Evaluation
Online learning
links with optimization and games
Université Paris–Saclay

a simple bluffing game

1) Let $\mathscr{T}$ be a treeplex as defined in the course. For $\theta \in \Theta$ and $a \in \mathscr{A}_\theta$, denote $\mathbb{1}_{[\theta,a]}$ the element in $\Delta(\mathscr{A}_\theta)$ that is the Dirac at a. Prove that $\mathscr{T}$ is the set of convex combinations of the points from the following set

$$\mathscr{T}_0 = \left\{ x \in \mathbb{R}_+^\Sigma,\ \forall \theta \in \Theta,\ \exists a \in \mathscr{A}_a,\ x_{[\theta]} = x_{[p(\theta)]} \mathbb{1}_{[\theta,a]}. \right\} .$$

Therefore, an useful consequence is that for all $v \in \mathbb{R}^\Sigma$,

$$\max_{x \in \mathscr{T}} \langle v, x \rangle = \max_{x \in \mathscr{T}_0} \langle v, x \rangle .$$

We consider the following two-player zero-sum game. Player 1, can be one of two types: *strong* (with probability $p \in [0,1]$) or *weak* (with probability $1-p$). Player 1 knows his type and Player 2 does not know the type of Player 1. Player 2 does not have a type. Then, Player 1 chooses a signal to send to Player 2: either *strengh* or *weakness*. Then, Player 2 chooses *to fight* or *not to fight*. If Player 2 chooses *not to fight*, each player gets payoff 0. If Player 2 chooses *to fight* whereas Player 1 is *strong*, Player 1 gets payoff 1 (and Player 2 gets payoff -1). If Player 2 chooses *to fight* whereas Player 1 is *weak* and has signalled *weakness*, Player 1 gets payoff −1 (and Player 2 gets payoff 1). If Player 2 chooses *to fight* whereas Player 1 is *weak* and has signalled *strengh*, Player 1 gets payoff −2 (and Player 2 gets payoff 2).

2) Formally write the game as an extensive-form game.

3) Solve the game using the CFR and CFR+ algorithms. Plot the convergence. Describe the obtained approximate solutions. Try different values for p.