14.04 Recitation 13: Externalities

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1 Externalities: Pollution as an Example

Let's talk about externalities and the failure of the welfare theorems. Recall that the welfare theorems tells us (1) when a competitive equilibrium is Pareto optimal and (2) when a Pareto optimal allocation can be implemented as a competitive equilibrium with price transfers. Today we will talk about a particular scenario in which the welfare theorem fails.

Let's consider the following economy. There is one agent that consumes 2 goods. (Think of good 1 as veggies and good 2 as meat.) A firm transforms good 1 into good 2, according to a production function $y_2 = F(-y_1)$. (We need some veggies to make meat, or we can eat the veggies directly.) However, making good 2 generates pollution $P(y_2)$, that affects the agents utility. (Meat production creates methane, which smells bad and hurts the environment.) Preferences are given by $u(x_1, x_2, P)$.

The Pareto problem is

$$\max_{x_1, x_2, y_1, y_2} u(x_1, x_2, P(y_2))$$

$$x_1 = 1 + y_1$$

$$x_2 = y_2$$

$$y_2 \leq F(-y_1)$$

The Lagrangian is

$$\mathscr{L} = u(x_1, x_2, P(y_2)) + \gamma_1 (1 + y_1 - x_1) + \gamma_2 (y_2 - x_2) + \gamma_3 (F(-y_1) - y_2)$$

This yields FOCs

$$\frac{\partial u}{\partial x_1} - \gamma_1 = 0$$

$$\frac{\partial u}{\partial x_2} - \gamma_2 = 0$$

$$\gamma_1 - \gamma_3 \frac{\partial F}{\partial y_1} = 0$$

$$\frac{\partial u}{\partial P} \frac{\partial P}{\partial y_2} + \gamma_2 - \gamma_3 = 0$$

Combining, we get

$$\frac{1}{\partial F/\partial y_1} = \frac{\frac{\partial u}{\partial x_2} + \frac{\partial u}{\partial P}\frac{\partial P}{\partial y_2}}{\frac{\partial u}{\partial x_1}},$$

Or

$$MRS^{PO} = \frac{\partial u/\partial x_2}{\partial u \partial x_1} = \frac{1}{\partial F/\partial y_1} - \frac{1}{\partial u/\partial x_1}\left(\frac{\partial u}{\partial P}\frac{\partial P}{\partial y_2}\right).$$

In a Walrasian equilibrium, however, the marginal rate of substitution will be different. Let p be the relative price of good 2. Note that the firm solves

$$\max_{y_1,y_2} py_2 - y_1$$

$$y_2 \leq F(-y_1)$$

This yields FOC

$$p = \frac{1}{\partial F/\partial y_1}$$

Similarly, we have that the consumer maximizes utility

$$\max_{x_1,x_2} u(x_1,x_2,P)$$

$$x_1 + px_2 \leq 1 + \pi$$

This gives FOC:

$$MRS^{eq} = \frac{\partial u/\partial x_2}{\partial u \partial x_1} = p = \frac{1}{\partial F/\partial y_1}$$

Note that if $\frac{\partial u}{\partial P} < 0$, then

$$MRS^{eq} > MRS^{PO}$$

This means x_1 is too big under the competitive equilibrium. Why does this happen? Because there is an extra commodity, called "pollution," which affects consumer utility but has **no market**. A remedy would be to create a market for pollution rights – e.g. via cap and trade.