Evaluation
Online learning
links with optimization and games
Université Paris–Saclay

approachability-based optimization

We first reduce online linear optimization on a convex compact set to an approachability problem and consider Blackwell and greedy Blackwell algorithms in this context. Those algorithms are then converted into optimization algorithms.

Let $\mathcal{A} \subset \mathbb{R}^d$ be a nonempty convex compact set. We consider online linear optimization on $\mathcal{A}$: for sequences $(a_t)_{t\geqslant 0}$ and $(u_t)_{t\geqslant 0}$ in $\mathcal{A}$ and $\mathbb{R}^d$ respectively, the quantity of interest is the following regret:

$$\max_{a\in\mathcal{A}} \sum_{t=0}^{\mathrm{T}} \langle u_t, a - a_t\rangle\,, \qquad \mathrm{T} \geqslant 0. \tag{1}$$

Consider the auxiliary approachability problem where $\mathcal{A}$ and $\mathcal{B} = \mathbb{R}^d$ are the actions sets, and where outcome function $g : \mathcal{A} \times \mathcal{B} \to \mathbb{R}^{d+1}$ is defined as

$$g(a, u) = (u, \langle u, x\rangle), \qquad a \in \mathcal{A},\ u \in \mathbb{R}^d.$$

Consider $\mathcal{X}_0 = \mathcal{A} \times \{-1\}$ and $\mathcal{C} = \mathcal{X}_0^\circ$.

1) Prove that $\mathcal{C}$ satisfies Blackwell's condition and give an associated oracle.

2) Relate the regret (1) with the above approachability problem.

3) In the approachability problem, consider Blackwell and greedy Blackwell algorithms and write corresponding guarantees. Deduce guarantees on the regret (1).

4) Convert the regret minimization algorithms thus obtained into constrained convex optimization algorithms on $\mathcal{A}$ and derive guarantees.

5) Conduct numerical experiments and compare the above algorithms to classical algorithms. *A possible setting is e.g. an SVM constrained on a closed Euclidean ball on a small (but not tiny) dataset—this is only a suggestion. Ideally, consider two or three different settings.*

6) Bonus. — Also apply to stochastic optimization.