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Control Engineering Practice 14 (2006) 1297–1311
CONTROL ENGINEERING PRACTICE
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A signal timing plan formulation for urban traffic control

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Received 5 November 2004; accepted 29 June 2005
Available online 15 August 2005

Abstract

This paper addresses urban traffic control using an optimization model for signalized areas. The paper modifies and extends a discrete time model for urban traffic networks proposed in the related literature to take into account some real aspects of traffic. The model is embedded in a real time controller that solves an optimization problem from the knowledge of some measurable inputs. Hence, the controller determines the signal timing plan on the basis of technical, physical, and operational constraints. The actuated control strategy is applied to a case study with severe traffic congestion, showing the effectiveness of the technique.

Keywords: Urban traffic control; Urban traffic model; Discrete time systems; Timing plan formulation; Optimization

1. Introduction

Traffic congestion of urban roads undermines mobility in major cities. Traditionally, the congestion problem on surface streets was dealt by adding more lanes and new links to the existing transportation network. Since such a solution can no longer be considered for limited availability of space in urban centres, greater emphasis is nowadays placed on traffic management through the implementation and operation of intelligent transportation systems (Di Febbraro & Sacco, 2004). In particular, traffic signal control on surface street networks plays a central role in traffic management. Despite the large research efforts on the topic, the problem of urban intersection congestion remains an open issue (Lo, 2001; Papageorgiou, 1999). Most of the currently implemented traffic control systems may be grouped into two principal classes (Papageorgiou, Diakaki, Dinopoulou, Kotsialos, & Wang, 2003; Patel & Ranganathan, 2001): (i) fixed time strategies, that are derived off-line by use of optimization codes based on historical traffic data; (ii) vehicle actuated strategies, that perform an on-line optimization and synchronization of the signal timing plans and make use of real time measurements. While the fixed time strategies do not use information on the actual traffic situation, the second actuated control class can be viewed as a traffic-responsive network signal policy employing signal timing plans that respond automatically to traffic conditions. The main decision variables in a timing plan are *cycle time*, *green splits* and *offset* (Diakaki, Papageorgiou, & Aboudolas, 2002). Cycle time is defined as the duration of time from the centre of the red phase to the centre of the next red phase. Green splits for a signal in a given direction of movement is defined as the fraction of cycle time when the light is green in that direction. Moreover, offset is defined as the duration from the start of a green phase at one signal to the following nearest start (in time) of a green phase at the other signal. A phase (or stage) is the time interval during which a given combination of traffic signals in the area is unchanged. In a real time control strategy, detectors located on the intersection approaches monitor traffic conditions and feed information on the actual system state to the real time controller, which selects the duration of the phases in the signal timing plan in order

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0967-0661/$ - see front matter
doi:10.1016/j.conengprac.2005.06.013

to optimize an objective function. Although the corresponding optimal control problem may readily be formulated, its real time solution and realization in a control loop has to face several difficulties such as the size and the combinatorial nature of the optimization problem, the measurements of traffic conditions and the presence of unpredictable disturbances (Papageorgiou et al., 2003). The first and most notable of vehicle actuated techniques is the British SCOOT (Hunt, Robertson, Beterton, & Royle, 1982), that decides an incremental change of splits, offsets and cycle times based on real time measurements. However, although SCOOT exhibits a centralized hardware architecture, the strategy is functionally decentralized with regard to splits setting (Diakaki et al., 2002). A formulation of the traffic signal network optimization strategy is presented in Wey (2000), that models traffic streams and includes constraints in the signal controllers. However, the resulting formulation leads to a complex mixed integer linear programming problem solved by branch and bound techniques. In addition, Lo (2001) adopts a cell-transmission macroscopic model that allows stating optimization problems providing dynamic signal timing plans. However, solving the resulting mixed integer program is computationally intensive and the formulation for real networks requires heuristics for solutions. Furthermore, Diakaki et al. (2002) propose a traffic-responsive urban control strategy based on a feedback approach involving the application of a systematic and powerful control design method. Based on the store and forward modelling approach and the linear-quadratic methodology, the technique proposed in Diakaki et al. (2002) designs off-line and employs on-line the traffic-responsive coordinated urban network controller. Despite the simplicity and the efficiency of the proposed control strategy, such a modelling approach can not directly consider the effects of offset for consecutive junctions and the time-variance of the turning rates and the saturation flows. On the other hand, a traffic-responsive plan is proposed in Lei and Ozguner (2001). Such a method needs as inputs the data relevant to traffic flows approaching the intersections, provided in Di Febbraro, Giglio, and Sacco (2004) by a hybrid Petri net model.

This paper proposes an urban traffic actuated control strategy to determine in real time the green splits for a fixed cycle time in order to minimize the number of vehicles in queue in the considered signalized area. The aim of the paper is to give a contribution in facing the *apparently insurmountable difficulties* (Papageorgiou et al., 2003) in the real time solution and realization of the control loop governing an urban intersection by traffic lights. To this aim, the paper pursues simplicity in the modelling and in the optimization procedure. Moreover, the macroscopic model introduced in Barisone, Giglio, Minciardi, and Poggi (2002) is revisited and modified to describe the urban traffic network (UTN). Although such a model is compatible with real time optimization and is suitable for vehicle actuated signal setting, it does not take into account realistic situations like the changing traffic scenarios, the different types of vehicles in the signalized area, the presence of vehicles in upstream junctions that reduce the travelling time of downstream vehicles, the pedestrian movements, the amber phases and the intergreen times. Hence, to give a more accurate and valid representation of real traffic systems, this paper considers in the model the presence of pedestrians, the classification of vehicles in the area, a proper evaluation of the travelling times and different levels of traffic congestion. Describing the system by a discrete time model with the sampling time equal to the cycle, the timing plan is obtained through the solution of a mathematical programming problem that minimizes the number of vehicles in the considered urban area. The minimization of the objective function is subject to linear constraints derived from the intersection topology, the fixed cycle duration and the minimum and maximum duration of the phases commonly adopted in practice. The optimization problem is solved by a standard optimization software on a personal computer, so that practical applications are possible in a real time control framework. The green phase durations are optimized on the basis of the real traffic knowledge and the technique requires traffic measurement in a prefixed set of cycles. In addition, the problem of synchronization of subsequent intersections is addressed to allow uninterrupted traffic flow. Indeed, an incorrect synchronization between successive intersections in the same direction may cause spillback phenomena: vehicles proceeding from one intersection to the downstream junction find only the concluding part of their green phase, and therefore line up in a queue which may produce oversaturation, blocking the upstream junction.

Finally, the actuated control strategy is applied to a case study representing a real signalized area with severe traffic congestion, located in the urban area of Bari (Italy), which includes two coordinated intersections. On the basis of limited traffic observations, appropriate selections of offset and optimal choice of the green phases are performed under different congestion scenarios. Several defined performance indices show that the introduced control strategy is able to reduce congestion even in oversaturated conditions and is attractive for use in real applications.

This paper is organized as follows. Section 2 describes the UTN model. Furthermore, Section 3 presents the actuated traffic control strategy and Section 4 presents the heuristic strategy for coordinating local traffic control signals. Moreover, Section 5 describes the case study and reports the results of the optimization performed under different traffic scenarios. Finally, Section 6 summarizes the conclusions.

2. The urban traffic network model

2.1. The revisited model

This section revisits the macroscopic model proposed in Barisone et al. (2002) for control and optimization purposes. The peculiarity of the model is that it is simple to implement and suited for real time control. However, it might not accurately describe real urban intersections, particularly in traffic congestion conditions. Hence, to overcome the drawbacks of the model, it is modified by introducing some relations that allow to take into account the presence of pedestrians, the evaluation of the travelling time, different levels of traffic congestion, the vehicle classification and the amber and intergreen times.

A UTN is defined as a set $L = \{L_i | i = 1, \ldots I\}$ of I links (see Fig. 1) and each link represents a lane available between two subsequent intersections. Moreover, if a road is composed by several lanes, then a single link is associated to each lane and lane changing is not considered. The link set L can be classified as: (1) input links, pertaining to the set $L_{in} \subset L$, that are controlled by a traffic light located at their end, (2) output links of the set $L_{out} \subset L$ from which vehicles exit freely, (3) intermediate links of the set $L_{int} = L \backslash (L_{in} \cup L_{out})$, also equipped with a traffic light. A capacity N_i is associated to each link $L_i \in L_{int}$, denoting the maximum number of vehicles that can be accommodated in the link. On the other hand, each link $L_i \in L_{in} \cup L_{out}$ is supposed to be of infinite capacity. In addition, the following sets of indices are defined: $L_{no} = L \backslash L_{out}$, i.e., the set of non-output links, and $L_{nin} = L \backslash L_{in}$, i.e., the set of non-input links. Accordingly, the sets of indices $I_{in} = \{h : L_h \in L_{in}\}$, $I_{out} = \{h : L_h \in L_{out}\}$, $I_{int} = \{h : L_h \in L_{int}\}$, $I_{nin} = \{h : L_h \in L_{nin}\}$, $I_{no} = \{h : L_h \in L_{no}\}$ are introduced. Moreover, for a generic link $L_i \in L$ with $i = 1, \ldots, I$, the sets L^i_{in} and L^i_{out} represent, respectively, the sets of incoming and outgoing links for L_i, and $I^i_{in} = \{h : L_h \in L^i_{in}\}$ and $I^i_{out} = \{h : L_h \in L^i_{out}\}$ denote the corresponding index sets.

