14.04 Recitation 12: Gorman Aggregation and Representative Agents

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Today, we will discuss representative agents. Here we study two questions:

1. Given a set of individual demand functions $x_i(p,w)$, can we find a fictitious household i^* such that the aggregate demand function can be thought of being generated by the choices of household i^*, which has as wealth the total wealth of the economy?

2. Given a set of individual demand functions $x_i(p, p\omega_i)$, can we find a fictitious household i^* such that Pareto optimal allocations in the economy can be thought as maximizing the utility of the representative household?

The first question is about the "positive" representative household because the question concerns describing aggregate consumption behavior. The second question is about the "normative" representative household: i.e. can we use the representative household to analyze the social efficiency of the economy?

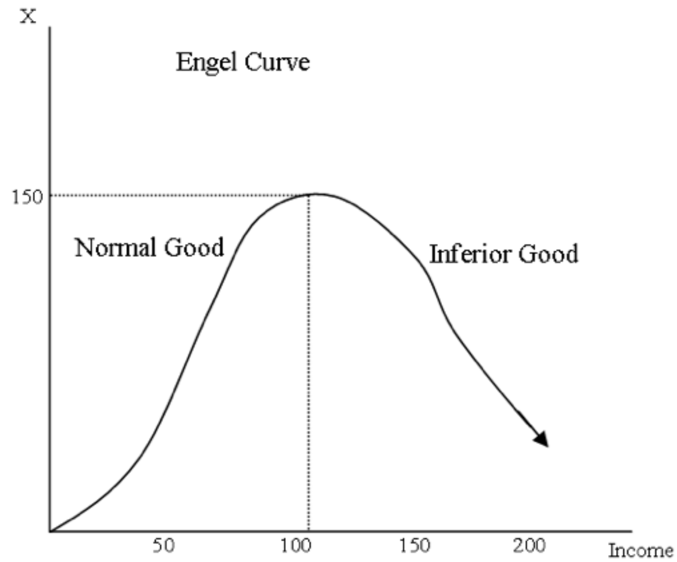
These two questions are important because many – although not all – macroeconomic models assume the existence of a representative household. As we will see, this assumption can be very problematic. I will be an attempt to give you an intuition for why, focusing on the first question.

1 Recap: Income Effects are Messy!

Remember Slutsky's equation?

$$\frac{\partial x_i}{\partial p_i} = \underbrace{\frac{\partial h_i}{\partial p_i}}_{\text{substitution effect}} - \underbrace{\frac{\partial x_i}{\partial w}}_{\text{income effect}} x_i$$

In general, we know the sign of the own-price substitution effect ($\frac{\partial h_i}{\partial p_i}$) – it is always negative. (This follows from the convexity of the indifference curve. How?) However, economic theory has no prediction about the sign of the income effect ($\frac{\partial x_i}{\partial w}$), which could be either positive or negative. If it's positive, we say the good is "normal;" if negative, we say it is "inferior." For example, we may want to consume less \$1 Burgers as we get richer. In this case, the income effect is negative. But we may want more Big Macs as we get richer. We can represent this relationship in the follow graph:



(In fact, if the income effect is very strongly negative, we might even have a "Giffen good." A Giffen good is something we want to consume more of as its price rises, because the price increase makes us poorer which makes us want this good a lot more. This is very weird, but possible according to consumer theory. It's an open question, however, whether Giffen goods exist at all!)

So, income effects generate messy predictions about consumer behavior. One consequence of this messiness is that aggregate consumer behavior may not be correspond to the behavior of a single representative consumer if income effects are heterogenous across consumers. In order to rule out "weird" aggregate demand functions, we will need special preference functions which restricts the relationship between individual demand and individual income. We will formalize this intuition below.

2 Parallel Linear Wealth Expansion Paths

Let's start with the definition of aggregate demand:

$$\overline{x}(p, w_1, w_2, \ldots. w_I) = \sum_i x_i(p, w_i)$$

Question: Under what condition can we ensure the existence of an aggregate demand function $X(p, \overline{w})$ that depends solely on the total wealth $\overline{w} = \sum_i w_i$ of the population, for all possible wealth profiles $w = (w_1, w_2, \ldots. w_I)$?

