed form.

3. A non-smooth solution scheme

Due to the non-differentiability of the solution set $S(\cdot)$ in (7), in this section, we propose a non-smooth solution scheme for effectively exploring a near optimum for problem (7). In the followings, we suppose the objective function $\Theta(\cdot)$ is semi-smooth which defined by [10] and locally Lipschitz. Therefore the directional derivatives of $\Theta(\cdot)$ can be characterized by the generalized gradient as follows:

$$\partial\Theta(\Psi^*) = \mathrm{conv}\left\{\lim_{k\to\infty} \nabla\Theta(\Psi^k) : \Psi^k \to \Psi^*, \nabla\Theta(\Psi^k)\text{exists}\right\}. \tag{13}$$

According to [6], the generalized gradient (13) is a convex hull of all points of the form $\lim \nabla\Theta(\Psi^k)$ where the subsequence $\{\Psi^k\}$ converges to the limit value Ψ^*. The component $g_\Psi \in \partial\Theta(\Psi^k)$ in (13) can be expressed as follows:

$$\nabla_\Psi P_0(\Psi^k, q^k) + \nabla_q P_0(\Psi^k, q^k) g_{q(\Psi^k)}, \tag{14}$$

where the directional derivatives g_q can be obtained from (4).

3.1. A Quasi–Newton Subgradient Projection method (QSP)

Consider the non-smooth problem (7), a general solution by an iterative subgradient method [12] can be developed as follows. Let $\mathrm{Pr}_\Omega(\Psi) \in \Omega$ denote the projection of Ψ on set Ω such that

$$\|\Psi - \mathrm{Pr}_\Omega(\Psi)\| = \inf_{v\in\Omega} \|\Psi - v\|. \tag{15}$$

Since Polyak' s subgradient method is a non-descent method with slow convergence as commented from the literature, in this section, we propose a new iterative scheme called Quasi–Newton Subgradient Projection (QSP) method to effectively solve the OPSF problem (7), where the current iterate is guided to move along the search direction progressively to approach a near optimum with global convergence. Regarding the details of the proposed QSP, it can be expressed as follows.

Lemma 1 (Quasi–Newton method with subgradients). *Let Ψ_1 locally solve (7), and Q_1 positive definite symmetric matrix. For $k = 1, \dots, n$, let*

$$\Psi^{k+1} = \Psi^k + d^k, \tag{16}$$

where

$$d^k = -Q_k g_k \tag{17}$$

is a descent direction and $g_\Psi \in \partial\Theta(\Psi^k)$. *For* $k = 1, \ldots, n-1, Q_k$ *is given as follows*:

$$Q_{k+1} = Q_k - \frac{Q_k u_k u_k^t Q_k}{u_k^t Q_k u_k} + \frac{v_k v_k^t}{v_k^t u_k}, \tag{18}$$

where $u_k = \Psi^{k+1} - \Psi^k$ *and* $v_k^t = g_{k+1} - g_k$. *If* $0 \notin \partial\Theta(\Psi^k)$ *for* $k = 1, \ldots, n$, *then* $Q_1, \ldots, Q_n$ *are symmetric and positive definite so that* $d^1, \ldots, d^n$ *are descent directions.*

Theorem 1 [A Quasi–Newton Subgradient Projection method (QSP)]. *In problem* (7), *suppose* $\Theta(\cdot)$ *is lower semi-continuous on the domain set* Ω. *Given a* Ψ^1 *such that* $\Theta(\Psi^1) = \alpha$, *the level set* $S_\alpha(\Omega) = \{\Psi : \Psi \in \Omega, \Theta(\Psi) \leqslant \alpha\}$ *is bounded and* Θ *is locally Lipschitzian and semi-smooth on the convex hull of* S_α. *A sequence of iterates* $\{\Psi^k\}$ *can be generated in accordance with*

$$\Psi^{k+1} = \mathrm{Pr}_\Omega(\Psi^k + l d^k), \quad k = 1, 2, \ldots, \tag{19}$$

where $l \in (0, 2)$ *is the step length which minimize* Θ^k . *Let* d^k *be the search direction obtained from* (17) *at current iterate k. Let* Ψ^* *be a minimum point on the level set* S_α, *then* $\|\Psi^k - \Psi^*\|$ *is monotonically decreasing and* $\|\Psi^k - \Psi^*\| \leqslant \|\Psi^1 - \Psi^*\|$, *i.e. the sequence of points* $\{\Psi^k\}$ *generated by QSP is bounded.*

