14.04 Recitation 7: Private Information and Mechanism Design

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1 Mechanism Design

In theses notes we will talk about private information and "mechanism design."

What is a mechanism? A mechanism is an allocation algorithm that takes messages from participating agents and spits out an allocation. Many things can be broadly thought about as mechanisms. For instance, a school or job assignment scheme such as the National Residency Matching Program or the Boston Public School assignment policy are mechanisms. Other mechanisms include auctions that assigns a good to some bidder with an associated transaction price (e.g. the FCC wireless spectrum auction), tax codes (which take tax forms and spits out tax levies), and regulatory rules in certain industries.

The basic idea is that the participating agents have private information about their type θ, which is unknown to the mechanism. The agents are allowed to communicate a message m to the mechanism after learning their own types θ. Then the mechanism mechanically spits out an allocation according to the prespecified algorithm. In this course, the allocation just involve a transfer $f(m)$, which we can think of as a tax or a rent.

The central issue in mechanism design is that agents may have private information. This creates an incentive for the agent to pretend to have a type different than his actual type. For example, since tax schemes are typically more generous towards low income individuals, taxpayers may face an incentive to report a lower income than their actual income. As a consequence, the government will want to design tax policies keeping in mind incentive constraints that arise when there is private information.

As you can imagine, mechanism design is an entire subfield of economics which we will only briefly touch on in this course. However, we will give you a flavor of it today.

2 The Relevation Principle

Imagine a pure exchange economy with one period, two agents, and one good. The endowment of agent 1 is known by agent 1 alone, while the endowment of agent 2 is public information. Let $\theta \in \Theta$ denote the endowment of agent 1, realized with some probability $p(\theta)$. We will think of agent 1 as a villa and agent 2 as the monastery.

Suppose agent 1 and 2 agree ex ante to some resource allocation rule $f(m)$ specifying some transfer from agent 1 to agent 2 given some message m sent by villa 1 to monestary 2. Under these scheme, agent

1 waits to see output θ before sending message $m \in M$. Thus agent 1's decision problem is, for every θ, to maximize $u_1(\theta - f(m))$ by choice of m.

Suppose there exists a unique maximizing solution to this problem, denoted

$$m^*(\theta) \equiv \arg\max_m u_1(\theta - f(m)).$$

Then given $\theta \in \Theta$,

$$u_1(\theta - f(m^*(\theta))) \geq u_1(\theta - f(m)) \tag{1}$$

for all possible m. Notice that the previous equation implies that, given $\theta \in \Theta$,

$$u_1(\theta - f(m^*(\theta))) \geq u_1\left(\theta - f\left(m^*\left(\tilde{\theta}\right)\right)\right)$$

for any $\tilde{\theta} \in \Theta$. That is, the maximizing message is superior to the maximizing message that would have been if the agent instead had type $\tilde{\theta}$.

Now consider an alternative mechanism in which villa 1 announces its type θ directly instead of some messsage $m \in M$. In other words, now the message space is Θ rather than some arbitrary space M. Furthermore, suppose announced types $\tilde{\theta}$ effects transfers $g\left(\tilde{\theta}\right) = f\left(m^*\left(\tilde{\theta}\right)\right)$. Note that this defines a new mechanism with message space Θ and transfer rule g. By substitution, we have that

$$u_1(\theta - g(\theta)) \geq u_1\left(\theta - g\left(\tilde{\theta}\right)\right). \tag{2}$$

Notice the two mechanisms we considered have equivalent transfers and incentive constraints. The intuition for this equivalence is formalized in a general result in mechanism design called the *relevation principle*, which states that: If a social choice function can be implemented by an arbitrary mechanism, then the same function can be implemented by an incentive-compatible-direct-mechanism (i.e. in which players truthfully report their type) with the same outcome.

3 Taxation and Risk-sharing in Villages

We now derive the Pareto optimal outcome in this economy subject to the incentive compability constraint (also known as the truth-telling constraint) which arises from the private information of agent 1.

$$\begin{aligned} \max_{g(\cdot)} \quad & \lambda_1 \sum_\theta p(\theta) u_1(\theta - g(\theta)) + \lambda_2 \sum_\theta p(\theta) u_2(W + g(\theta)) \\ \text{s.t.} \quad & u_1(\theta - g(\theta)) \geq u_1\left(\theta - g\left(\tilde{\theta}\right)\right) \forall \theta, \tilde{\theta} \end{aligned}$$

Notice a few things: First, the relevation principle greatly simplifies the analysis of mechanism, since it allows us to work with equation (2) instead of equation (1). Second, we are maximizing over the function $g(\cdot)$, that is, we are choosing an optimal payment scheme. Finally notice that the incentive compability constraint when θ is single dimensional can be rewritten as

$$g(\theta) \leq g\left(\tilde{\theta}\right) \forall \theta, \tilde{\theta} \tag{3}$$

This means that the only transfer rule possible is some constant, independent of output. In other words, there is no risk-sharing in this economy! This is the consequence of private information.

