

EVALUATION
ONLINE LEARNING
LINKS WITH OPTIMIZATION AND GAMES
UNIVERSITÉ PARIS–SACLAY



A HYBRID OF MIRROR DESCENT AND DUAL AVERAGING

This project studies an algorithm which shares similarities with mirror descent with step-sizes and dual averaging with parameters.

Let $d \geq 1$, \mathcal{X} a nonempty closed convex subset of \mathbb{R}^d , H a mirror map compatible with \mathcal{X} , $(u_t)_{t \geq 1}$ be a sequence in \mathbb{R}^d , $(\eta_t)_{t \geq 1}$ a positive sequence, and $x_1 \in \mathcal{X} \cap \text{dom } H$. Then define

$$x_{t+1} = \arg \max_{x \in \mathcal{X}} \left\{ \langle \nabla H(x_t) + \eta_t u_t, x \rangle - \frac{\eta_t}{\eta_{t+1}} H(x) \right\}, \quad t \geq 1.$$

One special case of interest is when $(\eta_t)_{t \geq 1}$ is nonincreasing.

- 1) Prove that the above can be seen as UMD iterates.
- 2) Is there cases where the above are MD with step-sizes? DA with parameters?
- 3) Derive regret bounds.
- 4) Derive guarantees for Lipschitz convex optimization.

- 5) For a Lipschitz convex optimization problem, perform numerical experiments to compare the performance of the above algorithm (in the case where H is the Euclidean mirror map) with corresponding MD with nonincreasing step-sizes and DA with nonincreasing parameters. *A possible setting is an SVM with no regularization, constrained in e.g. a Euclidean ball, but this is only a suggestion.*
- 6) BONUS. — Derive guarantees and perform numerical experiments for (constrained) smooth convex optimization. *Possible settings are least-squares linear regression and logistic regression.*