Proof. For any x and y in the set Ω, by definition of the projection, we have

$$\|\mathrm{Pr}_\Omega(x) - \mathrm{Pr}_\Omega(y)\| \leqslant \|x - y\|, \tag{20}$$

thus for Ψ^{k+1} we have

$$\begin{aligned} \|\Psi^{k+1} - \Psi^*\|^2 &= \|\mathrm{Pr}_\Omega(\Psi^k + l d^k) - \Psi^*\|^2 \leqslant \|\Psi^k + l d^k - \Psi^*\|^2 \\ &= \|\Psi^k - \Psi^*\|^2 + l^2\|d^k\|^2 + 2l(\Psi^k - \Psi^*)^t d^k, \end{aligned} \tag{21}$$

let $d^k = \Psi^* - \Psi^k$, thus in (21)

$$\begin{aligned} \|\Psi^{k+1} - \Psi^*\|^2 &\leqslant \|\Psi^k - \Psi^*\|^2 + l^2\|d^k\|^2 - 2l(\Psi^k - \Psi^*)^t(\Psi^k - \Psi^*) \\ &= \|\Psi^k - \Psi^*\|^2 + l^2\|(\Psi^* - \Psi^k)\|^2 - 2l(\Psi^k - \Psi^*)^t(\Psi^k - \Psi^*) \\ &= \|\Psi^k - \Psi^*\|^2 + l(l-2)\|(\Psi^* - \Psi^k)\|^2 = \|\Psi^k - \Psi^*\|^2 + l(l-2)\|d^k\|^2 \end{aligned} \tag{22}$$

since $0 < l < 2$, we have

$$\|\Psi^{k+1} - \Psi^*\|^2 < \|\Psi^k - \Psi^*\|^2, \tag{23}$$

for $k = 1, 2, ..$ It implies $\|\Psi^k - \Psi^*\|$ is monotonically decreasing and $\|\Psi^k - \Psi^*\| \leqslant \|\Psi^1 - \Psi^*\|$. □

Theorem 2 (Convergence of QSP). *If* Ω^* *for problem* (7) *is nonempty*, *then the method QSP is globally convergent*, *i.e.*

$$\lim_{k\to\infty} \|\Psi^k - \Psi^*\|^2 = 0 \quad \forall \Psi^* \in \Omega^*. \tag{24}$$

Proof. Since $\Psi^* \in \Omega^*$, following the results in Theorem 1, it is easy to check that every sequence $\{\Psi^k\}$ generated by the QSP is bounded. We proof this theorem by contradiction. Suppose

$$\lim_{k\to\infty} \|\Psi^k - \Psi^*\|^2 = \delta_0 > 0, \tag{25}$$

then

$$\{\Psi^k\} \subset S_0 = \{\Psi \in \Omega : \delta_0 \leqslant \|\Psi - \Psi^*\|^2,\ \|\Psi - \Psi^*\|^2 \leqslant \|\Psi^0 - \Psi^*\|^2\ \forall \Psi^* \in \Omega^*\}, \tag{26}$$

and S_0 is a closed bounded set.

From (22) with $l \in (0, 2)$, we have

$$\|\Psi^{k+1} - \Psi^*\|^2 \leqslant \|\Psi^k - \Psi^*\|^2 - l(2-l)\|\Psi^k - \Psi^*\|^2. \tag{27}$$

Because $S_0 \cap \Omega^* = \phi$, then on S_0 we let

$$T(\Psi^*) = l(2-l)\|\Psi^k - \Psi^*\|^2. \tag{28}$$

Since $T(\Psi^*)$ is Lipschitz continuous on S_0, we have

$$\inf_{\Psi^* \in S_0} T(\Psi^*) = \varepsilon_0. \tag{29}$$

From (25), there is a $k_0 > 0$ such that for all $k > k_0$,

$$\|\Psi^k - \Psi^*\|^2 < \delta_0 + \frac{\varepsilon_0}{2}. \tag{30}$$

On the other hand, from (27) and (29),

$$\|\Psi^{k+1} - \Psi^*\|^2 \leqslant \|\Psi^k - \Psi^*\|^2 < \delta_0 - \frac{\varepsilon_0}{2}. \tag{31}$$

This contradicts (25) and completes this proof. □

3.2. An iterative solution scheme

Consider the solution for problem (7), a new globally convergent solution scheme is established in the following steps.

Step 1.0 (Initials). Set parameter $l \in (0,2)$ and termination threshold ε_0.
  1.1 Start with Ψ^i and index $i = 0$.
Step 2.0 (Compute subgradients)
  2.1 Solve a parametric traffic assignment problem with Ψ^i in (2).
  2.2 Find the first-order direction derivatives g_q by solving an affine variational inequality (4).
  2.3 Compute the generalized gradients $g_\Psi \in \partial\Theta(\Psi^i)$ in (14).

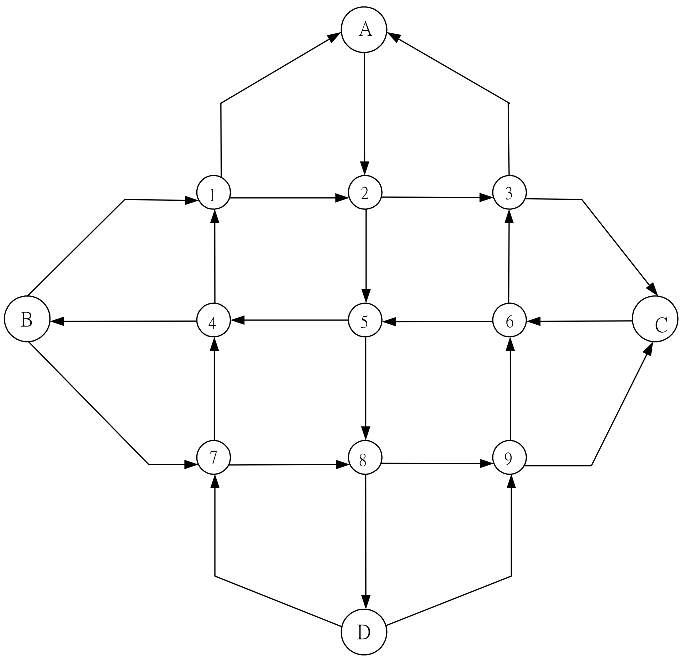

Fig. 1. Example network.

