MODULE 1: SOLAR HOUSE DESIGN AND SIMULATION

QUANTITATIVE ENGINEERING ANALYSIS 3

Passive Solar House Heat Modelling using Partial Differentiation

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Contents

Chapter 1

Introduction

1.1 Introduction

This report will provide a description of a passive solar house, along with a modeled heat transfer diagram of the house using differential equations and the use of MATLAB. With the modelled design, we will answer some questions that may be answered by the change of materials and dimensions of the house, and in turn optimize the passive solar house for a comfortable living temperature. The main questions posed are:

- How does the storage unit thickness affect the house's thermal behavior and why?
- How does the insulation thickness affect the house's thermal behavior and why?
- What is an optimal material to use for making wall in our model?

In this report, there will be a description of a mathematical representation of a system of heat transfer between the sunlight, the house and the outdoor environment. Then using the model we have created using the relationship between them, we will test the outcomes of the questions posed above. The pretense of the project is that the house is to be a passive solar house in Boston, with an outdoor temperature of -3 degrees Celsius, and with the sunlight angle at 25 degrees. The modeling is based on the winter of the Boston, and the primary goal of this project is to design a passive solar house that has reasonably comfortable indoor temperature of about 17-25 Celsius.

Chapter 2

Background

2.1 Passive Solar Houses

Solar houses are homes that are heated using the sun - without any other active sources of heat. Instead of using heats or other forms of heat producing methods, passive solar houses use the sunlight and the house's natural capability of storing the heat for keeping the indoors warm in winter and cool in the summer.

There are multiple factors that are needed in a passive solar house such as

- Aperture
- Absorber
- Thermal Mass
- Heat Distribution
- Control

Aperture is the method of taking in a certain amount of light. The house has to receive the most amount of sunlight during winter and the least amount of light during summer, the best way of achieving this is through a very large window facing south - if the house is located in a colder area. This will allow for maximum sunlight in the winter. However for summer this large window may be a big problem. This is solved through creating a shade that takes advantage of the height difference of the sun in winter and summer, where for summer, the angle of the sun is larger than of winter, letting an optimized shade be able to block the sun in summer and let it in when it is winter.
While letting in sunlight is the most important for a passive solar house, it is useless if the house cannot absorb any of the heat. Therefore the floor, wall, or partition of the house can be materials that can absorb heat, like a black painted tile.
With the heat absorbed into the house, the heat must be maintained - the best method for this is by using thermal mass. Materials that are particularly good at maintaining heat, such as stone, can be used in the house as a heat capacitor or heat storage unit. This will allow the heat that the floor or wall absorbed maintain the heat for a long period of time, which can stabilize the temperature in the house for longer periods of time.
(Reardon) While the heat can transfer in the natural way from the heat capacitors to the walls, there are methods where the heat will travel through ducts and other means of heat transportation. This will allow heat to travel around the house and heat it up uniformly, or visa versa.
Finally, other factors that does not involve the house can be utilized for heat conservation or simply for blocking out additional heat. For example, the use of a tree in front of the house will let less heat enter into the house thought the window in summer due to its leaves and more heat enter due to the leaves falling off in the winter. (Energy)

2.2 Mathematical Equations

There are several differential equations we implemented in order to model this house.

Heat Transfer Equation:

$$Q = m * c * \Delta T \tag{2.1}$$

where m is the mass of the material, c is the specific heat of the material, and ΔT is the temperature difference of the material.

Heat Transfer Arithmetic:

$$Q_{total} = Q_{in} - Q_{out} \tag{2.2}$$

Heat Transfer (convection) with Temperature as variables:

$$Q = h * A * (T_{in} - Tout) \tag{2.3}$$

where h is the heat transfer coefficient and A is the area of the material.

Convection Resistance Equation:

$$R_{conv} = \frac{1}{h_x * A} \tag{2.4}$$

where h is the heat transfer coefficient and A is the area of the material.

Conduction Resistance Equation:

$$R_{cond} = \frac{L}{k_x * A} \tag{2.5}$$

where L is the width of the material, k_x is the thermal conductivity, and A is the area of the material.

Thermal Resistance in Series :

$$R_{series} = \sum_i R_i \tag{2.6}$$

where R_i are resistance of the resistors that are connected in series.

Thermal Resistance in Parallel :

$$R_{parallel} = (\sum_i R_i{}^{-1})^{-1} \tag{2.7}$$

where R_i are resistance of the resistors that are connected in parallel.

Total Heat Transfer Equation with Resistance:

$$Q = \frac{Q_{in} - Q_{out}}{R_{tot}} \tag{2.8}$$

where R_{tot} is the total resistance in the model.

2.3 Heat Transfer in Closed Areas

Heat is transferred through 2 different ways in this project. Then first in convection, and the second is conduction. Convection is a heat transfer method through some type of movement within a fluid due to the rising of hotter materials paired with decreasing cooler materials. This usually happens between the surface of a material to the air. Conduction is a heat transfer that happens within solids. This occurs within the materials that make up the heat transfer network. These two types of heat transfer will be the main types of heat transfer we will be considering in this report. We will calculate the change in heat within the room using a series of heat transfer equations and differential equations that will monitor the change in heat within the system.

Chapter 3

Design

3.1 Design of the House

The design of the basic passive solar house is very simple. This house will be set in the winter climate with its single, very wide window facing south, so it will receive the most amount of sunlight. the house will have the dimensions of the figure below, with a width of 5 meters, a length of 5.1 meters and a height of 3 meters. The shade will have a dimension of 0.9 meters, with the window's size 2.6 meters in height, the length of 5.1 meters. The shade will act as a buffer for the sunlight in the summer, but act not in the winter, where the sun's rays are angled at 25 degrees.

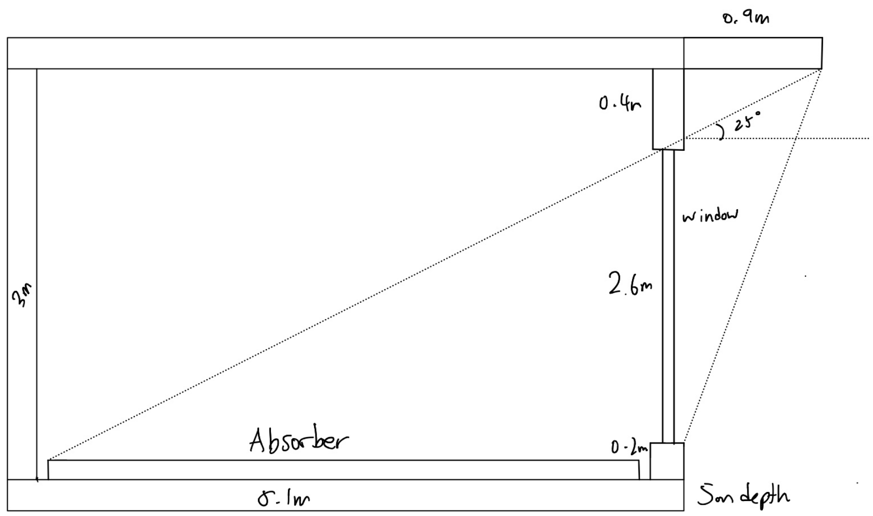

Figure 3.1: Design of the House

Below are the assumptions of our model:

- Heat storage unit (tile) has uniform temperature.
- The amount of air that flows into and out of the house is negligible.
- All solar radiation hitting window is absorbed by heat storage unit without losing any.
- The wall can't store heat, which means that there's no heat capacity.

