Intro to Interactive Theorem Proving

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October 5th, 2017

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- Proofs are too long to check carefully!
- Seeing what we proved so far helps during the proof process.

Examples

- Four color theorem (required checking 1,936 cases)
  Appel & Haken 1976 with computer assistance
  Werner & Gonthier 2015 with Coq

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  Appel & Haken 1976 with computer assistance
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- Kepler conjecture (hexagonal close packing of spheres)
  Hales 1998 with computer assistance (C++ etc.)
  Hales 2014 with HOL Light and Isabelle (Flyspeck)

How?

- Curry-Howard isomorphism

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- Calculus of inductive constructions

Curry-Howard Isomorphism

for simply typed λ-calculus and intuitionistic prop logic,

- Types are propositions!

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$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \to B} \qquad \frac{\Gamma, x : A \vdash t : B}{\Gamma \vdash \lambda x.t : A \to B}$$

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- Extension of Curry-Howard to a higher-order type theory
- Allows ∀ and ∃ in your types

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- Allows $\forall$ and $\exists$ in your types

Note

CIC is constructive, but you can add the law of excluded middle as an axiom and prove classical theorems.

Some Proof Assistants

- Isabelle (1986) (tactics!)
- Coq (1989) (tactics!)
- Agda (2007)