Effect of Schooling on Wages

$y_i =$ log wages
$x_i =$ years of schooling
$a_i =$ ability

Short $\quad y_i = \alpha + x_i\beta + \varepsilon_i$
Long $\quad y_i = \alpha + x_i\beta + a_i\gamma + \eta$
$\Rightarrow \varepsilon_i = a_i\gamma + \eta_i$

i) $\eta_i \sim N(0,\sigma^2)$
ii) $E[\eta_i | x_i, a_i] = 0$

$$\hat\beta = \frac{\sum(x_i-\bar x)(y_i-\bar y)}{\sum(x_i-\bar x)^2}$$
$$= \frac{\sum(x_i-\bar x)(\beta(x_i-\bar x)+\varepsilon_i)}{\sum(x_i-\bar x)^2}$$
$$= \beta + \frac{\sum(x_i-\bar x)(a_i\gamma+\eta_i)}{\sum(x_i-\bar x)^2}\left(\frac{1/n}{1/n}\right)$$
$$= \beta + \frac{\frac{1}{n}\sum(x_i-\bar x)a_i\gamma}{\hat\sigma^2_x} + \frac{\frac{1}{n}\sum\eta_i}{\hat\sigma^2_x}$$
$$E[\hat\beta | a_i, x_i] = \beta + \underbrace{\frac{\frac{1}{n}\sum(x_i-\bar x)a_i\hat\gamma}{\sigma^2_x}}_{\text{Bias}} + 0$$

1) $\sigma^2_x > 0$
2) $\frac{1}{n}\sum(x_i-\bar x)a_i \approx \text{cov}(x_i,a_i) \gtrless 0$
3) $\gamma \lesseqgtr 0$

In this application
1) $\text{cov}(x_i,a_i) > 0$ on average "signaling"
2) $\gamma > 0$

$\rightarrow \hat\beta$ positively biased