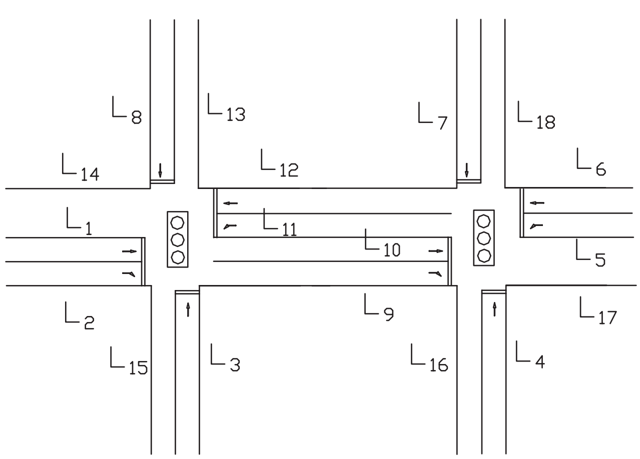

Fig. 1. Example of urban area comprising junctions pertaining to a common signal timing plan.

A generic signalized urban area is modelled as a UTN that comprises a number of junctions controlled by traffic lights pertaining to a common signal timing plan. In order to obtain a traffic actuated control strategy that determines in real time the correlated green splits of all the intersections, it is assumed that the UTN timing plan is characterized by a fixed cycle time of duration C seconds and divided in F phases. Moreover, the UTN is modelled as a discrete time system in which the state is described by the variables $n_i(k)$ with $i = 1, \ldots, I$, where $n_i(k) \in N^+ \cup \{0\}$ represents the number of vehicles in L_i at the beginning of the kth cycle and N^+ is the set of positive integer numbers. Hence, the fixed signal timing plan duration is assumed to be the time unit. To model the dynamics of each link L_i, the variables $u_i(k)$ and $y_i(k)$ are introduced, denoting respectively the number of vehicles entering the link within the kth cycle and the number of vehicles leaving it in the same time interval. Hence, the vehicle balance equation of link L_i for $i = 1, \ldots, I$ in the kth cycle is as follows:

$$n_i(k+1) = n_i(k) + u_i(k) - y_i(k) \quad \text{for } i = 1, \ldots, I \text{ and } k \in N^+. \tag{1}$$

Moreover, $u_i^f(k)$ and $y_i^f(k)$ denote the number of vehicles, respectively, entering and leaving L_i in the fth phase of the kth cycle, with $f = 1,\ldots,F$ and $k \in N^+$. Hence, it holds

$$u_i(k) = \sum_{f=1}^{F} u_i^f(k) \quad \text{for } i = 1, \ldots, I \text{ and } k \in N^+, \tag{2}$$

$$y_i(k) = \sum_{f=1}^{F} y_i^f(k) \quad \text{for } i = 1, \ldots, I \text{ and } k \in N^+. \tag{3}$$

Obviously, variables $u_i^f(k)$ with $i \in I_{in}$ may be measured by detectors located at the beginning of the corresponding input link (Kamata & Oda, 1991) and hence they constitute the non-controllable inputs of the model. On the other hand, the output streams of all links and the input streams of non-input links can be determined as follows. Consider for the generic links $L_h, L_i, L_j \in L$, with $i \in I^h_{out}$ and $j \in I^i_{out}$, variables $S^f_{h,i}(k)$ and $S^f_{i,j}(k)$ with $f = 1, \ldots, F$ and $k \in N^+$, representing, respectively, the number of vehicles travelling from L_h to L_i and from L_i to L_j in the fth phase of the kth cycle. The variables $u_i^f(k)$ for each $i \in I_{nin}$ may be determined by the following equations:

$$u_i^f(k) = \sum_{h \in I^i_{in}} S^f_{h,i}(k) \quad \text{for each } i \in I_{nin},\ f = 1, \ldots, F \text{ and } k \in N^+. \tag{4}$$

Analogously, the variables $y_i^f(k)$ are determined as follows:

$$y_i^f(k) = \sum_{j \in I_{out}^i} S_{i,j}^f(k)$$

for each $i \in I_{no}$, $f = 1, \ldots, F$ and $k \in N^+$. (5)

Eq. (5) applies only to non-output links, since for output links the outflow of vehicles is not split in different streams. In addition, if L_i is an output link, then all vehicles can freely exit from the link, since no traffic light is present and queues may be disregarded according to the output links infinite capacity hypothesis. Hence, if $n_i^f(k)$ is the overall number of vehicles in L_i which can leave the link during the fth phase of the kth cycle leaving aside capacity and congestion constraints, the following equation completes the system outputs:

$$y_i^f(k) = n_i^f(k)$$

for each $i \in I_{out}$, $f = 1, \ldots, F$ and $k \in N^+$. (6)

From Eqs. (4) and (5), it is apparent that determining variables $S_{i,j}^f(k)$ is crucial for the model. In the following, a procedure is presented that allows to evaluate such variables.

Variable $\tau_i(k)$ denotes the travelling time of link L_i, i.e., the average time necessary for a generic vehicle to travel along the link during the kth cycle of the signal timing plan, that is a function of the number of vehicles in the link. Moreover, let $u_{goi}^f(k)$ and $u_{stopi}^f(k)$ be the number of vehicles entering L_i in the fth phase of the kth cycle, respectively, able and unable to leave the link during the same phase. Such numbers are evaluated by the following equations:

$$u_{go\ i}^f(k) = \begin{cases} ps_i^f\, u_i^f(k)\, \frac{t^f(k) - \tau_i(k)}{t^f(k)} & \text{if } \tau_i(k) \leqslant t^f(k), \\ 0 & \text{if } \tau_i(k) > t^f(k), \end{cases}$$

$i = 1, \ldots, I$, $f = 1, \ldots, F$, $k \in N^+$ (7)

$$u_{stop\ i}^f(k) = u_i^f(k) - u_{go\ i}^f(k),$$

$i = 1, \ldots, I$, $f = 1, \ldots, F$, $k \in N^+$ (8)

where $t^f(k)$ is the duration of phase f in the kth cycle and ps_i^f is the binary variable denoting the state of the traffic lights in L_i during phase f (0 for red light and 1 otherwise).

In order to model the real traffic behaviour in the area, consider two consecutive links L_i and L_j with $j \in I_{out}^i$, and define the effective time $t_{eff\ i,j}^f(k)$, representing the actual fraction of the fth phase duration $t^f(k)$ in the kth cycle available for vehicles to leave L_i for L_j. To this aim, the value $t_{eff\ i,j}^f(k)$ has to take into account the time necessary for vehicles with right of way with respect to vehicles in L_i travelling to L_j to clear the intersection, as well as crossing pedestrians. Hence, the effective time models precedence of vehicles and pedestrians in the area. Clearly, no precedence model has to be taken into account when L_i is an output link, hence the effective time $t_{eff\ i,j}^f(k)$ is defined only for non-output links L_i with $i \in I_{no}$.

