14.04 Recitation 9: International Trade

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1 Setup

Today, we study two questions that are important in the study of international trade:

1. How does a change in the world price of a good affect the equilibrium factor prices and factor allocations in an open economy?

2. How does a change in the abundance of a factor affect the equilibrium factor prices and factor allocations in an open economy?

Consider an economy with $J = 2$ industries. Each industry j produces a consumer good q_j from a vector of 2 factors, $z_j = (z_{1j}, z_{2j}) \geq 0$. Each industry j has a concave, strictly increasing, and differentiable production function $f_j(z_j)$ with **constant returns to scale**. The prices of the J produced consumption goods are fixed at $p = (p_1, ..., p_J) \gg 0$.

The economy has total endowments of the $L = 2$ factor inputs, $\bar{z} = (\bar{z}_1, \bar{z}_2) \gg 0$. These endowments are initially owned by consumers and cannot be consumed directly. Factor 1 is often thought of as labor and factor 2 as capital

This setup represents a small open economy whose trading decisions in the world markets for consumption goods have little effect on the world prices of these goods. Output is sold in world markets. Factors are immobile (but freely traded locally) and must be used for production within the country

Our goal is to characterize the Walrasian equilibrium factor prices w^* and allocation z^*. The Walrasian equilibrium of this economy given the fixed output prices p consists of an input price vector w^* and a factor allocation z^* such that industries receive their desired factor demands under prices (p, w^*) and all the factor markets clear. That is, for all industries j,

$$z_j^* \in \arg\max_{z_j} p_j f_j(z_j) - w^* \cdot z_j,$$

and for all l,

$$\sum_j z_{lj}^* = \bar{z}_l.$$

Because of the concavity of industries' production functions, first-order conditions are both necessary and sufficient for the characterization of optimal factor demands. Therefore, the $L(J+1)$ variables formed by the factor allocation z^* and the factor prices w^* constitute an equilibrium if and only if they satisfy the following $L(J+1)$ equations (we assume an interior solution here): That is, for all j,

$$p_j \frac{\partial f_j\left(z_j^*\right)}{\partial z_{lj}} = w_j^*$$

and for all l,

$$\sum_j z_{lj}^* = \bar{z}_l.$$

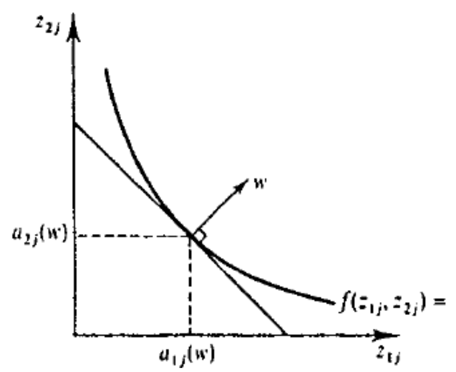
The equilibrium output levels are then $q_j^* = f_j\left(z_j^*\right)$ for every j.

2 Graphical illustrations

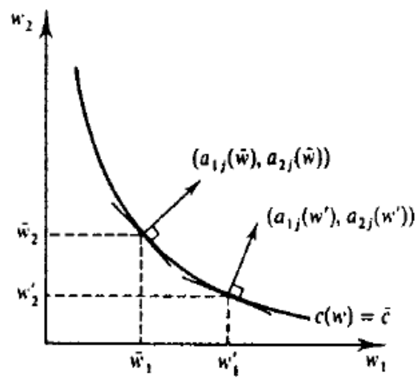
For every vector of factor prices $w = (w_1, w_2)$, we define

- $c_j(w)$ the minimum cost of producing one unit of good j
- $a_j(w) = (a_{1j}(w), a_{2j}(w))$ the input combination (assumed unique) at which this minimum cost is reached.

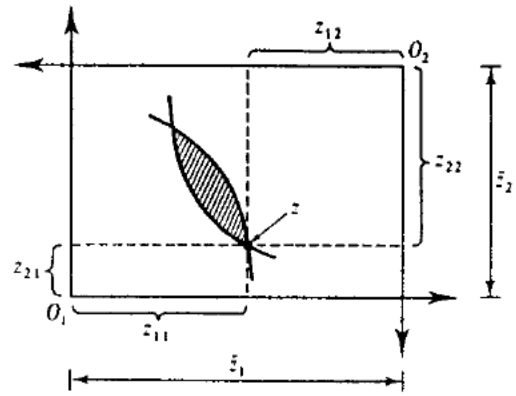
We first depict the unit isoquant of industry j, $\left\{(z_{1j}, z_{2j}) \in \mathbb{R}_+^2 : f_j(z_{1j}, z_{2j}) = 1\right\}$, along with the cost-minimizing input combination $(a_{1j}(w), a_{2j}(w))$:



