14.04 Recitation 14: Money, Money, Money

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"One of the most important forms of currency in England in Henry [II]'s (1154-1189) time were notched "tally sticks" used to record debts. Tally sticks were quite explicitly IOUs: both parties to a transaction would take a hazelwood twig, notch it to indicate the amount owed, and then split it in half. The creditor would keep one half, called "the stock" (hence the origin of the term "stock holder") and the debtor kept the other, called "the stub" (hence the origin of the term "ticket stub.)" Tax assessors used such twigs to calculate amounts owed by local sheriffs. Often, though, rather than wait for the taxes to come due, Henry's exchequer would often sell the tallies at a discount, and they would cirulate, as tokens of debt owed to the government, to anyone willing to trade for them.

Modern banknotes actually work on a similar principle... In fact this is precisely the logic on which the Bank of England—the first successful modern central bank—was originally founded. In 1694, a consortium of English bankers made a loan of £1,200,000 to the king. In return they received a royal monopoly on the issuance of banknotes. What this meant in practice was they had the right to advance IOUs for a portion of the money the king now owed them to any inhabitant of the kingdom willing to borrow from them, or willing to deposit their own money in the bank—in effect, to circulate or "monetize" the newly created royal debt. This was a great deal for the bankers (they got to charge the king 8 percent annual interest for the original loan and simultaneously charge interest on the same money to the clients who borrowed it) , but it only worked as long as the original loan remained outstanding. To this day, this loan has never been paid back. It cannot be. If it ever were, the entire monetary system of Great Britain would cease to exist.

- David Graeber, "Debt: The First 5,000 Years"

1 Introduction

Some objects - often referred to as monies - appear in exchange much more frequently than other objects. Today we'll talk about a model by Townsend and Wallace (1987) on "Circulating Private Debt" where some securities get traded frequently, i.e. circulate, whereas others do not. The securities in our model resemble historically observed bills of exchange.

The model will have intertemporal trade in spatially and informationally separated markets. Spatial and

Figure 1: Medieval English split tally stick (front and reverse view). The stick is notched and inscribed to record a debt owed to the rural dean of Preston Candover, Hampshire, of a tithe of 20d each on 32 sheep, amounting to a total sum of £2 13s. 4d.

informational separation make welfare-enhancing exchange difficult and correspondingly make circulating securities useful.

The difficulty of carrying out exchange under our assumptions shows up in two distinct ways: (1) market incompleteness: some physically feasible and beneficial trades cannot be accomplished. (2) a pure coordination game: there is no communication and no conflict and the problem facing the players is to choose strategies that are coordinated.

In other words, the utility-maximizing choices of quantities of securities are not in general unique but must somehow be coordinated across informationally separated markets if they are to be consistent with the existence of an equilibrium. This coordination problem arises only in some versions of our model, and it suggests that chaotic conditions sometimes arise in credit markets with unregulated issue of private securities – and that there may be a use for a centralized monetary system regulating the quantity of circulating securities.

2 Model

We start with a finite number of finite-lived people who meet deterministically at prescribed locations and at prescribed times. We will focus on an economy of four people who meet according to the pattern laid out in table V-1. Persons 1 and 4 always stay at those locations, whereas persons 2 and 3 switch locations each period.

Table V-1. Who Meets Whom When

Date	Location 1	Location 2
1	(1,2)	(3,4)
2	(1,3)	(2,4)
3	(1,2)	(3,4)
4	(1,3)	(2,4)

Let $J = 2$ denote the number of locations and T the number of dates. The commodity space has dimension JT. We can think of each commodity as consumption in a given location and time. We assume that each person gets utility from commodities and has positive endowments of commodities in a proper subset whose elements correspond to the location-date combinations that the person visits. That is, each person has zero endowment for the consumption good in at least one location and time. There is no transportation, production, or storage technology for goods.

At a particular time, a person can only trade securities with someone he or she meets. Second, securities can be transported, but they can move only with a person. Finally, we do not allow people to renege on their debts or to counterfeit others' debts. Securities or debts in our model take the form of promises to pay stated amounts of goods that are date and location specific. We assume that if the promise is presented at the relevant date and location, then it is honored.

2.1 Case: $T = 2$

If the economy lasts only two periods, then no trade is possible in the table V-I economy under our security trading rules. For example, person 1 cannot sell a promise to person 2 because person 2 can neither redeem it at date 2 nor pass it on to person 4, who has no use for it at date 2, the assumed last date.

Notice that there is a complete lack of double coincidence: no pair of persons cares about a common two-dimensional subspace of the commodity space. The securities we consider do not overcome this lack of double coindence. There is no trade.

There can, however, exist redistributions of the endowments that give rise to allocations that are Pareto-superior to the endowment allocation. Put differently, if all four people were together at some time zero and traded in complete (location and date contingent) markets, something we rule out, then the endowment would not necessarily be a competitive equilibrium.

2.2 Case: $T = 3$

Agents who meet at date 1 can trade debts due at date 3 when they meet again. For example, person 1 can issue a promise to pay a good at location 1, date 3, a promise that person 2 holds until he or she again meets person 1. Such securities do not circulate; they do not get traded in a secondary market and are not used to make third party payments. Such securities do no more than accomplish trades for which there IS a double coincidence.