The effective time may be employed to determine the maximum number of vehicles $Q_{i,j}^f(k)$ that may leave link L_i for L_j during the fth phase of the kth cycle due to rights of way, if the current phase is green for such vehicles. A linear approximation of $Q_{i,j}^f(k)$ as a function of the effective time $t_{eff\ i,j}^f(k)$ may be obtained by

$$Q_{i,j}^f(k) = \phi_{i,j}\, t_{eff\ i,j}^f(k),$$

for each $i \in I_{no}$, $j \in I_{out}^i$, $f = 1, \ldots, F$ and $k \in N^+$. (9)

In other words, Eq. (9) evaluates $Q_{i,j}^f(k)$ taking into account the precedence constraints and the area topology and parameter $\phi_{i,j} \in \Re^+$ is the linear approximation slope, measured in number of vehicles per seconds. Hence, different values of $\phi_{i,j}$ can represent different traffic scenarios and the higher $\phi_{i,j}$ the lower congestion in links L_i and L_j. Moreover, let $\beta_{i,j}$ be the rate of vehicles travelling from L_i to L_j (turning rates or splitting rates), obtained off-line by some suited algorithm (e.g. see Willumsen, 1991). It holds

$$\sum_{j \in I_{out}^i} \beta_{i,j} = 1 \quad \text{for each } i \in I_{no}. \tag{10}$$

On the basis of knowledge of $n_i^f(k)$, representing the overall number of vehicles in L_i which can leave the link during the fth phase of the kth cycle leaving aside capacity and congestion constraints, the following equation approximates the number of vehicles in a particular stream from L_i to L_j:

$$S_{i,j}^f(k) = min \begin{cases} \beta_{i,j}\, n_i^f(k), \\ ps_i^f\, \phi_{i,j}\, t_{eff\ i,j}^f(k), \\ N_j - n_j(k) - \sum_{\varphi=1}^{f-1} u_j^\varphi(k) \\ \quad - \sum_{(h,j) \in P_{i,j}} S_{h,j}^f(k) + \sum_{\varphi=1}^{f} y_j^\varphi(1) \end{cases}$$

for $i \in I_{no}$, $j \in I_{out}^i$, $f = 1, \ldots, F$ and $k \in N^+$. (11)

In (11) $S_{i,j}^f(k)$ is determined by the minimum of three factors: the number of vehicles directed to L_j that are in L_i during the current phase; the maximum number of vehicles that may travel from L_i to L_j taking into account the state of the traffic light and the effective time due to precedence constraints; the number of vehicles that can enter link L_j before reaching the capacity value N_j. However, Eq. (11) is simplified if $L_j \in L_{out}$; in such a case the link capacity is infinite and the above expression may be re-written as follows:

$$S^f_{i,j}(k) = min\{\beta_{i,j}\, n^f_i(k),\ ps^f_i\ \phi_{i,j}\ t^f_{eff\ i,j}(k)\}$$
$$\text{for } i \in I_{no},\ j \in I^i_{out},\ L_j \in L_{out},\quad f = 1,\ldots,F \text{ and } k \in N^+. \tag{12}$$

If the travelling time $\tau_i(k)$ in L_i is shorter than the duration of the fth phase of the kth cycle, then variable $n^f_i(k)$ can be evaluated as follows:

$$n^f_i(k) = \begin{cases} n_i(k) + u^f_{go\ i}(k) \\ \quad \text{if } \tau_i(k) \leqslant t^f(k) \text{ and } f = 1, \\ n^{f-1}_i(k) - y^{f-1}_i(k) + u^f_{go\ i}(k) + u^{f-1}_{stop\ i}(k) \\ \quad \text{if } \tau_i(k) \leqslant t^f(k) \text{ and } f = 2,\ldots,F \end{cases}$$
$$\text{for } i = 1,\ldots,I,\ f = 1,\ldots,F \text{ and } k \in N^+. \tag{13a}$$

Eq. (13a) states that the overall number of vehicles that can leave a link during a phase of a generic cycle equals the number of vehicles in the previous phase, minus the number of exited vehicles during the previous phase, plus the number of vehicles that can exit the link in the same phase. In particular, in Eq. (13a) it holds $u^f_{go\ i}(k) \neq 0$ and $u^{f-1}_{stop\ i}(k) \neq 0$ only if correspondingly $t^f(k)$ and $t^{f-1}(k)$ are green phases for vehicles in L_i. However, if the travelling time is higher than the duration of the fth phase of the kth cycle, then it holds $u^f_{go\ i}(k) = 0$ and $u^f_{stop\ i}(k) \neq 0$. Therefore, if there exists $W \in \{1,\ldots,F-1\}$ such that $t^f(k) \leqslant \tau_i(k) \leqslant W\, t^f(k)$, $n^f_i(k)$ may be re-written as follows:

$$n^f_i(k) = \begin{cases} n_i(k) & \text{if } f = 1 \\ n^{f-1}_i(k) - y^{f-1}_i(k) & \text{if } f = 2,\ldots,W \\ n^{f-1}_i(k) - y^{f-1}_i(k) + u^{f-W}_{stop\ i}(k) & \text{if } f = W+1,\ldots,F \end{cases}$$
$$\text{for } i = 1,\ldots,I,\ f = 1,\ldots,F \text{ and } k \in N^+. \tag{13b}$$

2.2. Evaluation of the effective time

The effective time $t^f_{eff\ i,j}(k)$ has to take into account the time necessary for vehicles with right of way with respect to vehicles in L_i travelling to L_j to clear the intersection, as well as crossing pedestrians. Hence, if $P_{i,j}$ is the set of pairs (h,z) such that $z \in I^h_{out}$ and $S^f_{h,z}(k)$ has right of way over $S^f_{i,j}(k)$, vehicular precedence may be modelled as follows (Barisone et al., 2002):

$$t^f_{eff\ i,j}(k) = t^f(k) - \left[\sum_{(h,z)\in P_{i,j}} S^f_{h,z}(k)\,\frac{x_{h,z}}{v_{h,z}}\right]$$
$$\text{for each } i \in I_{no},\ j \in I^i_{out},\ f = 1,\ldots,F \text{ and } k \in N^+, \tag{14}$$

where $x_{h,z}$ and $v_{h,z}$ are the area crossed by a vehicle and its average speed, respectively, while driving from L_h to L_z. However, Eq. (14) correctly models the effective time only when pedestrians do not obstruct vehicular traffic. If a vehicle and a pedestrian stream are simultaneously permitted in crossing directions, vehicles must give right of way to pedestrians. Consequently, the green phase for the former is reduced. Since the two opposite streams of pedestrians obstructing vehicular traffic are continuous, they are regarded as a unique flow with the total number of pedestrians moving in one direction. If two simultaneously crossing vehicle and pedestrian streams are present in L_i during the fth green phase of the kth cycle, the phase is reduced and the effective time for vehicles leaving L_i towards L_j may be modified as follows:

$$t^f_{eff\ i,j}(k) = t^f(k) - \left[\sum_{(h,z)\in P_{i,j}} S^f_{h,z}(k)\,\frac{x_{h,z}}{v_{h,z}}\right] - \left[p^f_{T_i}(k)\,\frac{l_{c_i}}{v_p}\right]$$
$$\text{for each } i \in I_{no},\ j \in I^i_{out},\ f = 1,\ldots,F \text{ and } k \in N^+. \tag{15}$$

Moreover, symbol $p^f_{Ti}(k)$ denotes the total number of pedestrians crossing the streams of vehicles in L_i during phase f of the kth cycle, l_{ci} is the crosswalk length of link L_i and v_p equals the average pedestrians speed. However, Eq. (15) may be used only for infrequent pedestrian crossings, whereas in signalized urban areas pedestrians usually move being grouped in platoons. In particular, pedestrians proceed with speed v_p in rows spaced 1 s in time, whereas the minimal length of each row equals 0.75 m (Rinelli, 2000). Denoting by p_{ri} the average number of pedestrians in one row crossing the streams in L_i and $n^f_{ri}(k)$ the number of such rows during phase f of the kth cycle, it holds

$$p_{r_i} = \left\lceil \frac{l_{c_i}}{0.75} \right\rceil; \quad n^f_{r_i}(k) = \left\lceil \frac{p_{T_i}(k)}{p_{r_i}} \right\rceil,$$
$$i = 1,\ldots,I,\ f = 1,\ldots,F \text{ and } k \in N^+, \tag{16}$$

where symbol $\lceil\rceil$ indicates the rounding up operation. Moreover, the first row of pedestrians that finds a green light crosses the street in the time,

$$t_{p1_i} = \frac{l_{c_i}}{v_p}, \quad i = 1,\ldots,I. \tag{17}$$

The successive rows, distanced 1 s in time, cross in a time,

$$t_{p2_i} = \frac{l_{c_i}}{v_p} + 1; \quad t_{p3_i} = \frac{l_{c_i}}{v_p} + 2; \ \ldots\ t_{pf_{r_i}} = \frac{l_{c_i}}{v_p} + (n^f_{r_i}(k) - 1),$$
$$i = 1,\ldots,I,\ f = 1,\ldots,F \text{ and } k \in N^+. \tag{18}$$

Hence, the total crossing time for the overall number of pedestrians in link L_i during the fth phase of the kth cycle is

$$t_{p^f_{r_i}}(k) = \left(\frac{l_{c_i}}{v_p} + (n^f_{r_i}(k) - 1)\right) sign(n^f_{r_i}(k)),$$
$$i = 1,\ldots,I,\ f = 1,\ldots,F \text{ and } k \in N^+, \tag{19}$$

where $sign(n^f_{ri}(k))$ describes the state of the pedestrian flow, i.e., $sign(n^f_{ri}(k))=1$ models the presence of pedestrians and $sign(n^f_{ri}(k))=0$ indicates no crossings. Summing up, the effective time is evaluated as follows:

$$t^f_{eff\ i,j}(k) = max\left(t^f(k) - \left[\sum_{(h,z)\in P_{i,j}} S^f_{h,z}(k)\, \frac{x_{h,z}}{v_{h,z}} \right] - \left(\frac{l_{c_i}}{v_p} + (n^f_{r_i}(k)-1) \right) sign(n^f_{r_i}(k)), 0 \right)$$

$$\text{for each } i \in I_{no},\ j \in I^i_{out},\ f = 1,\ldots,F \text{ and } k \in N^+, \quad (20)$$

where the maximum accounts for the case in which no time is left for vehicles to proceed from L_i to L_j after waiting for pedestrians and vehicles with priority. The measurements of pedestrian streams required by Eq. (20) may be obtained by employing image sensors based on closed circuit TV cameras (Kamata & Oda, 1991).

