EVALUATION

ONLINE LEARNING

LINKS WITH OPTIMIZATION AND GAMES

UNIVERSITÉ PARIS–SACLAY

AN ALTERNATIVE TO VOVK–AZOURY–WARMUTH

Consider the online linear regression problem, where the loss functions are of the form

$$\ell_t(x) = \frac{1}{2}(\langle w_t, x\rangle - z_t)^2, \quad w_t \in \mathbb{R}^d,\ z_t \in \mathbb{R},\ t \geqslant 0.$$

We consider

$$\mathrm{H}_t(x) = \frac{1}{2}x^\top \left(\lambda \mathrm{I} + \sum_{s=0}^{t+1} w_s w_s^\top\right), \quad x \in \mathbb{R}^d,\ t \geqslant 0,$$

and the algorithm starting at some $x_0 \in \mathbb{R}^d$ and giving:

$$x_{t+1} = \operatorname*{arg\,max}_{x\in\mathbb{R}^d} \left\{\langle \nabla \mathrm{H}_t(x_t) + z_t w_t, x\rangle - \mathrm{H}_t(x)\right\}, \quad t \geqslant 0.$$

The goal of the project is to compare the practical performance of Vovk-Azoury-Warmuth (VAW) from the course and the above algorithm.

1) Prove that the above algorithm is well-defined and that it can be written as UMD iterates.

2) Implement the above algorithm and VAW. *Note that each iteration of both algorithms is in itself an optimization problem.*

3) Choose at least two real-life labeled regression datasets of reasonable size and plot the regret performance of both algorithms. Try many different values for λ on a logarithmic grid.