2.3 Case: $T = 4$

Our security trading rules are now consistent not only with the existence of several noncirculating securities but also with the existence of several circulating securities. At date 1, person 1 can issue to person 2 a promise to pay a good at location 1, date 4. This promise can be redeemed by being passed from person 2 to person 4 at date 2, from person 4 to person 3 at date 3, and from person 3 to person 1- the issuer - at date 4. Similarly, each of persons 2, 3, and 4 can issue at date 1 a promise of a good at date 4 at some location.

3 Debt Equilibria

We now define a debt equilibrium in this spatially separated environment – this will look a lot like a competitive exchange equilibrium, except that securities will be traded instead of consumption goods. To write this out formally, we'll need a little more notation. Assume we have G individuals, each of whom lives T periods. At each time t, each person g pairs with some other person or with no one. These pairings occur at isolated locations. There will be $J \geq G/2$ locations.

If person g is at location i at time t, then he or she is endowed with some positive number of units, w_{it}^{g}, of the consumption good at location i, date t. Let w^{g} be the endowment (or wealth) vector. Similarly, let c^{g} be the corresponding consumption vector. Preferences are described by a utility function $U^{g}(c^{g})$. Assume U^{g} is continuous, concave, and strictly increasing.

Let d_{st}^{f} denote securities issued by person f at time $s$to pay d_{st}^{f} units of the consumption good where f will be at time t. Note d_{st}^{f} is well defined if there is a path of chain of pairings leading from where f is at s to where f is at t.

Let $p_{st}^{f}(i,u)$ be the price per unit of d_{st}^{f} at location i and date u, in units of good (i,u). There is a market in d_{st}^{f} at location $i$and date u if and only if there is a route such that the relevant security can be circulated to (i,u).

Let $d_{st}^{fg}(i,u)$ be the excess demand by g at (i,u) for d_{st}^{f}. Let d^{g} be the vector of debt demands of g over all securities that can be issued.

Our trading rules are:

1. All debts are paid back, i.e., no reneging, i.e. f must end up demanding as much as f issues:

$$\sum_{u=s}^{t} d_{st}^{ff}(\cdot,u) \geq 0 \; \forall f$$

2. Person $g \neq f$ cannot supply d_{st}^{f} without having previously acquired it:

$$\sum_{u=s}^{t'} d_{st}^{fg}(\cdot,u) \geq 0 \; \forall t' \geq s, \forall g \neq f$$

Furthermore, each person g has budget constraint:

$$c_{iu}^{g}+\sum d_{st}^{fg}\left(i,u\right)p_{st}^{f}\left(i,u\right)\leq w_{iu}^{g}$$

Definition 1. A **debt equilibrium** is a specification of consumption c^g and debt demands d^g for each g and positive security prices $p_{st}^{f}\left(i,u\right)$ such that

1. Individual maximization: c^g and d^g maximize $U^g\left(c^g\right)$ subject to trading rules and budget constraints.

2. Market clearing: $\sum_g c_{iu}^{g}=\sum_g w_{iu}^{g}$ for each $\left(i,u\right)$ and $\sum_g d_{st}^{fg}\left(i,u\right)=0$ for each $\left(i,u\right)$ and all potentially redeemable d_{st}^{f}.

4 Complete Markets Equilibria

A complete markets equilibrium is an allocation of consumption bundles if everyone could get together before date $t=1$ and trade contigent consumption bundles.

Let $e_{it}^{g}=c_{it}^{g}-w_{it}^{g}$ denote g's excess demand for consumption in period t at location i. Let s_{it} denote the price of the consumption good in period t at location i.

Definition 2. The individual consumption excess demands e_{it}^{g} and associated price s_{it} constitute a complete markets equilibrium if they satisfy:

1. Individual maximization: e_{it}^{g} maximizes utility subject to budget constraint $\sum_i\sum_t s_{it}e_{it}^{g}=0$.

2. Market clearing: $\sum_g e_{it}^{g}=0$.

5 Equivalence

Let's now return to our $T=4$, $J=2$ example. The following statements are true.

1. In the $T=4$, $J=2$ economy, any complete markets equilibrium can be implemented as a debt equilibrium.

2. Some complete markets equilibrium can only be implemented as a debt equilibrium with both circulating and noncirculating debt.

I won't prove these here, but you can take a took at the derivations in the Townsend and Wallace chapter. The equivalence result is powerful and shows how circulating private debt overcome market incompleteness created by spatial separation.

6 The Coordination Problem

Except there is a problem: Debt demands given prices are not unique. Given a set of debt prices, there may be multiple debt vectors which achieve a given vector of consumption for each individual. Furthermore, in order for the maximizing choices of individual consumption bundles to constitute a debt equilibrium, they must satisfy restrictions neither implied by utility maximization nor market clearing within each market. (I've avoided the technical derivation of this but you can take a look at Townsend and Wallace.)

In order words, market participants need to coordinate their individual debt issuance across spatially separated markets. This is difficult in our setting because individuals in different locations cannot communicate. Individuals do not find out whether they have issued too much or too little debt in $t = 1$ until $t = 2$, when the quantity of debt issued in the other market is revealed by the available demand of the traveling trader. This coordination problem might explain the boom and bust cycles that money markets frequently experience and suggests how institution of central banking may help.