7장 3| MIXED EFFECT MODEL

SAS를 이용한 실험 계획과 분산 분석 (자유아카데미)

MIXED EFFECT MODEL

- 혼합모형(Mixed Models) = 고정효과(fixed effect)와 임의효과(random effect)가 공존하는 모형

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk},$$

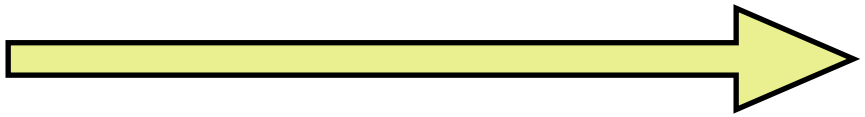
$$\alpha_i \sim \text{i.i.d. } N(0, \sigma_\alpha^2)$$ A 효과=random effect

$$\sum_{j=1}^{b} \beta_j = 0$$ B 효과=fixed effect

$$(\alpha\beta)_{ij} \sim \text{i.i.d. } N(0, \sigma_{\alpha\beta}^2)$$ AB효과=random effect

$$\epsilon_{ijk} \sim \text{i.i.d. } N(0, \sigma^2)$$

$\alpha_i,\ (\alpha\beta)_{ij},\ \epsilon_{ijk}$ are independent



```
proc glm data=a;
    class A B;
        model y=A B A*B;
        random A A*B / test;
run;
```

혼합모형에서 E(MS) 의한 검정

```
proc glm data=a;
     …
     random A A*B /test;
```

The GLM Procedure

Source	Type III Expected Mean Square
A	Var(Error)+2 Var(A*B)+4 Var(A)
B	Var(Error)+2 Var(A*B)+ Q(b)
A*B	Var(Error)+2 Var(A*B)

$$
\begin{aligned}
\alpha_i &\sim \text{i.i.d. } N(0, \sigma_\alpha^2) \\
\sum_{j=1}^{b} \beta_j &= 0 \\
(\alpha\beta)_{ij} &\sim \text{i.i.d. } N(0, \sigma_{\alpha\beta}^2) \\
\epsilon_{ijk} &\sim \text{i.i.d. } N(0, \sigma^2) \\
i &= 1, 2, \\
j &= 1, 2, \\
k &= 1, 2,
\end{aligned}
$$

$$
\begin{aligned}
E(MSA) &= \sigma^2 + 2\sigma_{\alpha\beta}^2 + 4\sigma_\alpha^2 \\
E(MSB) &= \sigma^2 + 2\sigma_{\alpha\beta}^2 + \frac{4}{2-1}\sum_{j=1}^{2} \beta_j^2 \\
E(MSAB) &= \sigma^2 + 2\sigma_{\alpha\beta}^2 \\
E(MSE) &= \sigma^2
\end{aligned}
$$

(공장에서 생산되는 라인에 불량품 수를 측정할 때, 기계의 종류(A1,A2,A3)간에 차이가 있는지, 혹은 기능공 간에 차이(B1, B2)가 있는지를 검정하고자 하여 공장내 작업하는 기계중 임의로 3개를 선택하고, 기능공 중에 2명을 임의로 선택하여 실험을 실시하였고, 각 처리조합당 3회의 반복실험을 하였다.

Possible Design??

```
proc glm data=a;  class A B;
        model y=A B A*B;
        random A B A*B / test;
    run;
```

	A1	A2	A3
B1	20	14	13
	18	18	16
	14	14	13
B2	19	12	9
	20	12	4
	20	9	4

Dependent Variable: y

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	369.8333333	73.9666667	15.13	<.0001
Error	12	58.6666667	4.8888889		
Corrected Total	17	428.5000000			

R-Square	Coeff Var	Root MSE	y Mean
0.863088	15.98373	2.211083	13.83333

Source	DF	Type I SS	Mean Square	F Value	Pr > F
A	2	229.3333333	114.6666667	23.45	<.0001
B	1	53.3888889	53.3888889	10.92	0.0063
A*B	2	87.1111111	43.5555556	8.91	0.0042

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A	2	229.3333333	114.6666667	23.45	<.0001
B	1	53.3888889	53.3888889	10.92	0.0063
A*B	2	87.1111111	43.5555556	8.91	0.0042

The GLM Procedure

Source	Type III Expected Mean Square
A	Var(Error) + 3 Var(A*B) + 6 Var(A)
B	Var(Error) + 3 Var(A*B) + 9 Var(B)
A*B	Var(Error) + 3 Var(A*B)

