14.04 Recitation 8: Moral Hazard and Limited Liability

Michael B Wong

Fall 2020

1 Introduction

In theses notes we walk through Paulson, Townsend, and Karaivanov (2006). The idea of the paper is to try to disentangle two potential sources of financial market imperfection – limited liability and moral hazard – that limit entry into entrepreneurship. Before we jump into the model in the paper, let us briefly discuss these underlying issues.

What are financial market imperfections? Roughly, this means that desirable loans or investments are prevented from happening. You can imagine that some individuals might have the ability to start a successful business, but do not have enough personal wealth to bootstrap the business without outside funding. Such entrepreneurs would ideally be able supplement their personal stake in entrepreneurial activities by borrowing. An imperfect financial market may fail to provide funding for such entrepreneurs.

In the economics literature, there are at least two theories for why capable entrepreneurs are prevented from securing funding. First is the idea of limited liability. Because borrowing to an entrepreneur is risky, the entrepreneur's personal wealth can play the role of collateral and limit her incentive to default. In an environment with limited liability, low-wealth households may be prevented from borrowing enough to become entrepreneurs, and others that are able to start businesses may be constrained in investment.

Another source of financial constraints arises from moral hazard. Since entrepreneurial effort is unobserved and repayment is feasible only if a project is successful, poor borrowers have little incentive to be diligent, increasing the likelihood of project failure and default. In order to break even, lenders charge higher interest rates to low-wealth borrowers. Some low-wealth potential entrepreneurs will be unable, or unwilling at such high interest rates, to start businesses at any scale. Low wealth entrepreneurs who do succeed in getting loans will be subject to a binding incentive compatibility constraint that ensures that they exert the appropriate level of effort. In contrast to the limited-liability case, when there is moral hazard and wealth increases, constrained entrepreneurs will increasingly self-finance and borrowing diminishes. In a moral hazard environment, all entrepreneurs who borrow will be constrained.

As you can see from this brief discussion, the predictions of the two theories are different. We now formalize this into a model, which PTK take to the data in order to see whether limited liability can be distinguished from moral hazard in structural estimates using cross-sectional data from a sample of households from Thailand.

2 Model

2.1 Households

Households are assumed to derive utility from consumption c and disutility from effort z:

$$U(c,z) = c - \kappa \frac{z^{\gamma_2}}{\gamma_2}.$$

Here we assume the agents are risk-neutral.

There are three parameters that characterize each household. These are initial wealth A, entrepreneurial talent θ, and years of education S. The distribution of (A,θ,S) is drawn ex ante and is observable by everyone.

2.2 Occupation Choice

There are two occupations: entrepreneur or wage worker.

- Entrepreneurs produce output q from their own effort z and from capital k. Output q can take on two values, 0 or θ. The probability of success is given by

  $$\Pr(q = \theta \mid z, k > 0) = \frac{k^{\alpha} z^{1-\alpha}}{1 + k^{\alpha} z^{1-\alpha}}.$$

  Output can be costlessly observed by everyone.

- Wage workers need zero capital, i.e. $k = 0$. The wage depends on the worker's effort, and is equal to w with probability $z/(1+z)$ or 0 otherwise.

2.3 Interest Rates

We will assume that all households can borrow at a gross cost of $r(A,\theta)$. Entrepreneurs who do not borrow and wage workers earn the given, riskless gross interest rate r.

2.4 Putting it together

For each household, the social planner makes a recommendation for effort z and capital level k to solve

$$\begin{aligned}
&\max_{z,k} \left\{ w\frac{z}{1+z} - \kappa \frac{z^{\gamma_2}}{\gamma_2} + rA \right\} \text{ if } k = 0 \\
&\max_{z,k} \left\{ \theta \frac{k^{\alpha} z^{1-\alpha}}{1 + k^{\alpha} z^{1-\alpha}} - \kappa \frac{z^{\gamma_2}}{\gamma_2} + r(A-k) \right\} \text{ if } k > 0, k \leq A \\
&\max_{z,k} \left\{ \theta \frac{k^{\alpha} z^{1-\alpha}}{1 + k^{\alpha} z^{1-\alpha}} - \kappa \frac{z^{\gamma_2}}{\gamma_2} + r(A,\theta)(A-k) \frac{k^{\alpha} z^{1-\alpha}}{1 + k^{\alpha} z^{1-\alpha}} \right\} \text{ if } k > 0, k > A
\end{aligned}$$

The first term in the maximand is the expected utility of a wage worker: expected wages minus the cost of effort, plus a riskless return on wealth. The second term is the expected utility of a entrepreneur who does not need to borrow to carry out the recommended k: expected output minus the cost of effort, plus a riskless return on any wealth not invested in the project. The final term is the expected utility of an entrepreneur who must borrow to carry out the assigned k: expected output minus the cost of effort, minus the expected cost of repaying the loan.

