Evaluation
Online learning
links with optimization and games
Université Paris–Saclay

iterates based on squared mahalanobis norms

Let $d \geqslant 1$. Recall that for a symmetric positive definite matrix A of size $d \times d$, the associated Mahalanobis norm is defined as

$$\|x\|_{\mathrm{A}} = \sqrt{\langle x, \mathrm{A}x \rangle}, \qquad x \in \mathbb{R}^d.$$

Let $\mathscr{X} \subset \mathbb{R}^d$ be a nonempty closed convex set, $(\mathrm{A}_t)_{t \geqslant 0}$ a sequence of symmetric positive definite matrices of size $d \times d$, and $(u_t)_{t \geqslant 0}$ a sequence in $\mathbb{R}^d$.

1) Let $x_0 \in \mathscr{X}$ and

$$x_{t+1} = \operatorname*{arg\,min}_{x \in \mathscr{X}} \left\| (x_t + \mathrm{A}_t^{-1} u_t) - x \right\|_{\mathrm{A}_t}, \qquad t \geqslant 0.$$

   Prove that the above are UMD iterates and derive bounds on the regret $\sum_{t=0}^{\mathrm{T}} \langle u_t, x - x_t \rangle$ for $\mathrm{T} \geqslant 0$ and $x \in \mathscr{X}$.

2) Let $y_0 \in \mathbb{R}^d$ and now define

$$x_t = \operatorname*{arg\,min}_{x \in \mathscr{X}} \left\{ - \left\langle y_0 + \sum_{s=0}^{t-1} u_s, x \right\rangle + \frac{1}{2} \langle x, \mathrm{A}_t x \rangle \right\}, \qquad t \geqslant 0.$$

   Same question as above.