The GLM Procedure
Tests of Hypotheses for Random Model Analysis of Variance

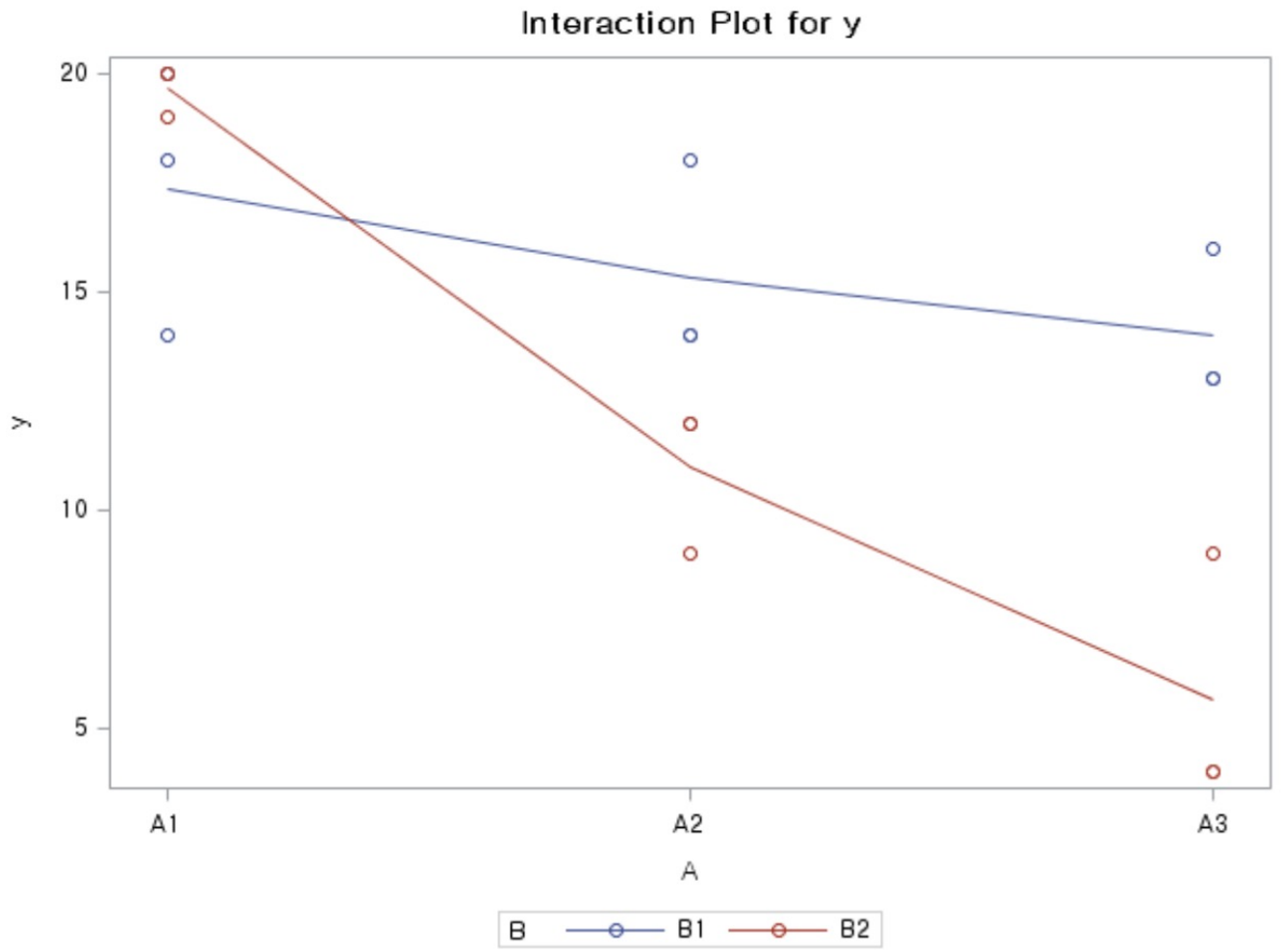
Dependent Variable: y

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A	2	229.333333	114.666667	2.63	0.2753
B	1	53.388889	53.388889	1.23	0.3836
Error: MS(A*B)	2	87.111111	43.555556		

Source	DF	Type III SS	Mean Square	F Value	Pr > F
A*B	2	87.111111	43.555556	8.91	0.0042
Error: MS(Error)	12	58.666667	4.888889		