From the tile, which is the heat absorber, to the outside temperature, the thermal resistance network of the house can be expressed as figure 3.2. The capacitor in the figure is the heat stored in the tile, and the first resistance refers to the thermal convection from the floor to the air in the house. Then, the next thermal resistances are connected in parallel because the heat flows to two different materials, wall and window to get to the outside air. One of the way to that heat flows to the outside is through wall, which goes through the thermal convection from air to wall, conduction in the wall, and the convection from wall to the outside. Another way is to go through the window, which heat transfers from air to window through convection, convection in the double-paned window, and the convection from window to air.

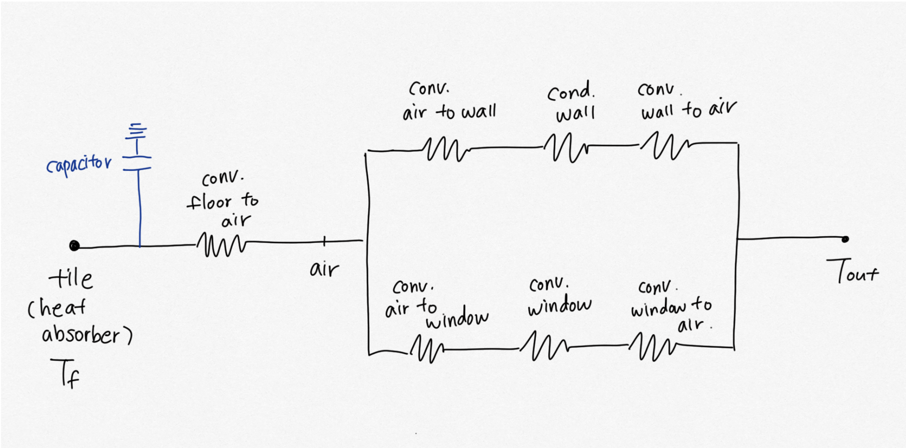

Figure 3.2: Thermal Resistance Network of the House

3.2 Specifics of Materials Used

The starting material used for wall in this design is concrete. The thermal conductivity of concrete is 0.4 W/m-K and the thickness of the wall to start with is 0.25m For the heat storage, the density and the specific heat of tile for energy storage is 3000 kg/m3, 800 J/kg-K, respectively. The thickness of the heat storage unit is 1.0m.

Chapter 4

Modeling

4.1 MATLAB Implementation

Algorithm 1: MATLAB code

Input: $tspan = [01] * 2400 * 3600$; %time in s
$T_f = -3$; %initial floor temp in C
$T_{out} = -3$; %temperature of outside air
$h_{in} = 15$; %heat transfer coefficients for indoor surfaces(W/m^2K)
$h_{out} = 30$; %heat transfer coefficients for outdoor surfaces(W/m^2K)
$h_{eq} = 0.7$; %heat transfer coecient of double-paned window (W/m^wK)
$A_f = 25.5$; %area of the floor (heat absorber) (m^2)
$A_{wall} = 73.1$; %area of the walls (m^2)
$A_{window} = 13$; %area of the window (m^2)
$L_{wall} = 0.2$; %thickness of the wall
$k_{wall} = 0.4$; %thermal conductivity of the wall

Equations: $R1 = 1/(h_{in} * A_f)$; %resistance of convection from floor to air
$R2 = 1/(h_{in} * A_{wall})$; %resistance of convection from air to wall
$R3 = L_{wall}/(k_{wall} * A_{wall})$; %resistance of conduction in the wall
$R4 = 1/(h_{out} * A_{wall})$; %resistance of convection from wall to outside
$R5 = 1/(h_{in} * A_{window})$; %resistance of convection from air to window
$R6 = 1/(h_{eq} * A_{window})$; %resistance of convection in double-paned window
$R7 = 1/(h_{out} * A_{window})$; %resistance of convection from window to outside
$R_p = 1/(1/(R2 + R3 + R4) + 1/(R5 + R6 + R7))$ %thermal resistance parallel
$R_{tot} = R1 + R_p$
$[t, T] = ode45(@(t, T)$timederiv$(t, T, R_tot), tspan, T_f)$;
$T_{air} = (R_p * T + R1 * T_{out})/(R1 + R_p)$;

Function $dTdt$ =*timederiv*(t, T, R_{tot})
  $Q_h = (-361 * cos(\pi * t/(12 * 3600)) + 224 * cos(\pi * t/(6 * 3600)) + 210)$; %W heater
  $T_{out} = -3$; % outdoor temp in C
  A = 13; % area of the window
  $C_f = 3000 * 800 * (25.5) * 1$; %heat capacity of the floor (J/K)
  dTdt= (Q$_h * A/C_f) - (T/(R_{tot} * C_f)) + (T_{out}/(R_{tot} * C_f))$;
end

4.2 Equations Used/Derived

Solar flux through south-facing window

$$Q_h = (-361 * cos(\pi * t/(12 * 3600)) + 224 * cos(\pi * t/(6 * 3600)) + 210) \tag{4.1}$$

where t is the time in seconds. Conservation of Energy

$$\frac{dU}{dt} = Q_{total} = Q_{in} - Q_{out} = m * c * \Delta T \tag{4.2}$$

where m is the mass of the material, c is the specific heat of the material, and ΔT is the temperature difference of the material. ODE(Ordinary Differential Equation) of the floor temperature

$$\frac{dT_f}{dt} = \frac{Q_h * A}{C_f} - \frac{T_f - T_{out}}{R_{tot} * C_f} \tag{4.3}$$

where T_f is the temperature of the floor, A is the area of the floor, C_f is the heat capacitance of the floor, T_{out} is the outside temperature, and R_{tot} is the total resistance of the thermal network. This equation is derived based on the equation of the conservation of energy. The equation of the indoor air temperature

$$T_{air} = \frac{R_p * T_f + R_1 * R_{tot}}{R_1 + R_p} \tag{4.4}$$

where R_p is the resistances that are connected in parallel, R_1 is the resistance of the convection from floor to wall, and R_{tot} is the total resistance of the thermal network. This equation is derived from the thermal network, where the heat transfer from the tile to the indoor air is equal to the heat transfer from the indoor air to the outdoor air.