2.3. Evaluation of the travelling time

A first approximation of the travelling time of a link L_i may be obtained assuming that the speed of vehicles in the link is constant in every cycle and equals the average speed v_i. Hence, the travelling time of link L_i is constant:

$$\tau_i(k) = \frac{l_i}{v_i}, \quad i = 1,\ldots,I \text{ and } k \in N^+, \quad (21)$$

where l_i represents the link length. Since the travelling time $\tau_i(k)$ affects the computation of the number of vehicles entering the link and leaving it during the same phase in (7), expression (21) is realistic when traffic is not congested. On the contrary, when vehicles line up in queues in the link, the travelling time should change at every time instant, being a function of the link state $n_i(k)$.

With reference to Fig. 2, consider a generic link L_i in which there are at the beginning of the kth cycle $n_i(k)$ vehicles in number. A simple expression evaluating the travelling time $\tau_i(k)$ is the summation of the time necessary for the vehicle to reach the end of the moving queue $T_{fi}(k)$ (free travelling time) and the time necessary for all the vehicles in the queue to leave the link $T_{ci}(k)$ (clearance time):

$$\tau_i(k) = T_{fi}(k) + T_{ci}(k) \quad \text{for } i = 1,\ldots,I \text{ and } k \in N^+. \quad (22)$$

Note that (22) assumes that the first vehicle in the link is always at the stop line, hence the equation is approximated. On the other hand, vehicles kinematics is obviously disregarded in the present study, since the model is macroscopic in nature. Then, let t_a and t_r be the average vehicle acceleration time to reach the steady state speed and the driver reaction time, respectively. Considering the average vehicle length of 5 m, the clearance time $T_{ci}(k)$ necessary for all the $n_i(k)$ vehicles to leave L_i is

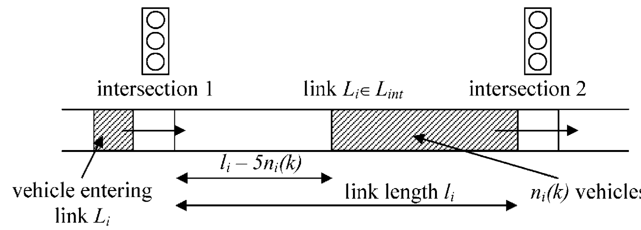

Fig. 2. Two subsequent junctions connected by a link L_i occupied by $n_i(k)$ vehicles at cycle K.

$$T_{c_i}(k) = t_a + t_r\, n_i(k) + \frac{5}{v_i}(n_i(k)-1), \quad i = 1,\ldots,I \text{ and } k \in N^+. \quad (23)$$

The vehicle entering L_i finds a free link section l_i-$5n_i(k)$. Hence, a first approximation of the free travelling time $T_{fi}(k)$ is

$$T_{f_i}(k) = \frac{l_i - 5n_i(k)}{v_i}, \quad i = 1,\ldots,I \text{ and } k \in N^+. \quad (24)$$

However, while vehicles enter L_i the queue shortens at its other end, and the free link space increases. In fact, during time $(l_i–5n_i(k))/v_i$ a number of $y_i(k)$ vehicles leave the link. According to (3) and (5) such a number depends on variables $S^f_{ij}(k)$, that are given by (11). In order to determine an approximation $\tilde{y}_i(k)$ of $y_i(k)$ without considering the actual links dynamics expressed by the recalled formulas, variable $\tilde{y}_i(k)$ is evaluated by (23) substituting $T_{ci}(k)$ with $(l_i - 5n(k))/v_i$ in the left-hand term and $n_i(k)$ with $\tilde{y}_i(k)$ in the right-hand terms. After some trivial manipulations one gets

$$\tilde{y}_i(k) = min\left(\left\lfloor \frac{l_i - 5(n_i(k)-1) - t_a v_i}{v_i t_r + 5} \right\rfloor, 0 \right), \quad i = 1,\ldots,I \text{ and } k \in N^+, \quad (25)$$

where symbol $\lfloor\rfloor$ indicates the rounding down operation and the minimum accounts for no vehicle able to leave the link. Hence, the free link section is now $l_i - 5(n_i(k) - \tilde{y}_i(k))$ and the free travelling time is modified as follows:

$$T_{f_i}(k) = \frac{l_i - 5(n_i(k) - \tilde{y}_i(k))}{v_i}, \quad i = 1,\ldots,I \text{ and } k \in N^+. \quad (26)$$

The travelling time is determined according to Eqs. (21)–(26) as follows:

$$\tau_i(k) = \frac{l_i - 5(n_i(k) - \tilde{y}_i(k))}{v_i} + t_a + t_r(n_i(k) - \tilde{y}_i(k)) + \frac{5}{v_i}(n_i(k) - \tilde{y}_i(k) - 1), \quad \text{for } i = 1,\ldots,I \text{ and } k \in N^+. \quad (27)$$

Experimental evidence shows that (27) determines a satisfactory approximation of the travelling time in the presence of congestion, i.e., when $l_i/v_i < T_{ci}(k)$. On the contrary, the travelling time may be approximated by (21) when few vehicles are in the link. Summing up, the following evaluation for the travelling time is obtained:

$$\tau_i(k) = \begin{cases} \dfrac{l_i - 5\left(n_i(k) - min\left(\left\lfloor \dfrac{l_i - 5(n_i(k)-1) - t_a v_i}{v_i t_r + 5}\right\rfloor, 0\right)\right)}{v_i} + t_a + t_r\left(n_i(k) - min\left(\left\lfloor \dfrac{l_i - 5(n_i(k)-1) - t_a v_i}{v_i t_r + 5}\right\rfloor, 0\right)\right) & \\ \quad + \dfrac{5}{v_i}\left(n_i(k) - min\left(\left\lfloor \dfrac{l_i - 5(n_i(k)-1) - t_a v_i}{v_i t_r + 5}\right\rfloor, 0\right) - 1\right) & \text{if } \dfrac{l_i}{v_i} < T_{c_i}(k) \\ \dfrac{l_i}{v_i} & \text{if } \dfrac{l_i}{v_i} \geqslant T_{c_i}(k) \end{cases}$$

with $i = 1, \ldots, I$ and $k \in N^+$. (28)

In order to use a constant value of $\phi_{i,j}$, (31) is averaged over K^* cycles and hence

$$\phi_{i,j} = \frac{1}{K^*}\sum_{k=1}^{K^*} \frac{v_i t^{\bar{f}}(k) + 5 - t_a v_i}{(v_i t_r + 5) t^{\bar{f}}(k)}$$

for each $i \in I_{no}$ and $j \in I^i_{out}$. (32)

2.4. Traffic congestion level evaluation

A key parameter in the model is the term $\phi_{i,j} \in \Re^+$ in the expression (9) of $Q^f_{i,j}(k)$ as a function of the effective time. Such parameter represents the traffic scenario and depends in principle on the considered cycle. A simple off-line procedure to evaluate a constant approximation of $\phi_{i,j}$ by way of observations in a chosen number of K^* cycles is as follows.

