Linear Programming (poggers)

Very shortly put, it is simply optimizing a linear function w.r.t. linear constraints.

The matrix form of a maximization problem would be:

$$\text{Maximize } c^T x \tag{1}$$
$$\text{Subject to } Ax \leq b \tag{2}$$

Or, they could be split into many constraints instead, with both $\leq$, $\geq$, and $=$ between them.

The decision variables are x. An assignment to them is a solution. A solution satisfying the constraints is feasible. A feasible solution that actually minimizes/maximizes is optimal (sometimes denoted by an asterisk, i.e. $P*$).

The set of feasible solutions, called F, is a convex polyhedron. That means it has n sides and has no two points such that a line may be drawn between them that crosses outside of the polyhedron. Usually, you need to traverse to a corner of the polyhedron to attain an optimal solution.

Two stinky cases: unbounded solutions, where an arbitrarily good solution may be found, and infeasible LP's, where the set of feasible solutions is empty.

I know what you're thinking at this point: "That's really cool, Jonathan, but what is the standard form of a linear program?" Well, here it is!

$$\text{Maximize } c_1x_1 + c_2x_2 + ... + c_nx_n \tag{3}$$
$$\text{Subject to} \tag{4}$$
$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n \leq b_1 \tag{5}$$
$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n \leq b_2 \tag{6}$$
$$... \tag{7}$$
$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n \leq b_m \tag{8}$$