Note that the loan is repaid only when the project is successful. The planner's problem is subject to a constraint that guarantees that the expected rate of repayment on such loans covers the cost of outside funds, so that lenders break even:

$$r = r(A,\theta)\,\frac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}} \qquad \text{for } k > A, \forall\theta, \forall A$$

3 Moral Hazard

When there is moral hazard, entrepreneurial effort is unobservable and the financial contract cannot specify an agent's effort. In terms of the planner's problem, this translates into a requirement that the capital assignment and the interest rate schedule are compatible with the effort choice that a borrowing entrepreneur would have made on his or her own.

Recall from above that the entrepreneur's own maximization problem is

$$\max_{z}\left\{\theta\frac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}} - \kappa\frac{z^{\gamma_2}}{\gamma_2} + r(A,\theta)(A-k)\frac{k^{\alpha}z^{1-\alpha}}{1+k^{\alpha}z^{1-\alpha}}\right\}$$

This gives first-order condition:

$$(\theta + r(A,\theta)(A-k))\,\frac{(1-\alpha)k^{\alpha}z^{-\alpha}}{(1+k^{\alpha}z^{1-\alpha})^2} - \kappa z^{\gamma_2-1} = 0,$$

which, in the presence of moral hazard, will be an additional constraint in the planner's maximization problem.

4 Limited Liability

To model limited liability, we assume that households can borrow up to some fixed multiple of their total wealth, but no more. If there is limited liability, this means that an additional constraint

$$k \le \lambda A$$

will be in planner's problem.

5 The Linear Programming Problem

Now we restate the occupational choice problem faced by an agent with wealth A, schooling S, and entrepreneurial talent θ as a contracting problem between the agent and a competitive financial intermediary.

The optimal contract between the two parties consists of prescribed investment, k, recommended effort, z, and consumption, c. Consumption can be contingent on the output realization, q. Agents assigned zero investment are referred to as "workers," and agents assigned a positive level of investment are called "entrepreneurs." Agents may now be risk averse, with risk neutrality embedded as a special case.

Nonconvexities arising from the incentive constraints, from the indivisibility of the choice between wage work and entrepreneurship, and from potential indivisibilities in k mean that, in general, standard numerical solution techniques that rely on first-order conditions will fail. By writing the principal-agent problem as a linear programming problem with respect to lotteries over consumption, output, effort, and investment, we can restore convexity and compute solutions.

Let the probability that a particular allocation (c,q,z,k) occurs in the optimal contract for agent (θ,A,S) be denoted by $\pi(c,q,z,k \mid \theta,A,S)$. The choice object, $\pi(c,q,z,k \mid \theta,A,S)$, enters linearly into the objective function as well as in every constraint.

We solve the following linear program:

$$\max_{\pi(c,q,z,k|\theta,A,S)} \sum_{c,q,z,k} \pi(c,q,z,k \mid \theta,A,S) U(c,z)$$

subject to (1) a Bayesian consistency constraint:

$$\sum_{c} \pi(c,q,z,k \mid \theta,A,S) = p(q \mid z,k,\theta) \sum_{c,q} \pi(c,q,z,k \mid \theta,A,S) \quad \forall q,z,k,$$

(2) a breakeven condition for the financial intermediary:

$$\sum_{c,q,z,k} \pi(c,q,z,k \mid \theta,A,S)(c-q) = r \sum_{c,q,z,k} \pi(c,q,z,k \mid \theta,A,S)(A-k),$$

(3) an incentive compability constraint (in the presence of moral hazard):

$$\sum_{c,q,z,k} \pi(c,q,z,k \mid \theta,A,S) U(c,z) \geq \sum_{c,q,z,k} \pi(c,q,z,k \mid \theta,A,S) \frac{p(q \mid z',k,\theta)}{p(q \mid z,k,\theta)} U(c,z') \quad \forall k > 0, z, z' \neq z$$

and (4) a constraint that the probabilities sum to one:

$$\sum_{c,q,z,k} \pi(c,q,z,k \mid \theta,A,S) = 1.$$

We can then solve this numerically and take the model to the data.