14.04 Recitation 2: How do Consumers Respond to Price Changes?

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1 Introduction

Today we will analyze consumer behavior responses to a price change. The basic intuition is that the consequences can be decomposed into two effects.

The first is that the subsitution effect: If the price of one good goes up, the consumers substitutes towards another good. E.g. if the price of Uber goes up, then we start using Lyft. The substitution effect represents a movement along an indifference curve.

The second is the income effect: If the price of one good goes up, the consumer is effectively poorer. E.g. if rents for apartments go up relative to household income, then people are effectively poorer because their budget set effectively shrinks. So the price change in fact moves up to a different indifference curve – one with a lower utility.

Intuitively, we might expect that the income effect is especially important for goods that take up a large share of income and which are difficulty to substitute away from. We will formalize this idea in the Slutsky equation today.

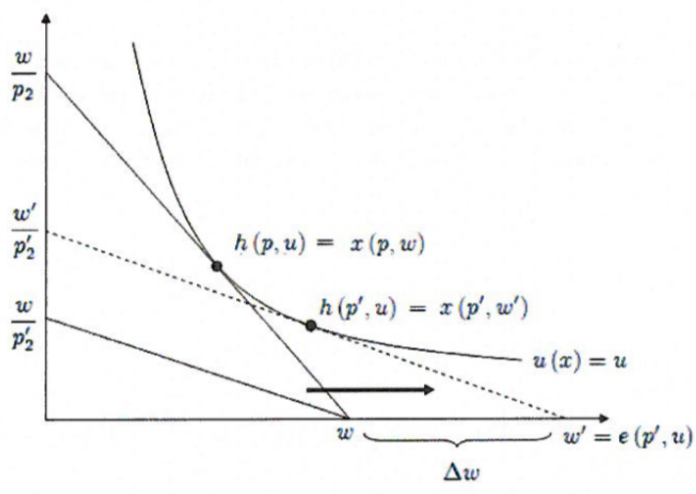


Figure 2: Compensated demand: case with $p_1 = 1$ and $p_2' > p_2$

2 Indirect Utility

The indirect utility function $v(p,w)$ is the utility of a utility-maximizing consumer facing prices p and wealth w.

$$\begin{aligned} v(p,w) &= \max_{x_1,x_2,\ldots,x_L} u(x_1,x_2,\ldots,x_L) \\ & \quad \text{s.t. } \sum_l p_l x_l = w \end{aligned}$$

3 Expenditure Function

We can reformulate the utility maximization problem as a cost minimization problem. The fact that we can do this comes from an important idea in convex optimization called "duality." We can then derive the expenditure function, which is the wealth (or expenditure) needed for a cost-minimizing agent to achieve a utility level u.

$$\begin{aligned} e(p,u) &= \min_{x_1,x_2,\ldots,x_L} \sum_l p_l x_l \\ & \quad \text{s.t. } u(x_1,x_2,\ldots,x_L) \geq u \end{aligned}$$

4 Marshallian Demand

Note that these two notions of optimization (utility maximization and cost minimization) correspond to two notions of demands. The first is called the uncompensated (or Marshallian) demand:

$$\begin{aligned} x(p,w) &= \arg\max_{x_1,x_2,\ldots,x_L} u(x_1,x_2,\ldots,x_L) \\ & \quad \text{s.t. } \sum_l p_l x_l = w \end{aligned}$$

5 Hicksian Demand

The second is called the compensated (or Hicksian) demand:

$$\begin{aligned} h(p,u) &= \arg\max_{x_1,x_2,\ldots,x_L} \sum_l p_l x_l \\ & \quad \text{s.t. } u(x_1,x_2,\ldots,x_L) \geq u \end{aligned}$$

6 Income and Substitution Effects

Now we can ask our first question: How does demand change when we change prices (holding w fixed)?

The resulting effect on the Marshallian demand is captured in the Slutsky equation:

$$\frac{\partial x_j}{\partial p_i} = \underbrace{\frac{\partial h_j}{\partial p_i}}_{\text{substitution effect}} - \underbrace{\frac{\partial x_j}{\partial w}x_i}_{\text{income effect}}$$

(What's the intuition behind this equation?)