4.3 Final Output

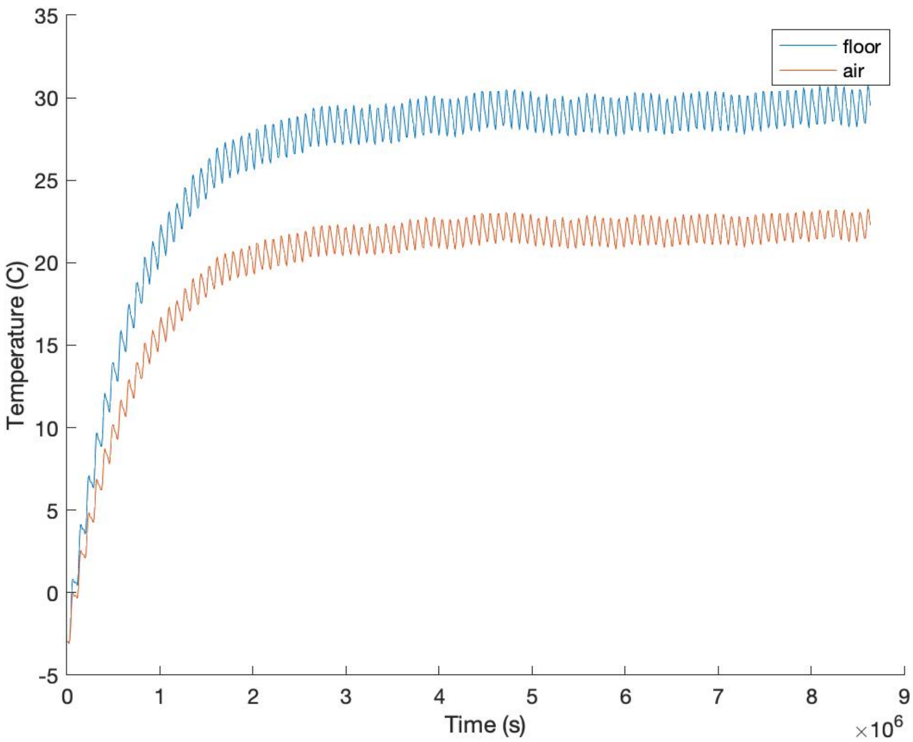

Figure 4.1: Time vs Temperature of floor and air

Figure 4.1 shows the plot of the temperature of the floor of the house and the air temperature inside the house in the span of $9 * 10^6$ seconds, or 100 days. The equilibrium of the temperatures reached by the model of the house seems to be at about 20 degrees Celsius to 23 degrees Celsius for indoor air temperature, and the floor seems to have a temperature between 26 to 29 degrees Celsius. Overall, this fits in the design requirements set by the outline of the project, and it is a very comfortable temperature to live in, considering that the location of the house is in Boston in the winter where the outdoor temperature is -3 degrees Celsius. The model shows that the equilibrium is reached at around the first 10 to 20 days, and the temperature is kept at a consistent comfortable temperature afterwards.

Chapter 5

Optimization

5.1 Material& Thickness Differences

Different materials have different heat capacity and the thermal conductivity. This means that the change in material will have a significant difference in the amount of heat it can maintain without losing it to the outdoor environment or can transfer in a certain amount of time. The materials of wall we will test consist of 4 completely different materials: Brick, Wood, Concrete and a type of Metal of out own choice: this chapter will outline the differences each material can make with different values of equilibrium temperature of the floor and the air. Also, the thickness of the walls or the heat storage unit (tile) pose a difference in the final equilibrium temperature, so we are going to investigate the differences the thickness of these parts of the house make in terms of the final equilibrium temperatures of the indoors.

5.2 Model Comparison

Equilibrium Temperature by Material		
Material	Thermal Conductivity ($W/m-K$)	Equilibrium Temperature (Air(x),Floor(y)) (C°)
Wood	0.04	$(115 \leq x \leq 120, 121 \leq y \leq 127)$
Concrete	0.4	$(20 \leq x \leq 23, 26 \leq y \leq 29)$
Clay brick	0.711	$(11 \leq x \leq 13, 18 \leq y \leq 21)$
Titanium	17	$(0.5 \leq x \leq 1.5, 7 \leq y \leq 9)$

Table 5.1: Material of wall is the default given by the design, Concrete [1]