Step 3.0 (Compute the Quasi–Newton subgradient)
  3.1 Compute the Quasi–Newton matrix Q_i by (18).
  3.2 Determine a search direction d^i via (17).
Step 4.0 (Conduct the QSP method)
  4.1 Compute the $T(\Psi^k)$ value via (28).
  4.2 If $T(\Psi^k) \leqslant \varepsilon_0$ then stop; otherwise continue.
  4.3 Find new iterate Ψ^{i+1} via (19) and increase index i by 1. Then go to Step 2.0.

4. Numerical calculations

In this section a general example network as given in Fig. 1 is chosen from [5] for numerical computations. In this test network it consists of 4-pair OD trips and 9 signalized junctions. Trip rates are set at 100 veh/h and link capacity is set 1800 veh/h. Numerical tests are conducted on three sets of distinct initials for the newly proposed GA method in [4], the proposed QSP and the system optimum solution (SO). Computational results are summarized in Table 1. As it seen from Table 1, the proposed QSP significantly improved the PI value for (7) by 32%, 19%, and 25% respectively for the three sets of initials without extra computational efforts in solv-

Table 1
Computational results for example network

Variable algorithm	1st set			2nd set			3rd set		
	SO	GA	QSP	SO	GA	QSP	SO	GA	QSP
Initial $1/\zeta$	75	75	75	90	90	90	135	135	135
Initial ϕ_{jm}/ζ	25	25	25	30	30	30	45	45	45
$1/\zeta$	80	90	85	88	110	90	100	105	98
ϕ_{11}/ζ	38	42	40	42	48	38	42	42	40
ϕ_{12}/ζ	40	40	42	45	50	36	38	50	38
ϕ_{13}/ζ	29	55	38	38	52	40	36	52	35
ϕ_{13}/ζ	42	52	38	40	48	42	44	55	41
ϕ_{14}/ζ	44	30	40	42	52	36	42	48	43
ϕ_{15}/ζ	36	38	45	48	44	38	45	50	51
ϕ_{16}/ζ	44	30	40	42	52	36	42	48	43
ϕ_{17}/ζ	36	38	42	48	44	38	45	50	51
ϕ_{18}/ζ	39	37	45	51	45	39	39	38	45
ϕ_{19}/ζ	40	43	44	44	49	43	42	44	44
PI	79.2	85.5	80.1	79.3	88.5	79.8	80.1	86.1	81.2
#	50	88	62	52	101	65	51	102	65

Where ϕ_{jm}/ζ denotes the duration of greens for signal group j at junction m measured in seconds, $1/\zeta$ denotes the common cycle time measured in seconds, PI denotes the performance index value measured in (veh-h/h), and # represents number of traffic assignment solved.

Table 2
Computational results on example network with various demand levels

Scaling factor	SO	GA	QSP	Relative gap 1	Relative gap 2
1.1	81.6	89.7	82.1	9.9	0.6
1.2	82.2	92.7	82.4	12.8	0.2
1.3	85.1	95.3	85.6	12.0	0.6
1.4	87.4	97.4	87.9	11.4	0.6
1.5	88.1	100.5	88.9	14.1	0.9
1.6	88.9	106.8	89.5	20.1	0.7
1.7	89.7	109.3	90.6	21.9	1.0
1.8	90.9	117.6	92.1	29.4	1.3
1.9	91.1	118.7	92.8	30.3	1.9
2.0	92.0	123.5	93.2	34.2	1.3

Where relative gap 1 and 2 are the relative differences measured in percentage respectively between SO and GA and SO and QSP.

ing traffic assignments. Also the relative differences between the proposed QSP and SO are within 1.5%, which are much less than those of 8%, 11.5%, and 7.5% did by GA. A series of 10 congested networks by increasingly scaling travel demands by 10% is performed continuously. As it seen from Table 2, again, the relative differences of the proposed QSP and SO are much less than those of GA and SO especially when the scaling factor increases, which obviously shows the robustness of the proposed QSP when the road networks become increasingly congested.

5. Conclusions and future work

This paper addresses a new scheme for an OPSF problem expressed as an MPEC program from a non-smooth approach. A Quasi–Newton subgradient projection approach (QSP) is proposed to effectively search for local optima with global convergence. Numerical calculations are conducted on an example network with distinct initials and scaled travel demands. As it shown, the proposed QSP achieved appreciably significant performance in solving the non-convex OPSF problem without extra computational efforts. Consider further work associated, ways in tackling congestion pricing for urban networks are being investigated together with explorations in meta-heuristics implementations.

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