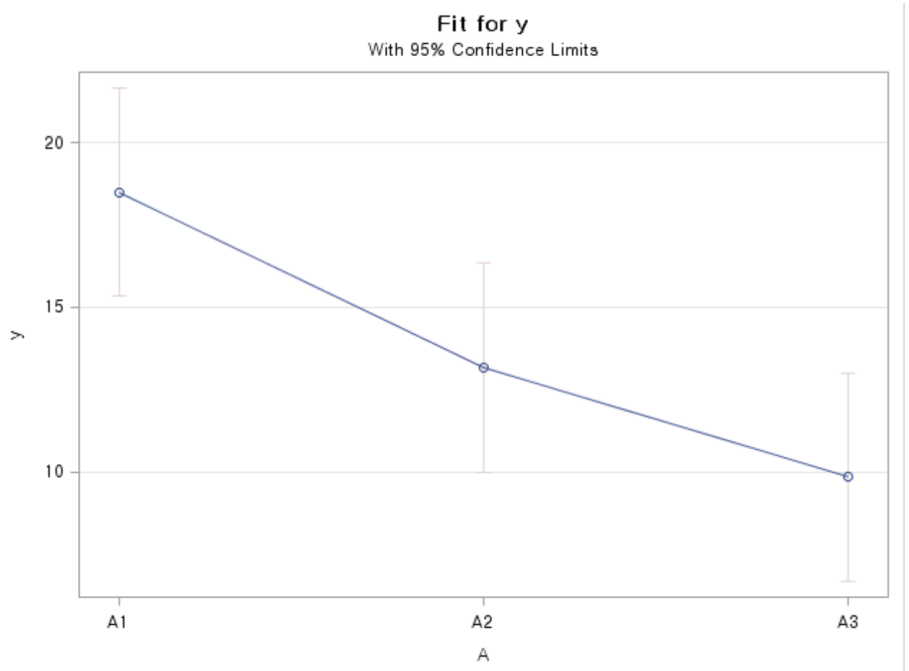
- A요인 효과(기계효과) : 유의하지 않음
- B요인 효과(기능공 효과) :유의하지 않음
- AB 상호작용효과 : 유의함