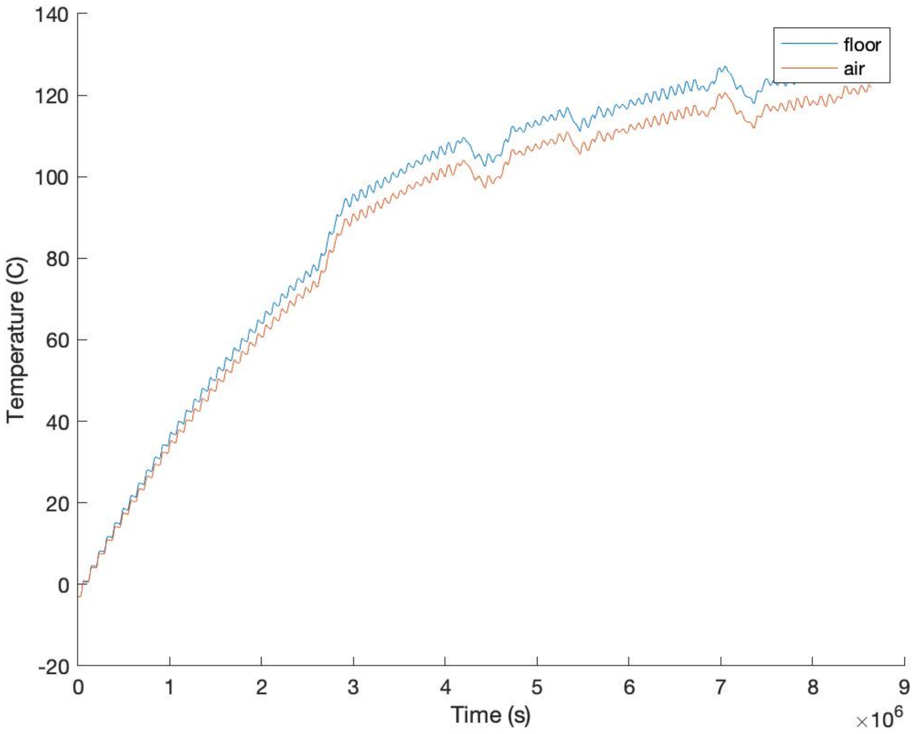

Figure 5.1: Time vs Temperature of floor and air when using wood for wall

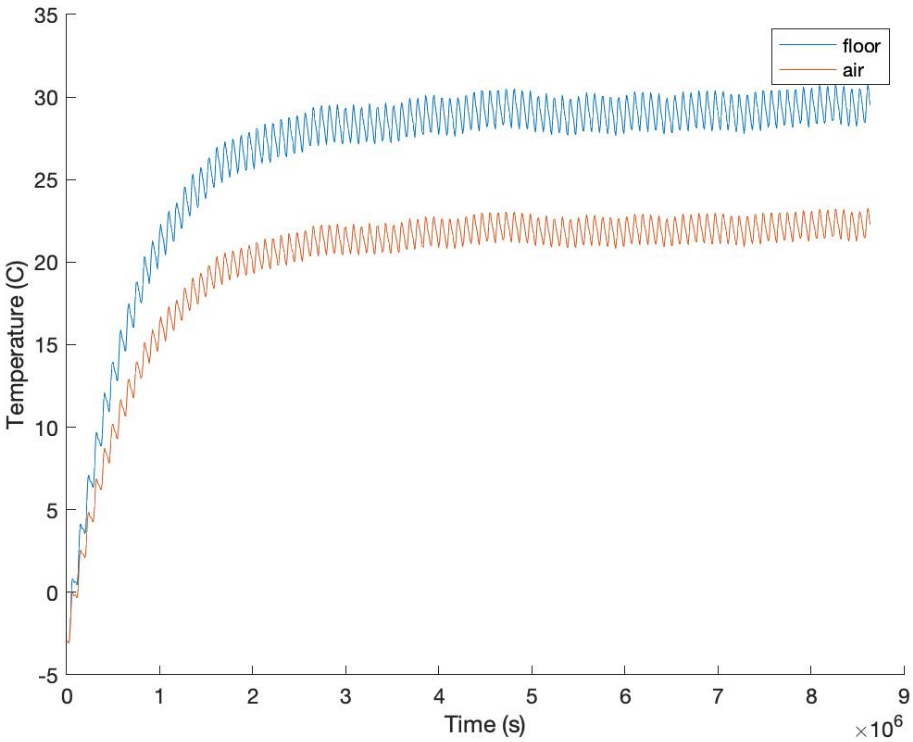

Figure 5.2: Time vs Temperature of floor and air when using Concrete for wall

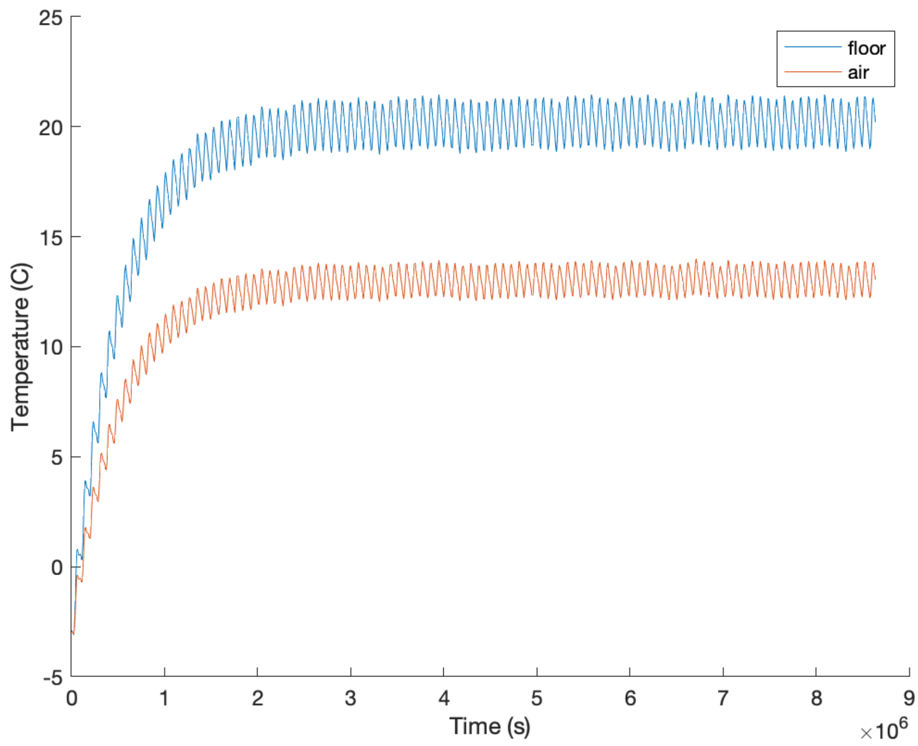

Figure 5.3: Time vs Temperature of floor and air when using clay brick for wall

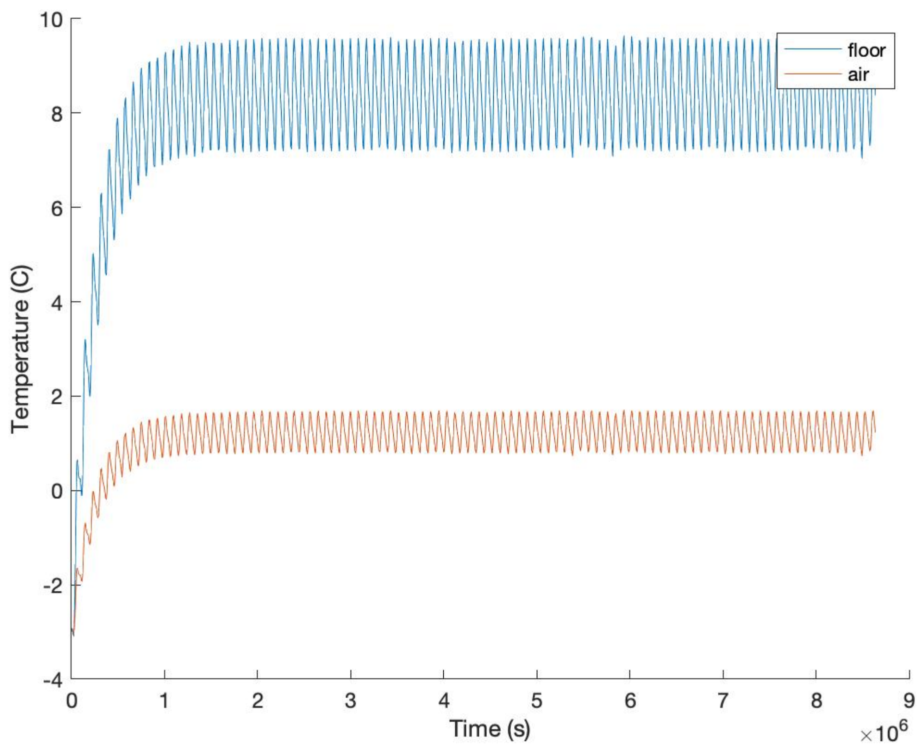

Figure 5.4: Time vs Temperature of floor and air when using Titanium for wall

Figure 5.1, 5.2, 5.3, and 5.4 are the graphs of time versus temperature of floor and air when using wood wall, concrete wall, clay brick wall, and titanium wall, respectively. The equilibrium temperature of the floor and the indoor air for each cases are in the table 5.1.

Observing the graphs, we can see two major relationships between the thermal conductivity of the wall and the temperature. The first thing that can be noticed is that the larger the thermal conductivity, the smaller the equilibrium temperature. This makes sense because thermal conductivity is the ability of the material to conduct heat. In other words, materials with high thermal conductivity are materials with high heat transfer rate, which means that the heat leaves the house faster, resulting the low equilibrium indoor temperature. Second, the larger the thermal conductivity, the faster the temperature reaches its equilibrium. For instance, the temperature reaches its equilibrium around the end of the 100th day of the simulation for wood wall, while the temperature reaches its equilibrium after around 10 days after the start. The relationship is reasonable since having higher thermal conductivity means that the heat transfer is faster, which makes the temperature to reach the equilibrium faster.

Equilibrium Temperature by Thickness of Insulator	
Thickness (m)	Equilibrium Temperature (Air(x),Floor(y)) ($C°$)
0.1	$(8 \leq x \leq 10, 15 \leq y \leq 17)$
0.25	$(20 \leq x \leq 23, 26 \leq y \leq 29)$
0.5	$(37 \leq x \leq 40, 43 \leq y \leq 45)$
0.75	$(55 \leq x \leq 58, 62 \leq y \leq 66)$

Table 5.2: The thickness of the tile is set as the default value given in the Design