Partial answer: The individual demand functions must have parallel linear wealth expansion paths.

Proposition 1. *Suppose that the individual demand functions $\{x_i(p, w_i)\}_i$ are differentiable. Then, if there exists a function $X(p, \overline{w})$ that satisfies*

$$X(p, \overline{w}) = \overline{x}(p, w) \tag{1}$$

for all w, then for all $w \in \mathbb{R}^I_+$ and all prices p:

$$\frac{\partial x_{il}(p, w_i)}{\partial w_i} = \frac{\partial x_{jl}(p, w_j)}{\partial w_j} \;\; \forall i, j = 1, 2, ..., I; \; \forall l = 1, 2, ..., L \tag{2}$$

Condition (2) is a necessary (but not sufficient) condition for the existence of a representative household. This is already a very strong restriction, because it needs to hold for all potential wealth levels w.

Here is a graphical representation of parallel linear wealth expansion paths.

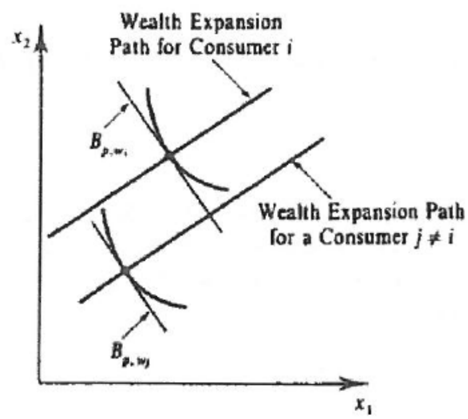


Figure 1: Parallel Linear Expansion Paths (MWG Figure 4.B.1)

3 Gorman Form and The Representative Household

Okay, now let's look for a necessary and sufficient condition for when we can represent the aggregate consumption behavior of a set of consumers as a positive representative household.

Definition 2. Let $\succeq$ be some rational and continuous preferences over $X \subseteq \mathbb{R}^L_+$. If $\succeq$ admits a utility function $u : X \to \mathbb{R}_+$ for which its indirect utility function $v(p, w)$ satisfies:

$$v(p, w) = a(p) + b(p)w$$

then the preferences are Gorman Form.

Definition 3. We say that a preference $\succeq$ defined on a consumption set $X \subseteq \mathbb{R}^L_+$ (with corresponding utility function u) corresponds to a Positive Representative Household if and only if the aggregate demand function satisfies (1) and moreover, it solves the utility maximization problem

$$\begin{aligned} X(p, \overline{w}) \;\; &= \;\; \arg\max_x u(x) \\ &\quad\;\; \text{s.t. } p \cdot x \leq \overline{w} \end{aligned}$$

Proposition 4. *Suppose that demand functions are differentiable in w. A necessary and sufficient condition for condition* (2) *to be satisfied is that preferences for all agents are Gorman form, and also:*

$$v_i(p, w_i) = a_i(p) + b(p)w_i \tag{3}$$

Moreover, if we can find preferences $\succeq$ *that admit the indirect utility function:*

$$v(p, w) = a(p) + b(p)w$$

with $a(p) = \sum_i a_i(p)$ *and* $b(p) = b(p)$*; then* $\succeq$ *correspond to a Positive Representative Household for this economy.*

Intuition for equation (3): $\partial v_i / \partial w_i = \partial v_j / \partial w_j$ for all p, w. That is, given prices, the **marginal utility of wealth does not vary across households**.

Now, a brief word about the "normative" representative household. The Normative Representative Household is a fictitious household that consumes the aggregate supply of the economy, and has the additional property that **any Pareto optimal allocation must maximize the utility of the Normative Representative Household that consumes the aggregate net supply of the economy at that allocation**. As it turns out, preferences that Gorman Aggregate give us both a Positive and Normative representative Household. We can use the representative household for both welfare analysis (Pareto Optimality of feasible allocations) and equilibrium determination (positive representative household).