상호작용 효과 그래프

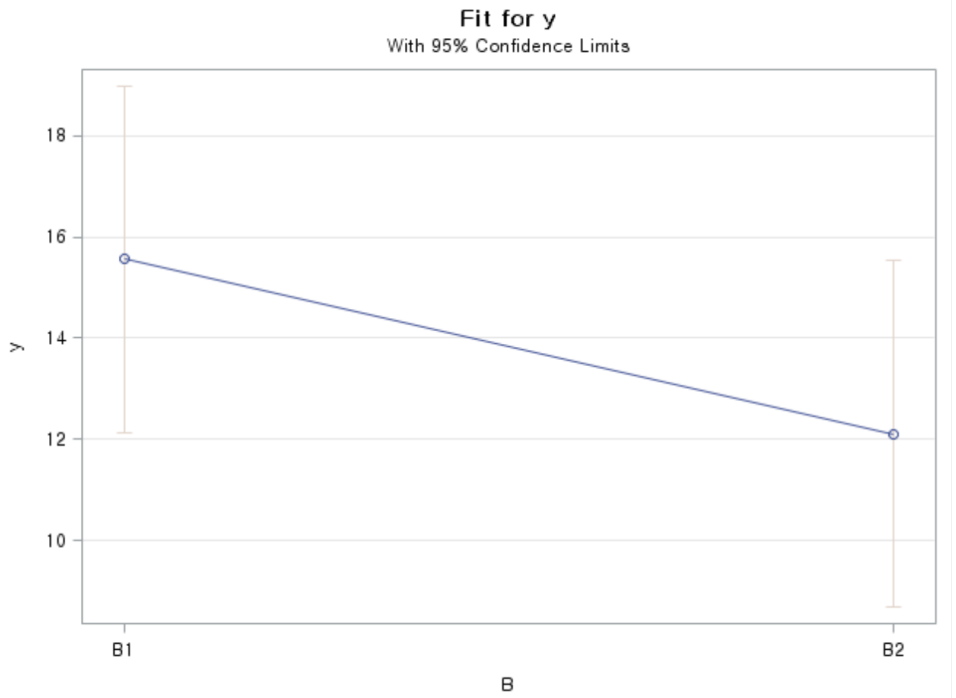


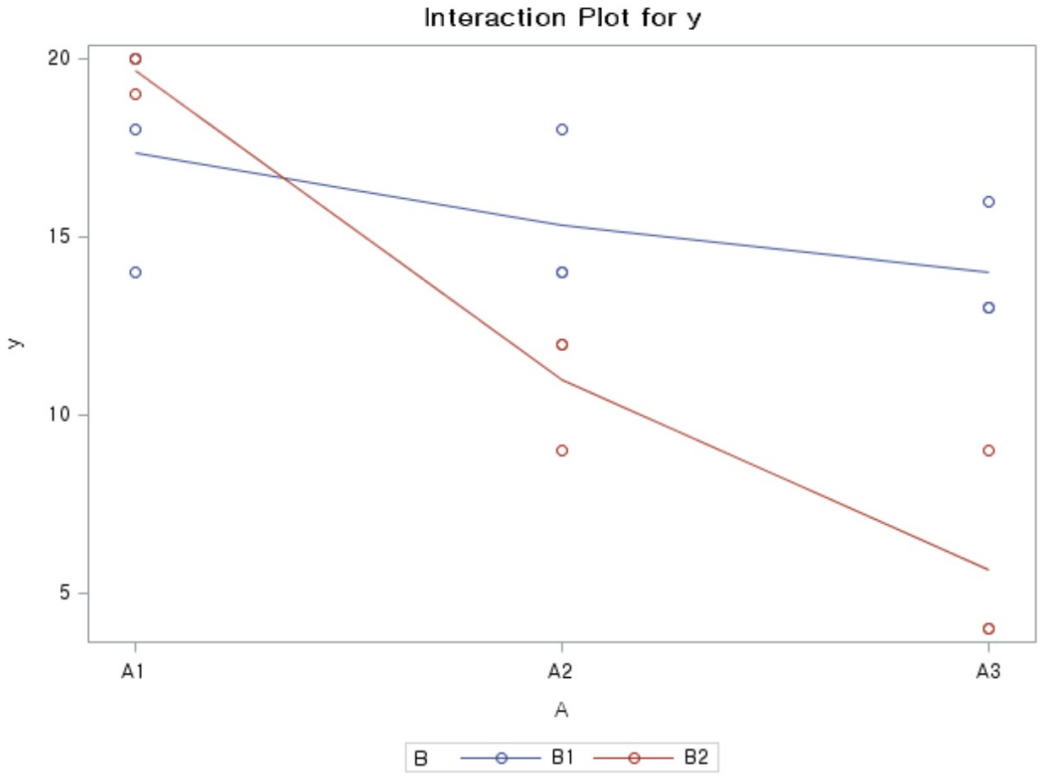
주효과 그래프

```
proc glm data=a;
    class A ;
        model y=A ;
        random A ;
     run;
```




```
proc glm data=a;
    class B ;
        model y=B ;
        random B ;
     run;
```