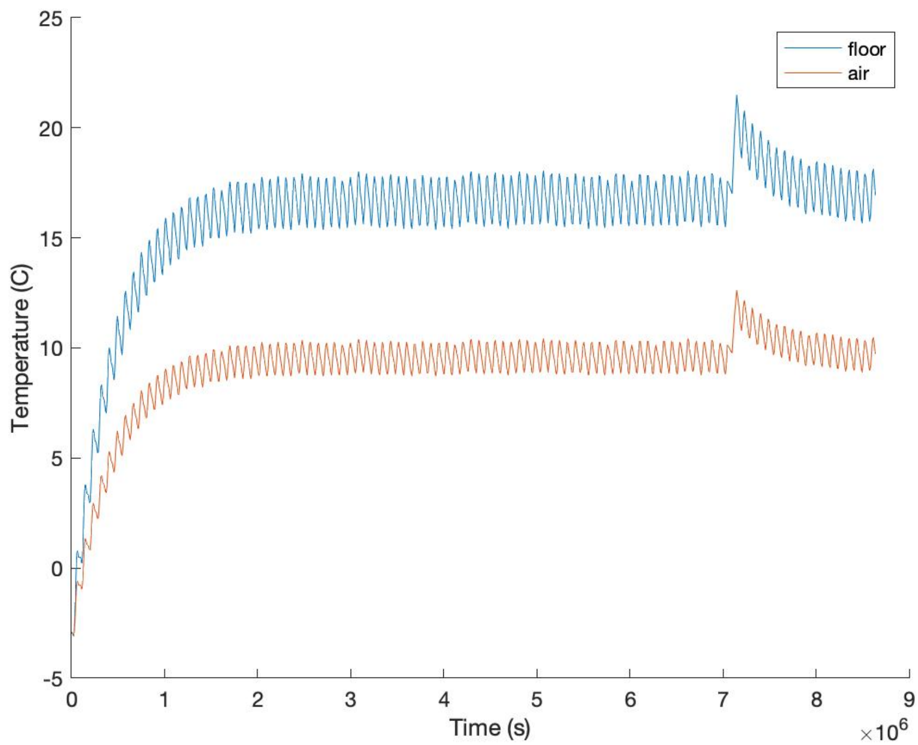

Figure 5.5: Time vs Temperature of floor and air when wall thickness is 0.1m

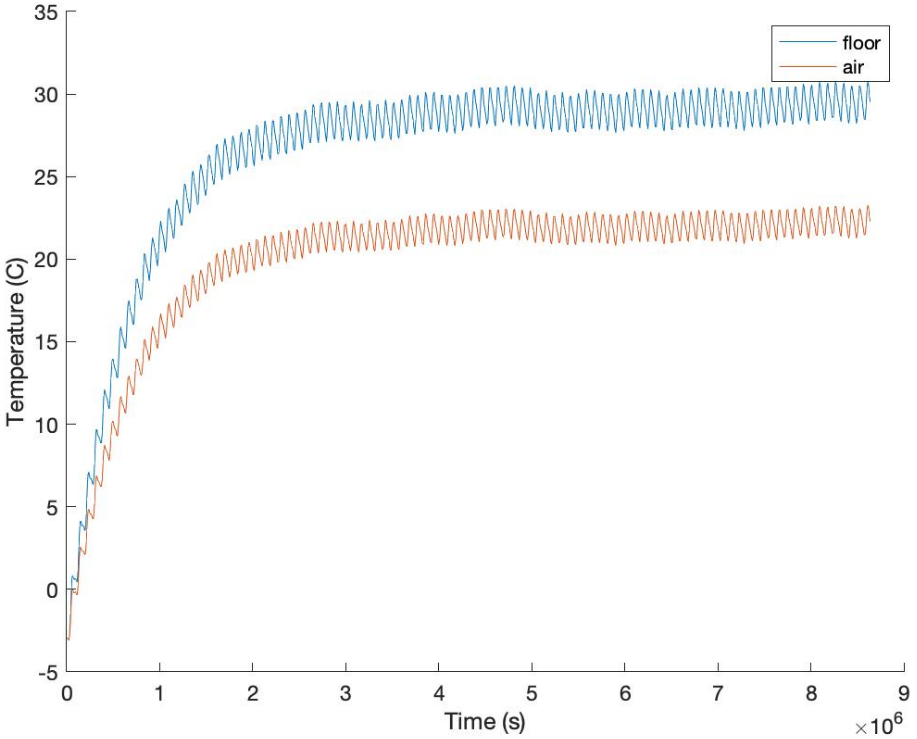

Figure 5.6: Time vs Temperature of floor and air when wall thickness is 0.25m

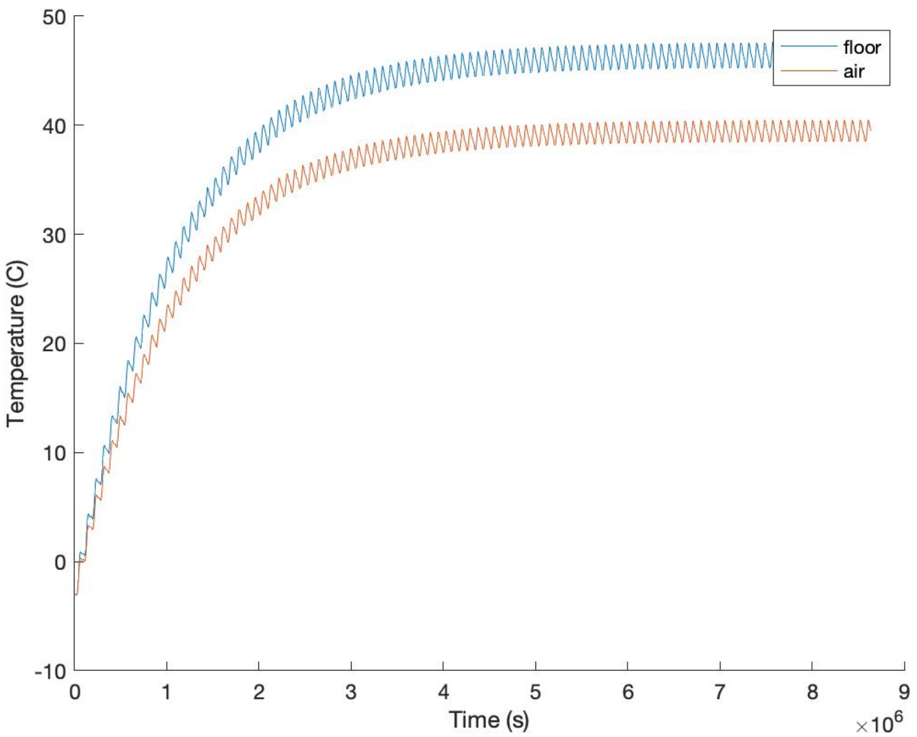

Figure 5.7: Time vs Temperature of floor and air when wall thickness is 0.5m

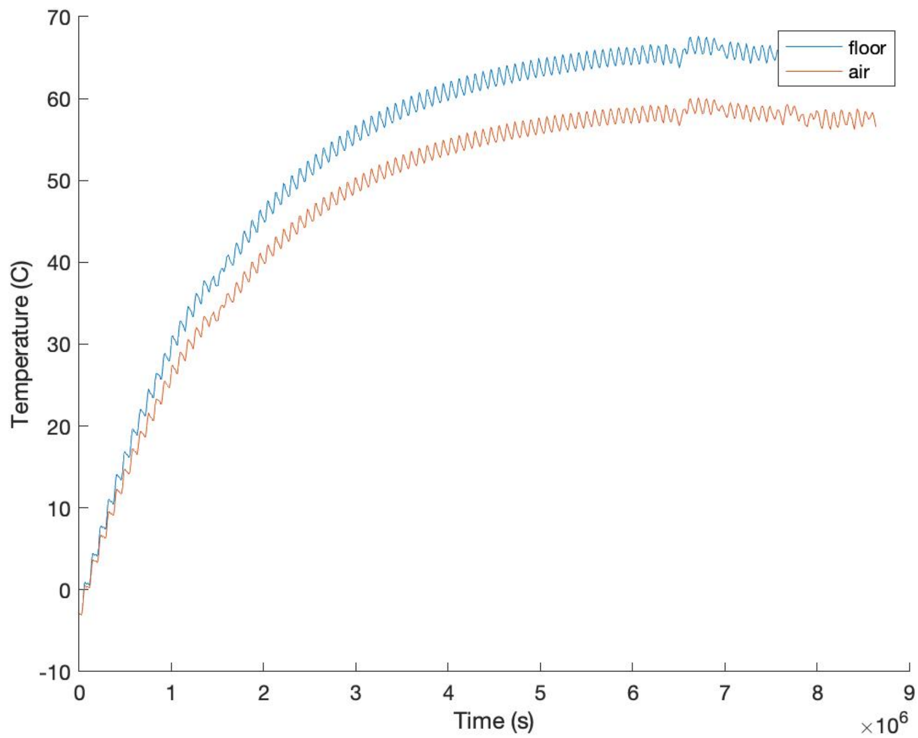

Figure 5.8: Time vs Temperature of floor and air when wall thickness is 0.75m

The result of testing different thicknesses of insulator seems to have a pattern of an increasing trend. When the thickness of the insulators increase, then the equilibrium temperature of both air and floor temperatures increase. Logically, the walls are a buffer for letting heat stay in the house rather than going out. If the volume of this increases, then there is more volume for the heat to pass through, therefore making it harder for the heat inside the house to leave. Therefore, the thicker the insulator, the hotter the equilibrium temperature becomes.

Looking at the graphs, the time to reach equilibrium for each thickness differs. To show the biggest difference, compare Figure 5.5 with Figure 5.8. The time to reach equilibrium is drastically different, where for the thickness of 0.1m, equilibrium is reached around at $2*10^6$ seconds, whereas for the thickness of 0.75m, the equilibrium is reached at around $8*10^6$. This pattern continues as the thickness rises, the time to reach equilibrium increases. The reason for this is highly likely because of the time it takes for the heat to leave the house, as there is significantly more caught inside than leaving the house. Therefore the balance between heat leaving and entering takes longer to achieve.

Equilibrium Temperature by Thickness of the Heat Storage unit	
Thickness (m)	Equilibrium Temperature $(C°)$
0.25	$(17 \leq x \leq 25, 24 \leq y \leq 33)$
0.5	$(19 \leq x \leq 23, 26 \leq y \leq 32)$
0.75	$(20 \leq x \leq 24, 27 \leq y \leq 31)$
1	$(20 \leq x \leq 23, 27 \leq y \leq 30)$

Table 5.3: The thickness of the wall is set as the default value given in the Design