The derivation of this equation follows from the following identity:

$$h_j(p,u) \equiv x_j(p,e(p,u))$$

If we now differentiate with respect to p_i and apply the chain rule, then we have that

$$\frac{\partial h_j}{\partial p_i} = \frac{\partial x_j}{\partial p_i} + \frac{\partial x_j}{\partial w}\frac{\partial e}{\partial p_i}.$$

The final step is to note that $\frac{\partial e}{\partial p_i} = h_i$. This follows from a result called the envelope theorem.

[An aside: We can rewrite the Slutsky equation in terms of elasticities. Elasticities are unit-less, which makes them particularly easy to compare. Note that we can rewrite Slutsky equation as

$$\underbrace{\frac{\partial x_j}{\partial p_i}\frac{p_i}{x_j}}_{\equiv \varepsilon_{ij}} = \underbrace{\frac{\partial h_j}{\partial p_i}\frac{p_i}{x_j}}_{\equiv \varepsilon_{ij}^H} - \underbrace{\frac{\partial x_j}{\partial w}\frac{w}{x_j}}_{\equiv \eta_j}\underbrace{\frac{p_i x_i}{w}}_{\equiv s_i}$$

This means that

$$\varepsilon_{ij} = \varepsilon_{ij}^H - s_i \eta_j$$

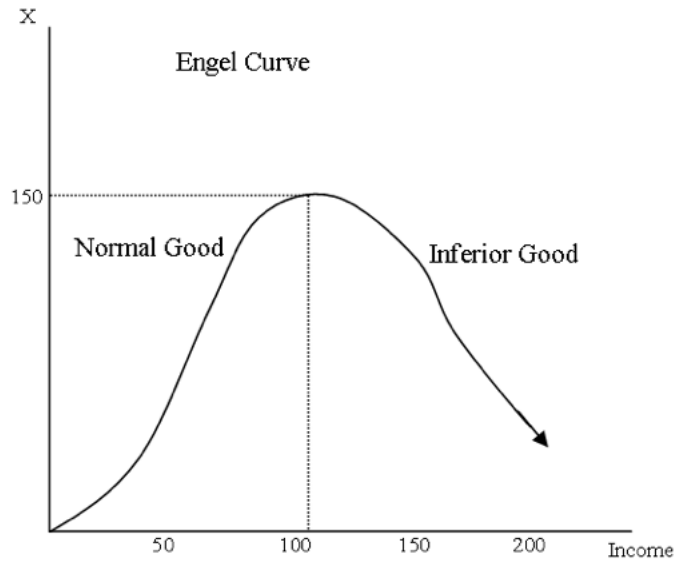
where ε_{ij} is the uncompensated elasticity of demand for good j respective to price i, ε_{ij}^H is the compensated elasticity of demand for good j respective to price i, s_i is the expenditure share of good i, and η_j is the income elasticity of demand for good j.]

7 Income Effects are Messy!

Let's go back to our discussion of income and substitution effects above. Remember Slutsky's equation?

$$\frac{\partial x_i}{\partial p_i} = \underbrace{\frac{\partial h_i}{\partial p_i}}_{\text{substitution effect}} - \underbrace{\frac{\partial x_i}{\partial w}x_i}_{\text{income effect}}$$

In general, we know the sign of the own-price substitution effect ($\frac{\partial h_i}{\partial p_i}$) – it is always negative. (This follows from the convexity of the indifference curve. How?) However, economic theory has no prediction about the sign of the income effect ($\frac{\partial x_i}{\partial w}$), which could be either positive or negative. If it's positive, we say the good is "normal;" if negative, we say it is "inferior." For example, we may want to consume less $1 Burgers as we get richer. In this case, the income effect is negative. But we may want more Big Macs as we get richer. We can represent this relationship in the follow graph:



(In fact, if the income effect is very strongly negative, we might even have a "Giffen good." A Giffen good is something we want to consume more of as its price rises, because the price increase makes us poorer which makes us want this good a lot more. This is very weird, but possible according to consumer theory. It's an open question, however, whether Giffen goods exist at all!)

So, income effects generate messy predictions about consumer behavior. One consequence of this messiness is that aggregate consumer behavior may not be correspond to the behavior of a single representative consumer if income effects are heterogenous across consumers. In order to rule out "weird" aggregate demand functions, we will need special preference functions which restricts the relationship between individual demand and individual income. We will come back to this in the future.