In the case where θ is multi-dimensional, however, this implication is lost. The intuition is simple: If crops were low the villa might be eager to give up some other good. Indeed, relatively bad crops might be associated with relatively abundant net labor, if shocks were perceived early on, and one can imagine the villa eager to supply labor in return for a lower output rent.

4 Implementation via Lotteries

An alternative formulation of the Pareto problem is to consider probabilistic mechanisms, which may make parameter-contingent allocation possible even if there is only one good. Let lottery $\pi(\tau \mid \theta)$ denote the probability of transfer τ as a function of announcement θ. Suppose for simplicity that there is a finite number of values τ. The Pareto problem then becomes

$$\begin{aligned}
\max_{\pi(\cdot|\cdot)} \quad & \lambda_1 \sum_\theta p(\theta) \left[\sum_\tau \pi(\tau \mid \theta) u_1(\theta - \tau)\right] + \lambda_2 \sum_\theta p(\theta) \left[\sum_\tau \pi(\tau \mid \theta) u_2(W + \tau)\right] \\
\text{s.t.} \quad & \sum_\tau \pi(\tau \mid \theta) u_1(\theta - \tau) \geq \sum_\tau \pi\left(\tau \mid \tilde{\theta}\right) u_1(\theta - \tau) \forall \theta, \tilde{\theta}
\end{aligned}$$

First, note that this program is concave so its solution can be easily characterized. Furthermore, since we are optimizing over probabilities $\pi(\tau \mid \theta)$, this is in fact a linear program, so the solutions can be computed numerically.

5 Costly State Verification

Suppose that at some cost K in terms of forgone consumption, agent 2, the monastery, could observe or verify all of agent l's actual output. That is, agent 2 could audit or monitor agent 1. Even supposing that this cost K might be considerable, we can still ask whether such auditing would take place. This would allow us to determine whether monitoring is a serious possibility, something which might well have happened in practice.

Let $\pi(d \mid \tilde{\theta})$ depote the probability of an audit conditioned on announced parameter $\tilde{\theta}$, either $d = 1$ for audit or $d = 0$ for no audit. Let $\pi(\tau \mid \tilde{\theta}, d = 0)$ denote the probability of transfer τ conditioned on announcement $\tilde{\theta}$ and no audit. Let $\pi(\tau \mid \tilde{\theta}, \theta, d = 1)$ denote the probability of transfers τ conditioned on announcement $\tilde{\theta}$ the fact of an audit, and revelation of actual parameter value θ.

The Pareto problem is to maximizing by choice of probabilities

$$\pi(d \mid \tilde{\theta}), \pi(\tau \mid \tilde{\theta}, d = 0), \pi(\tau \mid \tilde{\theta}, \theta, d = 1)$$

the objective function

$$
\begin{aligned}
&\lambda_1 \sum_\theta p(\theta)\left\{\pi(d=0 \mid \theta)\sum_\tau\left[\pi(\tau \mid \theta, d=0)u_1(\theta-\tau)\right]+\pi(d=1 \mid \theta)\sum_\tau\left[\pi(\tau \mid \theta,\theta, d=1)u_1(\theta-\tau)\right]\right\}\\
&+\lambda_2 \sum_\theta p(\theta)\left\{\pi(d=0 \mid \theta)\sum_\tau\left[\pi(\tau \mid \theta, d=0)u_2(W+\tau)\right]+\pi(d=1 \mid \theta)\sum_\tau\left[\pi(\tau \mid \theta,\theta, d=1)u_2(W+\tau-K)\right]\right\}
\end{aligned}
$$

subject to

$$
\begin{aligned}
&\sum_\theta p(\theta)\left\{\pi(d=0 \mid \theta)\sum_\tau\left[\pi(\tau \mid \theta, d=0)u_1(\theta-\tau)\right]+\pi(d=1 \mid \theta)\sum_\tau\left[\pi(\tau \mid \theta,\theta, d=1)u_1(\theta-\tau)\right]\right\}\\
&\geq \sum_\theta p(\theta)\left\{\pi(d=0 \mid \tilde{\theta})\sum_\tau\left[\pi(\tau \mid \tilde{\theta}, d=0)u_1(\theta-\tau)\right]+\pi(d=1 \mid \tilde{\theta})\sum_\tau\left[\pi(\tau \mid \tilde{\theta},\theta, d=1)u_1(\theta-\tau)\right]\right\} \quad \forall\theta,\tilde{\theta}
\end{aligned}
$$

These constraints show how audit probabilities and consumption allocations, conditioned on being audited, play a role in the solution. The $\pi(\tau \mid \tilde{\theta},\theta,d=1)$ can be set equal to unity at τ values implying extreme values of consumption, zero or subsistence, for agent 1. These probabilities appear only on the right-hand side of the incentive constraints, or, to put it another way, they are never brought into the solution. The agent never lies about his parameter values. Still, audits can occur with positive probability. In this way the agent is threatened with off-equilibrium behavior.

A striking feature of the solution to program is that the probability of audits is positive even for relatively large values of audit cost K. Even rare, costly audits can help alleviate the incentive problems of nonfixed rentals, that is, of θ-contingent transfers.