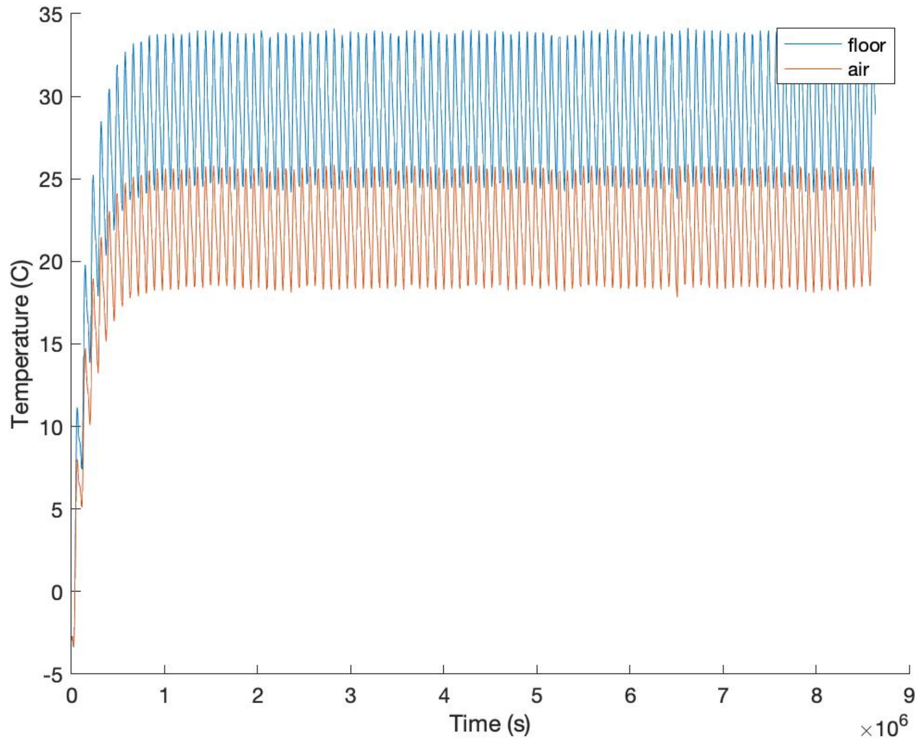

Figure 5.9: Time vs Temperature of floor and air when tile thickness is 0.25m

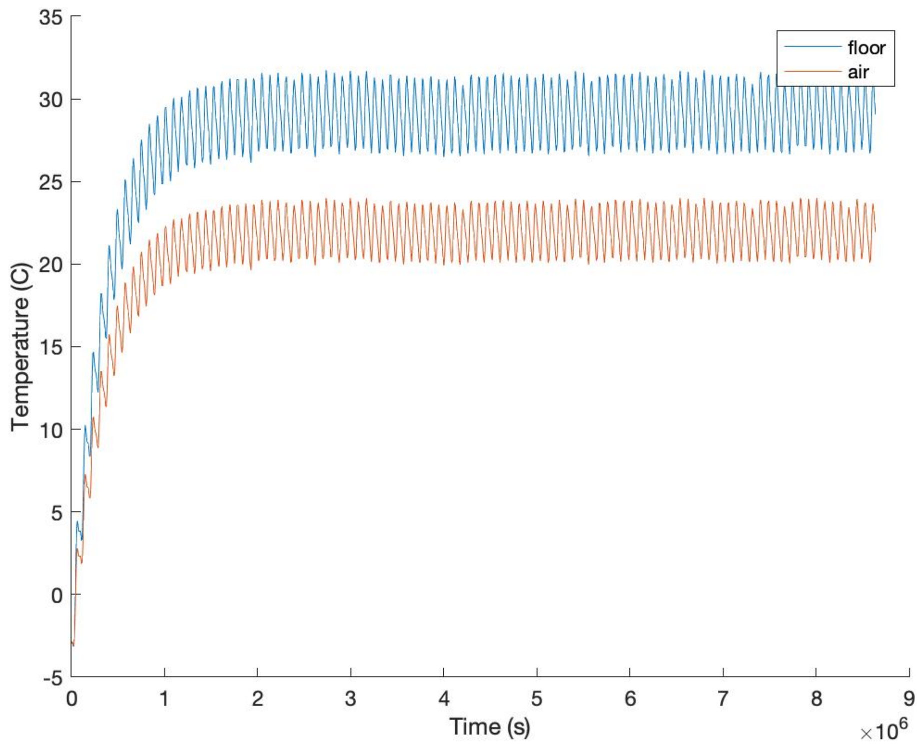

Figure 5.10: Time vs Temperature of floor and air when tile thickness is 0.5m

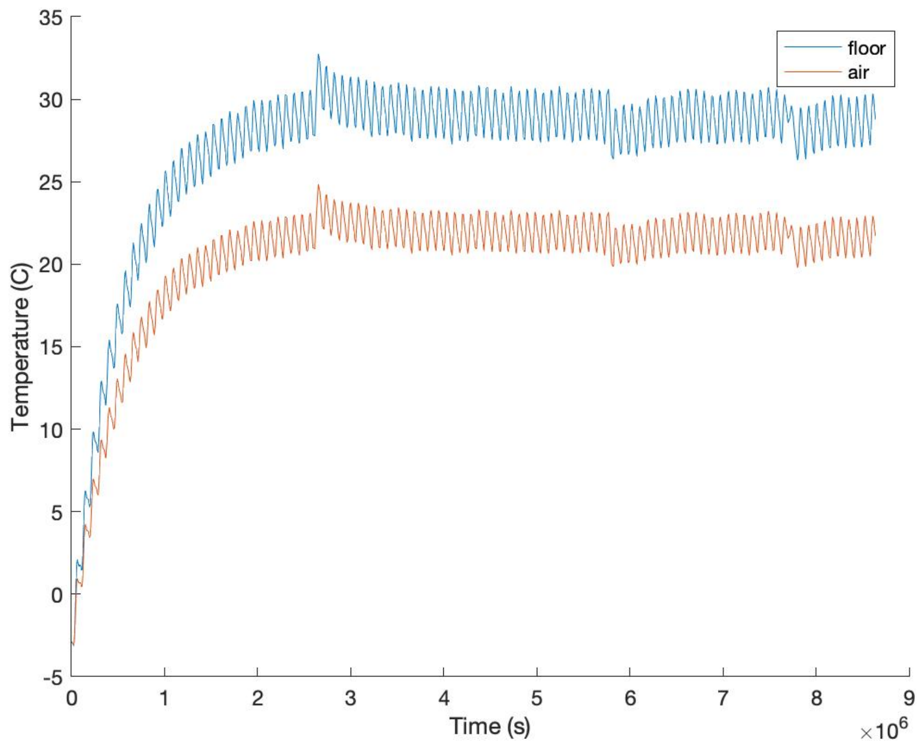

Figure 5.11: Time vs Temperature of floor and air when tile thickness is 0.75m

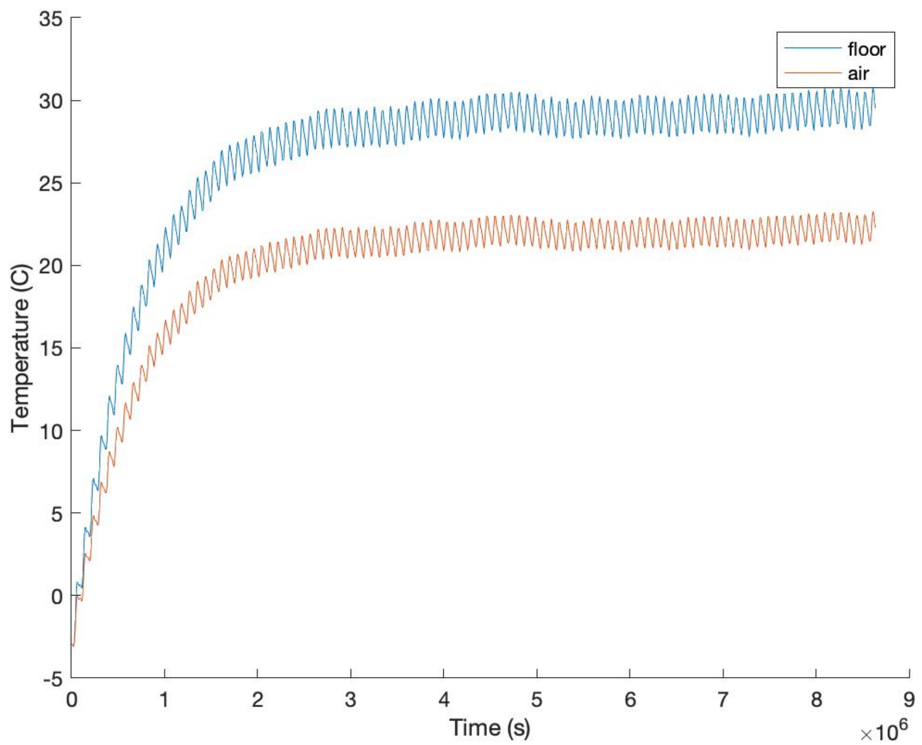

Figure 5.12: Time vs Temperature of floor and air when tile thickness is 1m

Table 5.3 a different trend than it did for the thickness of the insulators. As the thickness of the tiles increased, the range between the highest and lowest values of equilibrium became smaller - which means that the difference of temperature throughout the day is different. Also, as the thickness of the tiles increased, the time it took for the indoor temperature to reach its equilibrium took longer.

Logically, the floor(tiles) are used to take in heat and store it - which is why it is called the heat storage unit. If the volume of this heat storage unit increases, (Because the area of the floor tiles are identical and the only value that changes is the thickness) the amount of heat that tile can store also increases. If heat storage increases, there is more heat in the house that is absorbed and stored compared to the amount that is leaving, therefore letting the difference of temperature in the house during the day stay small, conversely, less heat means faster cooling, and therefore it has a vast difference of temperature. This can also be clearly seen in the Figures 5.9-5.12, where the amplitude of the graphs decrease as the thickness of the tiles increase.

The time it takes for the graph to reach equilibrium seems to have a very close correlation between the thickness of the heat absorber. As the heat absorber stores heat, it also releases heat to the indoor air - however the balance between heat that leaves the storage and heat that is absorbed during