Consider the generic non-output link L_i with $i \in I_{no}$ and an additional link in the area L_j, with $j \in I^i_{out}$. The interval time $t^{\bar{f}}(k)$ represents the duration of the green phase $\bar{f}$ for the stream of vehicles travelling from L_i to L_j in the kth cycle. More precisely, $t^{\bar{f}}(k)$ may include at least one amber time $t^{f_1}(k)$ with $f_1 \in \{1, \ldots, F\}$, in which a few vehicles cross the intersection. Then denoting by $\tilde{Q}_{i,j}(k)$ an approximation of $Q^{\bar{f}}_{i,j}(k)$ and assuming that the average length of a vehicle is 5 m, it holds:

$$t^{\bar{f}}(k) = t_a + \tilde{Q}_{i,j}(k)\, t_r + \frac{5(\tilde{Q}_{i,j}(k) - 1)}{v_i}$$

for each $i \in I_{no}$, $j \in I^i_{out}$ and $k \in N^+$ (29)

and trivial calculations lead to

$$\tilde{Q}_{i,j}(k) = \frac{v_i t^{\bar{f}}(k) + 5 - t_a v_i}{v_i t_r + 5}$$

for each $i \in I_{no}$, $j \in I^i_{out}$ and $k \in N^+$. (30)

Dividing (30) by $t^{\bar{f}}(k)$, according to (9) an approximation of $\phi_{i,j}$ for the kth cycle is obtained:

$$\phi_{i,j}(k) = \frac{v_i t^{\bar{f}}(k) + 5 - t_a v_i}{(v_i t_r + 5) t^{\bar{f}}(k)}$$

for each $i \in I_{no}$, $j \in I^i_{out}$ and $k \in N^+$. (31)

2.5. Amber lights and intergreen times

In order to realistically model American and most European signalized urban areas, the considered signal timing plans include green lights (signalling clear way), red lights (corresponding to a stop signal) and amber lights (corresponding to a caution signal after green and before red). In addition, the so-called lost time or intergreen times are taken into account, i.e., short duration phases in which all traffic lights in one intersection are red, in order to let vehicles, previously allowed to occupy the crossing area and late due to congestion, clear the junction. Including such intergreen times in the signal timing plan is crucial when modelling congested areas with a complex topology. According to the urban areas regulations of most European and American countries (Rinelli, 2000), fixed amber times from 1 to 5 s and fixed lost times from 1 to 2 s are selected.

2.6. Vehicle classification

Different vehicles in the area are classified adopting the passenger car as standard unit. Hence, all vehicles are assessed in terms of passenger car units (PCU) by vehicle profile classifier detectors (Kamata & Oda, 1991) and the proposed classification of vehicles is shown in Table 1. By expressing vehicles in the area in terms of PCU, the differences in dimensions are taken into account.

3. The actuated traffic control strategy

The proposed traffic controller is based on the knowledge of the actual traffic demand. Considering

Table 1
Passenger car units conversion factors

Type of vehicle	PCU
Private cars, taxis, light private goods vehicles and trucks under 5*t*	1.0
Motorcycles, scooters and mopeds	0.5
Buses, coaches and trucks over 5*t*	3.0
Multiple axles rigid or articulated trucks	5.0

an optimization horizon of K cycles, the controller estimates every C seconds the UTN state variables $n_i(k)$ with $i = 1,\ldots,I$ and $k = 1,\ldots,K$ and minimizes an objective function over the last KC seconds by determining the new green phases durations. Hence, at each decision time instant the controller chooses a strategy so as to minimize a performance index during the finite horizon and applies the selected phases to the signalized area starting from the next K cycles. In order to minimize the risk of oversaturation and the spillback of link queues, the following objective function is defined. For each input and intermediate link L_i, the mean number of vehicles over K cycles in PCU is

$$OF_i(K) = \frac{1}{K}\left[\sum_{k=1}^{K} n_i(k)\right], \quad \text{for each } i \in I_{no}. \tag{33}$$

The control objective is to minimize the number of vehicles in the UTN in the optimization horizon, i.e., to minimize the following objective function (Barisone et al., 2002):

$$\min_{t^f(k)} OF(K) = \min_{t^f(k)} \sum_{i \in I_{no}} OF_i(K), \tag{34}$$

subject to (1)–(10), (12), (13a) and (13b). In other words, solving the mathematical programming problem (34) means finding the phase durations (the control inputs) $t^f(k)$ for $k = 1\ldots,K$, that minimize the average number of vehicles over K cycles in the area.

An additional constraint is the following:

$$t^f_{min} \leqslant t^f(k) \leqslant t^f_{max},$$
$$\text{with } f = 1,\ldots,F,\ k = 1,\ldots,K, \tag{35}$$

where t^f_{min} and t^f_{max} are the minimum and maximum durations of phase f. Clearly, (35) forces the optimization problem to reject solutions proposing extremely short or long phases, which may be hard to put up with for regular drivers. In addition, if phase f corresponds to amber light or intergreen time, then it holds $t^f_{min} = t^f_{max}$, so that the phase duration is constant in the optimization problem. Finally, in order to fix the cycle time to the pre-set value C, the following constraint is included:

$$\sum_{f=1}^{F} t^f(k) = C, \quad k = 1,\ldots,K. \tag{36}$$

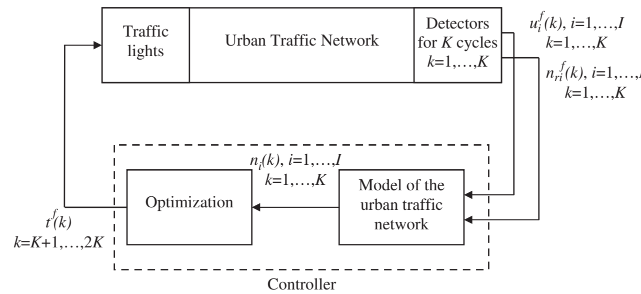

Fig. 3. The actuated traffic control strategy based on the proposed model.

The proposed traffic actuated control strategy is schematically represented in Fig. 3. The UTN is controlled in each time interval of KC seconds using as control inputs the phase durations determined by the controller at the previous decision time instant. During the interval of KC seconds, detectors feed into the controller the necessary measurements, i.e., the number of vehicles entering the input links and the number of rows of pedestrians crossing the streams of vehicles in each link. On the basis of such specific UTN input measurements that can be viewed as disturbances, the model evaluates the UTN state in each discrete time interval. Hence, a mathematical programming problem minimizes every KC seconds the objective function (34) subject to the defined constraints and the controller determines the optimized green phase durations $t^f(k)$ to be used for the next K cycles. The problem solution is obtained by a standard commercial optimization software using a generalized reduced gradient method (Lasdon, Waren, Jain, & Ratner, 1978; Lasdon & Smith, 1992). Despite the problem non-linearity, the limited number of required measurements as inputs to the controller and the short execution time of the optimization procedure make the presented traffic control strategy particularly attractive for use in real applications. However, note that the proposed approach may be employed not only on-line, i.e., to produce a traffic actuated signal timing plan, but also off-line, i.e., to generate some signal timing plans on the basis of historical data. More precisely, different traffic conditions can be characterized and a fixed optimized timing plan can be assigned to each traffic scenario. Consequently, the suited signal timing plan may be activated in fixed times of the day or dynamically selected on the basis of the current traffic conditions.

4. Coordination of traffic control signals

The control strategy presented in the previous section does not address the issue of coordinating local traffic control signals. This section discusses the problem of synchronizing intersections located in the same direction in a signalized urban area in order to allow uninterrupted flow of traffic, i.e., the offset specification. More precisely, the offset between two signalized intersections is defined as the duration from the start of a green phase at one signal to the start of the following nearest (in time) green phase at the other signal. Note that in its current form the proposed strategy to select offsets is designed to ensure vehicle progression in one direction.

Consider two successive intersections in one direction and suppose that link L_i in the area connects the two junctions, i.e., $L_i \in L_{int}$. In addition, $O_i(k)$ denotes the offset between the corresponding green phases during the generic cycle k. Most commonly in the literature the suggested offset value is constant as follows (Di Febbraro, Giglio, & Sacco, 2002):

$$O_i = \frac{l_i}{v_i} \quad \text{for each } i \in I_{int}. \tag{37}$$

Hence, the suggested offset equals the constant approximation of the travelling time in the link defined by (21). However, the previous expression is realistic when traffic is not congested. On the contrary, when vehicles line up in queues in the link, the offset should take into account the time to drain the link: in other words, the travelling time in the link increases due to congestion and (37) becomes impractical. Hence, a suitable selection of the offset is derived as follows.

Suppose that $n_i(k)$ is the state at the kth cycle of link $L_i \in L_{int}$ connecting two junctions and consider the clearance time $T_{ci}(k)$, defined by (23), and the free travelling time $T_{fi}(k)$, defined by (26). Hence, the following different situations may occur when a vehicle enters the link.

- $T_{fi}(k) > T_{ci}(k)$. The free link travelling time is greater than the clearance time. In such a case, in order to avoid disruption of the traffic flow in the link and create a green wave for vehicles entering , it is necessary to postpone the beginning of the green phase at the downstream junction and the recommended offset is $O_i(k) = (T_{fi}(k) - T_{ci}(k))$.
- $T_{fi}(k) \leqslant T_{ci}(k)$. The free link travelling time is lower than or equal to the clearance time. In other words, the link is congested. Therefore, in order to drain part of the queue in the link while vehicles from the upstream intersection proceed to the downstream one, the green lights in the successive junctions are simultaneously activated, i.e., $O_i(k) = 0$ is selected.

