Optimization Methods for Machine Learning

Project 1

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1 Introduction

The present work aims to reconstruct a two dimensional function $F : \mathbb{R}^2 \to \mathbb{R}$ from which we don't have the analytic expression. The reconstruction (in the region [-2;2]x[-3;3]) will be based on 250 points $(x^p; y^p)$ where y^p is defined as $F(x^p)$ in addition with a small random noise. In a nutshell, we will use neural networks based first on Multilayer Perceptron, then on Radial Basis Functions.

2 Question 1. (Full minimization)

In this section we construct two shallow Feedforward Neural Network : a MLP and a RBF network. The goal is to find a function $f(x)$ which approximates the true function F. The regularized training error will be calculated using the following formula :

$$E(\omega; \pi) = \frac{1}{2P} \sum_{p=1}^{P} (f(x^p) - y^p)^2 + \frac{\rho}{2} ||\omega||^2$$

where ω/π are the parameters/hyperparameters. Observe that the regularization parameter ρ belongs to π.

2.1 MLP

As activation function for the MLP network we use the hyperbolic tangent

$$g(t) = \frac{e^{2\sigma t} - 1}{e^{2\sigma t} + 1}$$

where the spread parameter σ will also belong to the set of hypeparameters π.

We searched for the best hyperparameters (N, σ, ρ) by performing a grid search. Our final values for these parameters are (respectively) : 32, 1 and 0.0009. In the Appendix

2.2 RBF

As Radial Basis Function for our network we choose the Gaussian function

$$\phi(||x - c_j||) = e^{-(||x - c_j||/\sigma)^2}$$

As for MLP, the spread parameter σ of the function ϕ will be added to the list of hyperparameters. After performing a grid search we decide to take as hyperparameters : N=32, σ=1, rho=0.0009.

The plot of the approximating function found (see Apendix)

3 Question 2. (Two blocks methods)

3.1 MLP

3.2 RBF

4 Question 3. (Decomposition method)

5 Bonus : Best Model

6 Conclusion : Final Table

Ex		settings
0	0	2

Appendix (Figures)

Question 1 (MLP)

Question 1 (RBF)