the day is harder to achieve when there is a higher capacity of heat. What this means is that if there is a higher heat capacity, the time it takes for the solar flux to store enough energy for it to balance the temperature inside the house is longer. To achieve a constant flow of input heat and output heat that is balanced, there needs to be a certain amount of storage in the heat absorber. However, this optimal capacity takes longer for larger storage.

5.3 Points of Interest

The connection between types of materials used for the walls changes the value of thermal conductivity in the equation given in the modeling chapter. Table 5.1 shows the difference in equilibrium temperature depending in the thermal conductivity. The pattern shows that there is a decreasing trend of temperature compared to an increasing trend of thermal conductivity. Also, the higher the thermal conductivity, the faster it arrived at equilibrium, looking at the graphs above.

On the pattern between thickness of insulators and floors with the equilibrium temperature, (based on the default values of the design created earlier), as the thickness of the insulator increases the equilibrium temperature increases. For the floor, the thickness altered the time it takes for the indoor temperature to reach equilibrium, and the amplitude of everyday temperature change. In the graphs for thickness of the insulators, there seems to be slight hiccups in the data for 0.75m, for unknown reasons.

5.4 Final Model

The relationships that were observed throughout the optimization were:

- The higher the thermal conductivity, the lower the equilibrium temperature and the faster the temperature reaches the equilibrium.
- The thicker the wall is, the higher the equilibrium temperature and the slower the temperature reaches the equilibrium.
- The thicker the heat storage unit, the bigger the amplitude in equilibrium temperature and the slower the temperature reaches its equilibrium.