Summing up, the obtained heuristic formula for the offset determination to be included in the controller specification is the following:

$$O_i = \frac{1}{K}\sum_{k=1}^{K} max((T_{fi}(k) - T_{ci}(k)), 0)$$
$$\text{for each } i \in I_{int} \tag{38a}$$

where the evaluated offset is averaged over the optimization horizon. Eq. (38a) represents the recommended choice for the offset between the green phases of two coordinated intersections. However, the proposed offset selection method may easily be extended to consider several successive junctions located in one direction. Since the offset represents the duration from the start of a green phase f_{i1} at one traffic light to the following start of a green phase f_{i2} at the other traffic light, (38a) may be rewritten as follows:

$$\sum_{f=f_{i1}}^{f_{i2}-1} t^f(k) = min\left(max\left(\frac{1}{K}\sum_{k=1}^{K} max((T_{fi}(k), -T_{ci}(k)), 0) \sum_{f=f_{i1}}^{f_{i2}-1} t^f_{min} \right), \sum_{f=f_{i1}}^{f_{i2}-1} t^f_{max} \right)$$
$$\text{with } k = 1, \ldots, K \quad \text{for each } i \in I_{int}, \tag{38b}$$

where the phases bounds (35) are taken into account. Obviously, (38b) becomes an additional constraint of the optimization problem (34).

5. The case study

5.1. Description of the signalized urban area

Fig. 4 illustrates the topology of a real signalized area comprising two successive intersections. The proposed case study is located in the urban area of Bari (Italy) and is characterized by severe traffic congestion. In particular, the area consists of two intersections, regularly crossed by cars, trucks, public transportation buses and mopeds. Adopting the formalism introduced in Section 3, the signalized area in Fig. 4 is modelled with 13 links, including 6 input links (L_1, L_2, L_3, L_4, L_5, L_6), 5 output links (L_8, L_9, L_{10}, L_{12}, L_{13}) and 2 intermediate links (L_7, L_{11}). The intermediate links exhibit the same capacity $N_7 = N_{11} = 22$ PCU. The whole area is usually congested in rush hours, with occasional spillback phenomena. In particular, links L_5 and L_7 are often congested, so that the last vehicles in the queues generally cross the corresponding intersection only after two cycles. Clearly, such congestion phenomena differ with the time of the day. Hence, in the following several timing plans are considered, each corresponding to a specific time interval in a weekday: the morning peak TS 1 (07.30–10.00 am, oversaturated traffic), day time TS 2

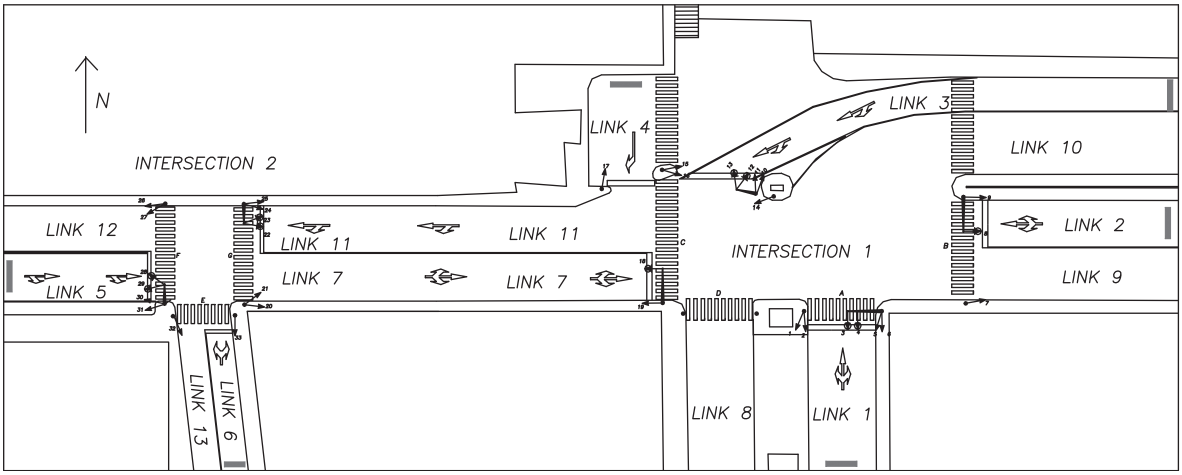

Fig. 4. Layout of the signalized area, including two coordinated intersections: grey rectangles indicate vehicles detectors.

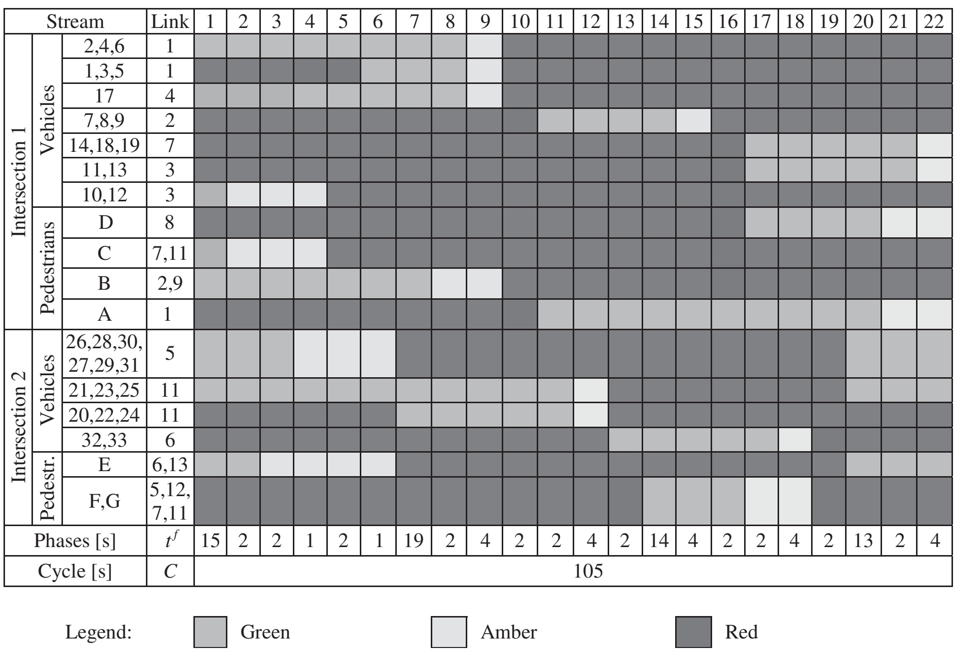

Fig. 5. Signal timing plan of the signalized area before optimization.

(10.00–12.00 am and 05.00–07.00 pm, under-saturated traffic), the noon peak TS 3 (12.00 am–02.00 pm, oversaturated traffic) and the evening peak TS 4 (07.00–09.00 pm, oversaturated traffic). All the other times of the day are disregarded in the present study, since they correspond to low demand traffic. The two junctions, respectively, named intersections 1 and 2 (see Fig. 4), are currently controlled by a heuristically determined timing plan with cycle time $C = 105\,\text{s}$.

Fig. 5 reports the fixed signal timing plan that is currently implemented in the area and Fig. 6 depicts the allowed turning movements. In particular, the streams allowed to proceed during the phases of the signal timing plan in Fig. 5 are depicted in Fig. 6 and labelled with letters and numbers corresponding to those indicated in Figs. 4 and 5. More precisely, pedestrian streams are depicted with letters from A to G, while vehicle streams are indicated with numbers from 1 to 33. Moreover, amber and intergreen times are taken into account in the present study, so that the considered fixed timing plan comprises 22 phases, including 13 fixed-length amber phases and 3 lost times (i.e., phases 10 and 16 for intersection 1 and phase 19 for intersection 2). Overall 6 green phase durations, namely $t^1(k)$, $t^7(k)$, $t^{11}(k)$, $t^{13}(k)$, $t^{14}(k)$ and $t^{20}(k)$, may be determined by the proposed control strategy with $t^1_{min} = t^7_{min} = t^{14}_{min} = t^{20}_{min} = 5\,\text{s}$, $t^{11}_{min} = t^{13}_{min} = 2\,\text{s}$. In addition, from keeping constant the cycle time, amber and lost times, $t^1_{max} =$