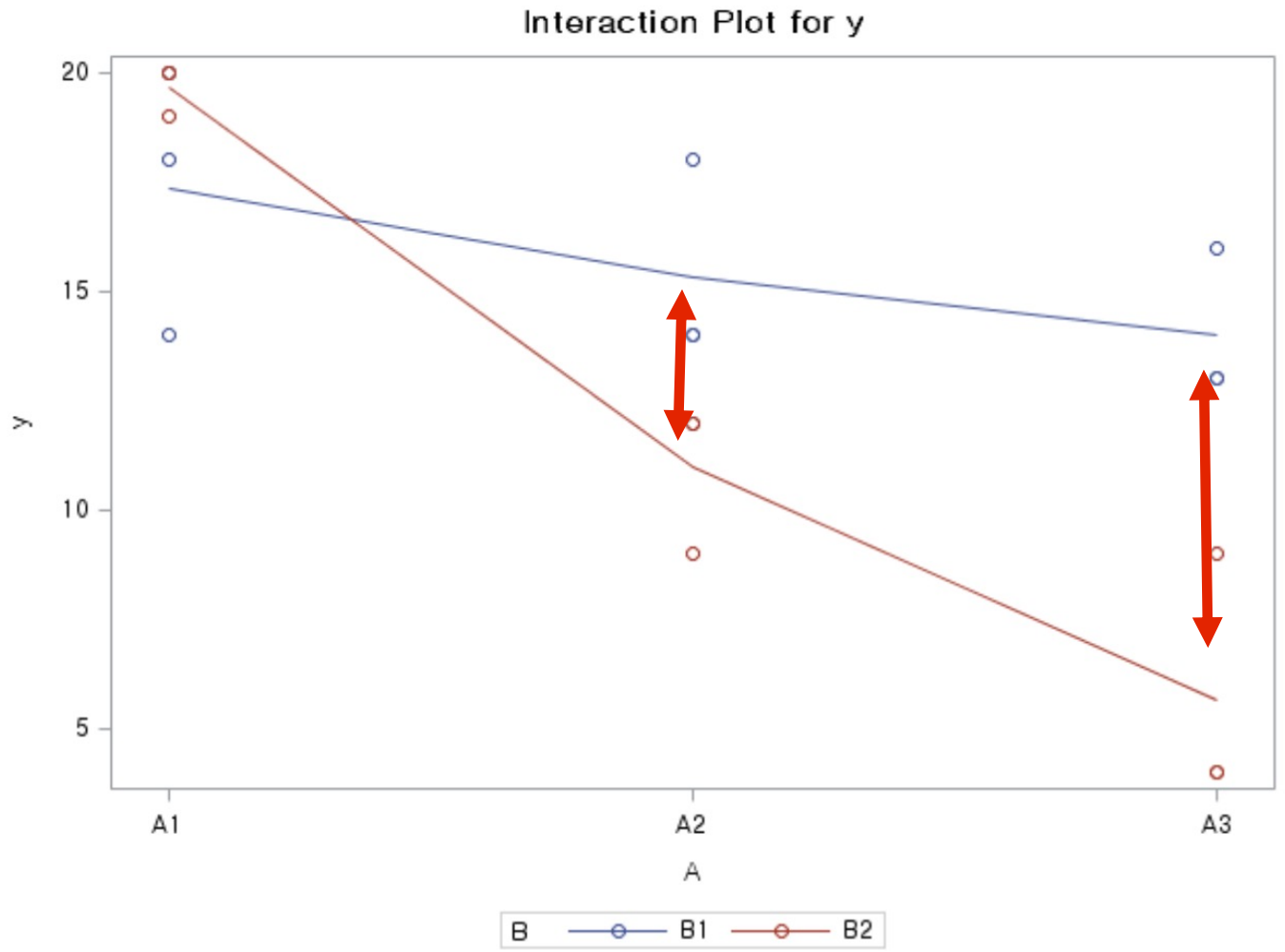
- ☑상호작용 존재
- ☑주효과 의미없음
- ☑조건부 주효과 해석 필요
- ☑slice option (SAS)

```
proc glm data=a;
class A B;
model y=A B A*B;
random A B A*B ;
lsmeans                    run;
```

The GLM Procedure
Least Squares Means

A*B Effect Sliced by A for y

A	DF	Sum of Squares	Mean Square	F Value	Pr > F
1	1	8.166667	8.166667	1.67	0.2205
2	1	28.166667	28.166667	5.76	0.0335
3	1	104.166667	104.166667	21.31	0.0006



이원배치법 다중비교

- AB 효과 유의하지 않을 때
  - ✓A효과 유의하면 : lsmeans A / pdiff=all;
  - ✓B효과 유의하면 : lsmeans B / pdiff=all;
- AB 효과 유의할 때
  - ✓lsmeans A*B / pdiff=all ;

```
proc glm data=a;
class A B;
model y=A B A*B;
random A B A*B ;
lsmeans A*B /pdiff=all adjust=tukey slice=A;
run;
```

The GLM Procedure
Least Squares Means
Adjustment for Multiple Comparisons: Tukey

A	B	y LSMEAN	LSMEAN Number
1	1	17.3333333	1
1	2	19.6666667	2
2	1	15.3333333	3
2	2	11.0000000	4
3	1	14.0000000	5
3	2	5.6666667	6

Least Squares Means for effect A*B
Pr > |t| for H0: LSMean(i)=LSMean(j)

Dependent Variable: y

i/j	1	2	3	4	5	6
1		0.7838	0.8691	0.0389	0.4748	0.0003
2	0.7838		0.2299	0.0045	0.0721	<.0001
3	0.8691	0.2299		0.2299	0.9728	0.0018
4	0.0389	0.0045	0.2299		0.5780	0.0975
5	0.4748	0.0721	0.9728	0.5780		0.0061
6	0.0003	<.0001	0.0018	0.0975	0.0061	

6	4	5	3	1	2
(A3,B2)	(A2,B2)	(A3,B1)	(A2,B1)	(A1,B1)	(A1,B2)
5.7	11	14	15.3	17.3	19.7