Based on the findings, the optimal material of the wall, thickness of the wall, and the thickness of the tile is chosen. Among the four different materials that can be used for a wall above, we selected concrete wall since the concrete leads to reasonably comfortable equilibrium temperature for both floor and the tile. Next, the wall thickness of 0.25 is selected to have the most reasonable temperature for both floor and air. The primary goal of this project was to make the temperature of the air around 17 to 25 Celcius, and this value of thickness of the wall enables the floor and air to reach optimal equilibrium temperature. Lastly, we selected 0.5m for the thickness of the heat storage unit, which has reasonable amplitude of the temperature and reasonable time to reach the equilibrium. If the storage unit is thicker, the amplitude is too big that the temperature difference will be huge throughout the day, which is not comfortable for people to live. If the storage unit is thinner, it takes more time to reach its equilibrium, which is also not an optimal situation.

Chapter 6

Evaluation

6.1 Long Period Plotting

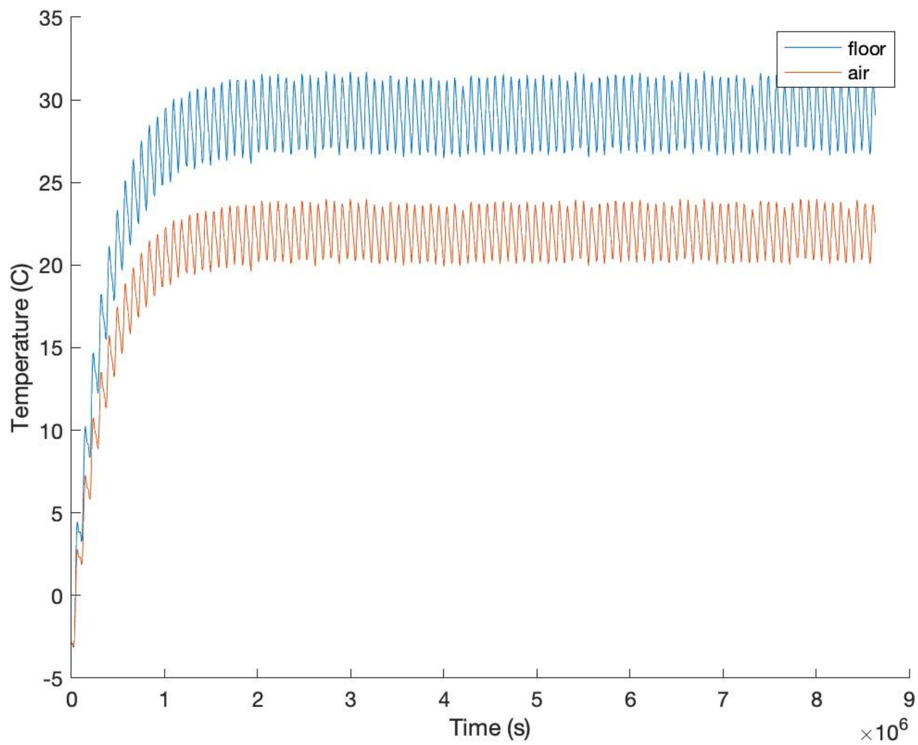

Figure 6.1: Time vs Temperature of floor and air for 100 days

Figure 6.1 shows a long term temperature change for 100 days, where the values of each variables are:

- $T_f = -3$; %initial floor temp in C
- $T_{out} = -3$; %temperature of outside air in C
- $h_{in} = 15$; %heat transfer coefficients for indoor surfaces(W/m^2K)
- $h_{out} = 30$; %heat transfer coefficients for outdoor surfaces(W/m^2K)
- $h_{eq} = 0.7$; %heat transfer coecient of double-paned window (W/m^wK)
- $A_f = 25.5$; %area of the floor (heat absorber) (m^2)
- $A_{wall} = 73.1$; %area of the walls (m^2)

- $A_{window} = 13$; %area of the window(m^2)
- $L_{wall} = 0.25$; %thickness of the wall (m)
- $k_{wall} = 0.4$; %thermal conductivity of the wall (W/mK)
- $C_f = 3000 * 800 * (25.5) * 0.5$; %tile thickness 0.5 ($m$)

The plot shows that after reaching equilibrium, which takes about $2 * 10^6$ seconds, the temperature of the air ranges between 20 to around 24 degrees Celsius, and the temperature of the floor ranges from 26 to 31 degrees Celsius.

6.2 24 hour Plot

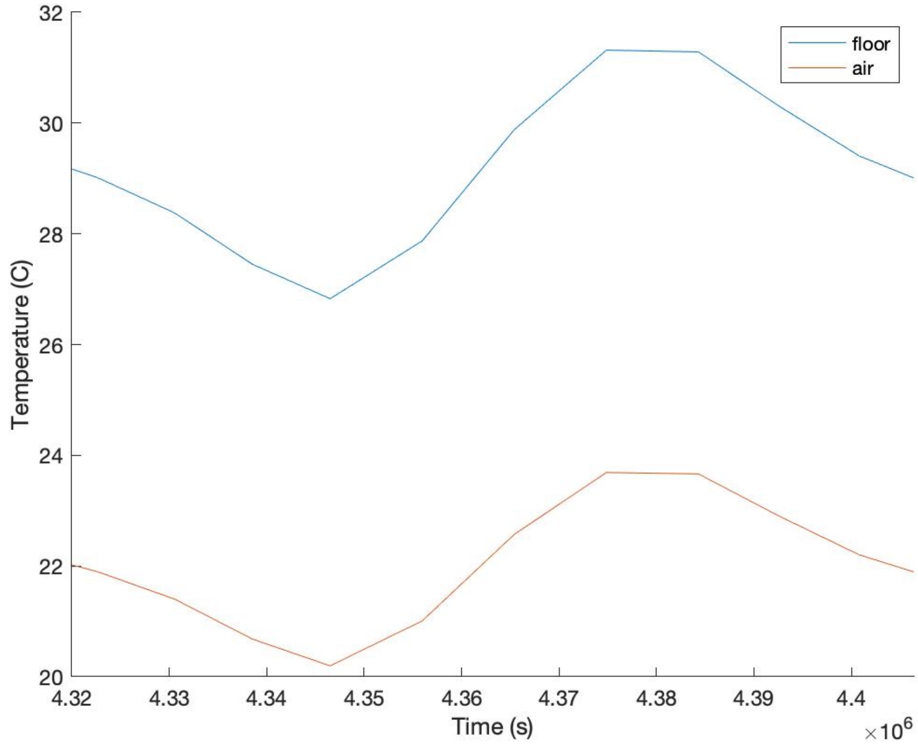

Figure 6.2: Time vs Temperature of floor and air in 24 hours

The 24 hour plot shows the highest and lowest temperatures in a 24 hour period. The highest temperature of air during the day is 23.69 and the lowest temperature of the day is 20.2 degrees Celsius. For the tile, the highest temperature was 31.32 and the lowest 26.83. Compared to the normal dwelling temperatures, it seems to fit within those boundaries - making it a very comfortable place to live.

6.3 Observation and Analysis

Considering our experiences in living in different temperatures, (Korea basically has a vast difference in temperatures between the four seasons ranging from -15 degrees Celsius to 40 degrees Celsius) the most comfortable living temperature lies around the 20 to 25 degrees Celsius. In the summer, the recommended indoor temperature is 26 degrees, and in the winter 18. With the house able to stay at a steady temperature between 20 and 24 degrees, the house will be very comfortable to live in during the winter. To make the house even better, we can consider outdoor design such as trees that can control the indoor temperatures by blocking out the sun in the summer and providing insulation in the winter.

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