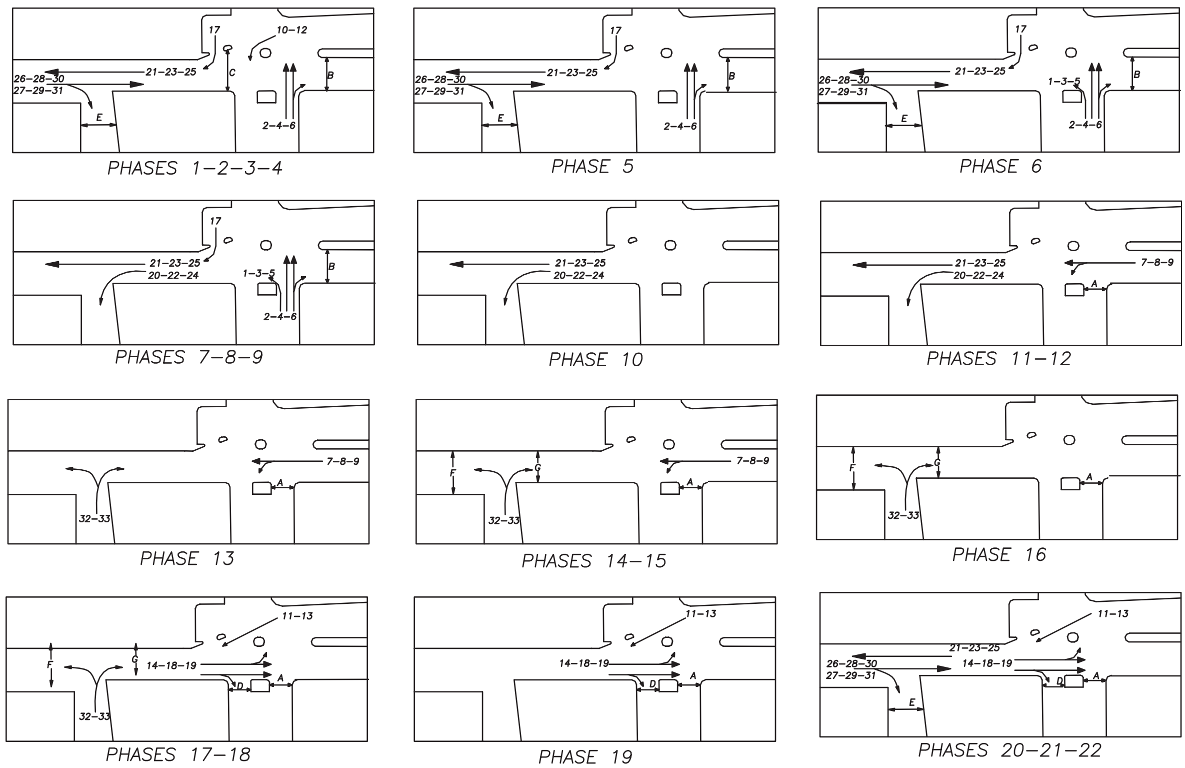

Fig. 6. Vehicle and pedestrian streams in each phase of the timing plan for the signalized urban area.

$t^7_{max} = t^{11}_{max} = t^{13}_{max} = t^{14}_{max} = t^{20}_{max} = 40\,\text{s}$ is obtained. Moreover, the parameters of the signalized area are: $t_a = 3\,\text{s}$, $t_r = 1.1\,\text{s}$ (Rinelli, 2000), $v_i = 50\,\text{km/h}$ for $i = 1, \ldots, I$; $v_{h,z} = 50\,\text{km/h}$ and $x_{h,z} = 7\,\text{m}$ for (h,z) such that $z \in I^h_{out}$ and $S^f_{h,z}(k)$ has right of way over $S^f_{i,j}(k)$; $v_p = 1\,\text{m/s}$; $l_{ci} = 3\,\text{m}$. Furthermore, the link lengths l_i with $i = 1, \ldots I$ are listed in Table 2, Table 3 reports the turning rates $\beta_{i,j}$ with $j \in I^i_{out}$ and Table 4 shows parameters $\Phi_{i,j}$ with $j \in I^i_{out}$.

5.2. The results

The introduced optimization model is applied to the case study on a personal computer in a standard commercial optimization software. For each of the considered times of the day and for $K = 15$ cycles, on the basis of real data (i.e., information on pedestrian and vehicle streams) collected on the signalized area the control strategy solves the optimization problem (34) subject to the recalled constraints to determine the signal timing plan. Using a 1 GHz Pentium III processor equipped with 256 MB RAM, the execution time of the optimization program for the case study equals 30 s.

To evaluate the obtained results, the following performance indices are used:

$$OF(15) = \sum_{i \in I_{no}} \frac{1}{15} \left[\sum_{k=1}^{15} n_i(k) \right], \tag{39}$$

Table 2
Link lengths [m]

l_1	l_2	l_3	l_4	l_5	l_6	l_7	l_8	l_9	l_{10}	l_{11}	l_{12}	l_{13}
80	50	110	8	110	20	55	80	50	100	55	110	20

Table 3
Turning rates

$\beta_{i,j}$	Output and intermediate links						
	$j = 8$	$j = 9$	$j = 10$	$j = 12$	$j = 13$	$j = 7$	$j = 11$
Input and intermediate links							
$i = 1$	0.12	0.04	0.57				0.27
$i = 2$	0.31		0.07				0.62
$i = 3$	0.60						0.40
$i = 4$							1.00
$i = 5$					0.10	0.90	
$i = 6$				0.30		0.70	
$i = 7$	0.08	0.52	0.40				
$i = 11$				0.90	0.10		

$$OF_i(15) = \frac{1}{15} \left[\sum_{k=1}^{15} n_i(k) \right] \quad \text{for each } i \in I_{no} \tag{40}$$

with $I_{no} = \{1, 2, 3, 4, 5, 6, 7, 11\}$. Index (39) denotes the average number of vehicles per cycle in the area and indices (40) are the average number of vehicles per cycle in each intermediate and input link. All the previous

Table 4
Traffic congestion level parameters

$\phi_{i,j}$	Output and intermediate links						
	$j=8$	$j=9$	$j=10$	$j=12$	$j=13$	$j=7$	$j=11$
Input and intermediate links							
$i=1$	1.20	1.89	1.76				1.24
$i=2$	1.09		1.08				1.08
$i=3$	0.90						1.35
$i=4$							0.54
$i=5$					1.08	1.08	
$i=6$				1.08		1.08	
$i=7$	1.08	1.08	1.08				
$i=11$				0.60	5.40		

Table 5
Green phase durations [s] after optimization with fixed offset

Phase number	TS1	TS2	TS3	TS4
1	10.4	5.3	9.5	16.6
7	16.0	17.8	13.8	5.0
11	8.0	2.2	2.0	2.0
13	2.0	2.4	2.0	2.0
14	6.6	11.9	9.8	9.2
20	22.0	25.4	27.9	30.2

performance indices are expressed in PCU. In addition, the average percentage utilization per cycle corresponding to each intermediate link is considered as follows:

$$P_i(15) = 100\frac{OF_i(15)}{N_i}$$

for each $i \in I_{int} = \{7, 11\}$. (41)

Table 5 reports the green phases obtained for the considered time slots after optimization and with fixed offset. More precisely, the offset between the beginning of the green phase of L_5 and the start of the green phase of L_7, i.e., between the beginning of $t^{17}(k)$ and the start of $t^{20}(k)$, is heuristically selected and fixed to $O_7 = t^{17}(k) + t^{18}(k) + t^{19}(k) = 8\,\text{s}$, i.e., the summation of three phases that are not changed by the control strategy (since phases 17 and 18 are amber and 19 is a lost time for intersection (2). Tables 6–9 show the results for each timing plan before and after optimization for all the different time slots. In particular, the second column in Tables 6–9 reports the performance indices (39)–(41) before optimization (i.e., adopting the fixed timing plan in Fig. 5), whereas the third column reports results after optimization. From an analysis of the second column in the tables it is apparent that several

Table 6
Morning peak (TS 1) results

Performance indices (PCU)	Before optimization	After optimization (fixed offset)	After optimization (variable offset)	Percentage variation
$OF(15)$	80.60	65.14	62.86	−22.0
$OF_1(15)$	25.68	29.09	29.5	14.9
$OF_2(15)$	8.49	9.87	12.18	43.5
$OF_3(15)$	1.51	1.61	1.49	−1.3
$OF_4(15)$	0.10	0.10	0.10	0.0
$OF_5(15)$	21.14	11.06	6.56	−69.0
$OF_6(15)$	4.48	3.01	2.24	−50.0
$OF_7(15)$	12.08	9.15	7.74	−35.9
$P_7(15)$	54.9%	41.6%	35.2%	−35.9
$OF_{11}(15)$	7.11	1.27	3.05	−57.1
$P_{11}(15)$	32.3%	5.8%	13.9%	−57.1

Table 7
Day traffic (TS 2) results

Performance indices (PCU)	Before optimization	After optimization (fixed offset)	After optimization (variable offset)	Percentage variation
$OF(15)$	46.73	36.36	36.25	−22.4
$OF_1(15)$	16.44	17.64	17.7	7.7
$OF_2(15)$	6.70	8.41	8.34	24.5
$OF_3(15)$	1.51	1.71	1.66	9.9
$OF_4(15)$	0.10	0.10	0.1	0.0
$OF_5(15)$	4.86	1.32	1.33	−72.6
$OF_6(15)$	0.98	0.87	0.87	−11.2
$OF_7(15)$	10.90	5.28	5.2	−52.3
$P_7(15)$	49.6%	24.0%	23.6%	−52.3
$OF_{11}(15)$	5.25	1.04	1.06	−79.8
$P_{11}(15)$	23.9%	7.7%	4.8%	−79.8

Table 8
Noon peak (TS 3) results

Performance indices (PCU)	Before optimization	After optimization (fixed offset)	After optimization (variable offset)	Percentage variation
$OF(15)$	67.95	52.42	46.16	−32.1
$OF_1(15)$	18.25	19.14	19.41	6.4
$OF_2(15)$	5.53	7.36	8.03	45.2
$OF_3(15)$	6.25	7.02	5.11	−18.2
$OF_4(15)$	0.07	0.07	0.07	0.0
$OF_5(15)$	12.06	6.27	2.5	−79.3
$OF_6(15)$	7.27	5.07	3.69	−49.2
$OF_7(15)$	11.49	5.52	5.08	−55.8
$P_7(15)$	52.2%	25.1%	23.1%	−55.8
$OF_{11}(15)$	7.02	1.96	2.27	−67.7
$P_{11}(15)$	31.9%	8.9%	10.3%	−67.7

Table 9
Evening peak (TS 4) results

Performance indices (PCU)	Before optimization	After optimization (fixed offset)	After optimization (variable offset)	Percentage variation
$OF(15)$	107.22	73.35	65.02	−39.4
$OF_1(15)$	21.60	22.80	22.64	4.8
$OF_2(15)$	5.55	7.47	8.3	49.5
$OF_3(15)$	10.76	11.11	8.24	−23.4
$OF_4(15)$	0.07	0.07	0.07	0.0
$OF_5(15)$	38.58	18.42	12.64	−67.2
$OF_6(15)$	9.21	4.63	3.96	−57.0
$OF_7(15)$	12.08	5.83	5.07	−58.0
$P_7(15)$	54.9%	26.5%	23.0%	−58.0
$OF_{11}(15)$	9.37	3.01	4.1	−56.2
$P_{11}(15)$	42.6%	13.7%	18.6%	−56.2

Table 10
Green phase durations [s] after optimization with variable offset

Phase number	TS1	TS2	TS3	TS4
1	9.8	5.0	10.0	11.4
7	16.2	17.7	12.0	8.2
11	2.4	2.5	2.0	2.0
13	2.4	2.6	2.0	2.0
14	10.6	11.7	6.3	7.3
19	3.9	2.3	9.3	9.3
20	21.7	25.2	25.4	26.8

links are often congested, especially in the time slots corresponding to rush hours (i.e., TS1, TS3 and TS4). However, the third column in Tables 6–9 shows that adopting the green phases determined with the proposed control strategy leads to a significant improvement in the signalized area congestion, i.e., to lower values of $OF(15)$, in each time slot. In addition, for most links performance indices $OF_i(15)$ with $i \in I_{no}$ and $P_i(15)$ with $i \in I_{int}$ decrease after optimization.

In order to improve the synchronization of the intersections, a second optimization test is performed with variable offset, i.e., letting $t^{19}(k)$ vary with $t^{19}_{min} = 2$s and appropriate values of $t_{\max}{}^f(k)$ for $f = 1, 7, 11, 13, 14, 19, 20$. Moreover, the corresponding values of the modified phases obtained in the different time slots employing the offset specified by (38b) are reported in Table 10. The fourth columns of Tables 6–9 show the obtained performance indices and the last column of Tables 6–9 reports the improvement in terms of percentage variation of the indices obtained with optimization and variable offset with respect to the values of the second column.

As a summary, Fig. 7 reports the values of performance index (39) for the different time slots before and after optimization and enlightens the obtained improvement.

6. Conclusion

This paper provides a contribution to the issue of signal control for UTN including coordinated

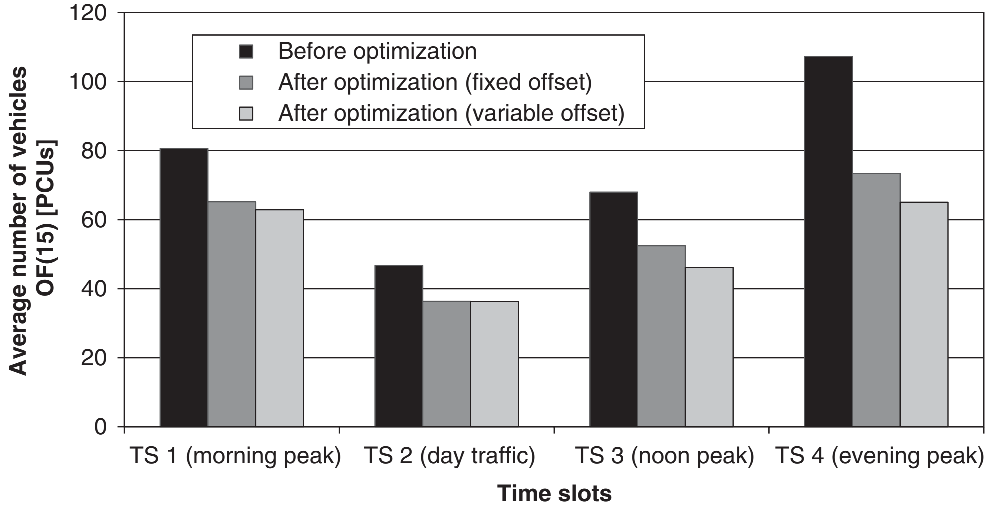

Fig. 7. Objective function values in the various time slots under different signal timing plans.

intersections. An actuated traffic control strategy is proposed in order to minimize the risk of oversaturation and spillback in links. Starting from a macroscopic model proposed in the related literature (Barisone et al., 2002), the UTN model is revised and extended to obtain a more accurate system description and to take into account the traffic scenarios, the different types of vehicles in the area, as well as pedestrians. In addition, a heuristic method determines the offset value to improve coordination of subsequent intersections, allowing uninterrupted flow of vehicles. The proposed optimization procedure works in real time and provides an actuated signal timing plan on the basis of a limited number of measurements, that allow to evaluate the discrete time state of the intersection area. The stated mathematical programming problem can be solved by a standard commercial optimization software with short execution time, so that the proposed traffic actuated control strategy is suited for real time applications.

Comparing the resulting methodology with the model-based traffic-responsive strategies present in the related literature, the following points are enlightened. Firstly, the traffic-responsive urban control problem is usually formulated as a combinatorial optimization problem and exponential-complexity algorithms determine a global minimum. On the contrary, the short execution time of the optimization procedure introduced in this paper allows an efficient real time application. Secondly, the method can be compared with the control strategy proposed in Diakaki et al. (2002), that requires the on-line accurate measurement of the number of vehicles in all the considered network links (e.g., through a video detection system) and sets the green phases on the basis of such measurements. Despite the efficiency of such a methodology, the control law can be difficult to implement if a dedicated high-level instrumentation is not available. On the contrary, even if the control strategy presented in this paper has a centralized architecture, the controller has to store few data (i.e., the number of vehicles entering the input links) to evaluate the complete state of the UTN. Finally, the optimization technique addresses new elements such as the presence of pedestrians and the synchronization of subsequent intersections.

The signal control technique is applied to an urban area located in the city of Bari (Italy), which includes two intersections. Results show the ability of the real time control strategy to optimize the signal timing plan on the basis of knowledge of the traffic condition, even for traffic scenarios with high congestion. The presented approach may be employed both on-line, i.e., to produce a traffic actuated signal timing plan, and off-line, i.e., to generate a signal timing plan on the basis of historical data, that may either be activated in fixed times of the day or dynamically selected on the basis of the actual traffic conditions.

Future perspectives include simulating the proposed traffic actuated control strategy on an extended signalized area and eventually comparing the proposed method with alternative techniques proposed in the related literature. Moreover, further research will address the optimization of offsets to improve vehicle progression in different directions of a traffic network.

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