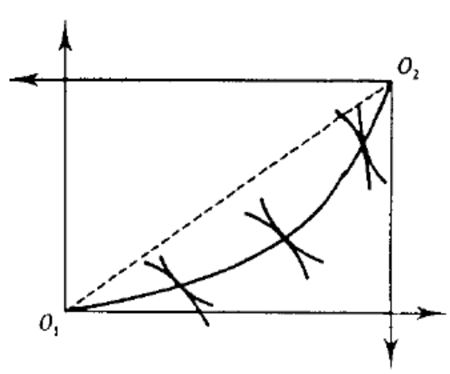
Next, we draw a level curve of the unit cost function: $\left\{(w_1, w_2) : c_j(w_1, w_2) = \bar{c}\right\}$. This curve is downward sloping because as w_1 increases, w_2 must fall in order to keep the minimized costs of producing one unit of good j unchanged. Moreover, the set $\left\{(w_1, w_2) : c_j(w_1, w_2) \geq \bar{c}\right\}$ is convex because of the concavity of the cost function $c_j(w)$ in w. Note that the vector $\nabla c_j(w)$, which is normal to the level curve at $\overline{w} = (\overline{w}_1, \overline{w}_2)$, is exactly $(a_{1j}(w), a_{2j}(w))$, As we move along the curve toward higher w_1 and lower w_2, the ratio $a_{1j}(w)/a_{2j}(w)$ falls.



The below figure represents the possible allocations of the factor endowments between the two industries in an Edgeworth box of size $\bar{z}_1$ by $\bar{z}_2$. The factors used by industry 1 are measured from the southwest corner; those used by industry 2 are measured from the northeast corner. We also represent the isoquants of the two industries in this Edgeworth box. Figure depicts an inefficient allocation z of the inputs between the two industries: Any allocation in the interior of the hatched region generates more output of both goods than does z



Finally, we draw the Pareto set of factor allocations, that is, the set of factor allocations at which it is not possible, with the given total factor endowments, to produce more of one good without producing less of the other. The Pareto set (endpoints excluded) must lie all above or all below or be coincident with the diagonal of the Edgeworth box. If it ever cuts the diagonal then because of constant returns, the isoquants of the two industries must in fact be tangent all along the diagonal, and so the diagonal must be the Pareto set.



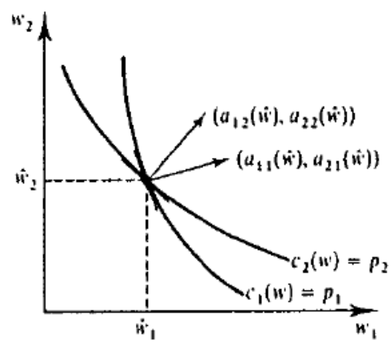
3 Solving for Equilibrium Factor Prices and Allocations

To determine the equilibrium factor prices, suppose that we have an interior equilibrium in which the production levels of the two goods are strictly positive. Given our constant returns assumption, a necessary condition for (w_1^*, w_2^*) to be the factor prices in an interior equilibrium is that it satisfies the system of equations:

$$c_1(w_1, w_2) = p_1 \text{ and } c_2(w_1, w_2) = p_2$$

Interpretation: prices must be equal to unit cost. This gives us two equations for the two unknown factor prices w_1 and w_2.

We depict the level curves for the two unit cost functions in the figure below. Note that the necessary condition for $(\hat{w}_1, \hat{w}_2)$ to be the factor prices of an interior equilibrium is that these curves cross at $(\hat{w}_1, \hat{w}_2)$. Moreover, the factor intensity assumption implies that whenever the two curves cross, the curve for industry 2 must be flatter (less negatively sloped) than that for industry 1. From this. it follows that the two curves can cross at most once.



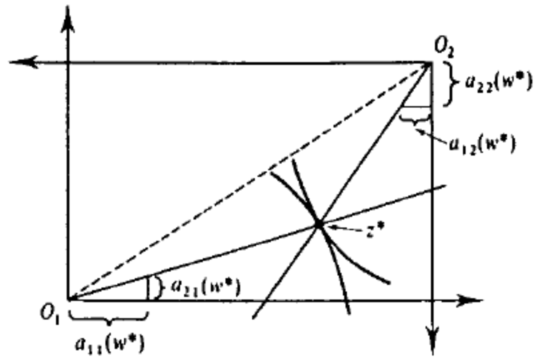
Once equilibrium factor prices w^* are known, the equilibrium output levels can be found by determining the unique point (z_1^*, z_2^*) such that

$$\frac{z_{11}^*}{z_{21}^*} = \frac{a_{11}(w^*)}{a_{21}(w^*)} \text{ and } \frac{z_{12}^*}{z_{22}^*} = \frac{a_{12}(w^*)}{a_{22}(w^*)}$$

and

$$z_{11}^* + z_{12}^* = \bar{z}_1 \text{ and } z_{21}^* + z_{22}^* = \bar{z}_2.$$

The equilibrium factor allocation are shown in the Edgeworth box below.



4 Factor Price Equilization Theorem

In the 2 x 2 production model, if the factor intensity condition holds, then as long as the economy does not specialize in the production of a single good, the equilibrium factor prices depend *only on the technologies of the two industries and on the output prices p*. Thus, the levels of the endowments matter only to the extent that they determine whether the economy specializes. This result is known in the international trade literature as the *factor price equalization theorem.* The theorem provides conditions (which include the presence of tradable consumption goods, identical production technologies in each country, and price-taking behavior) under which the prices of nontradable factors are equalized across nonspecialized countries.

5 Comparative Static Results

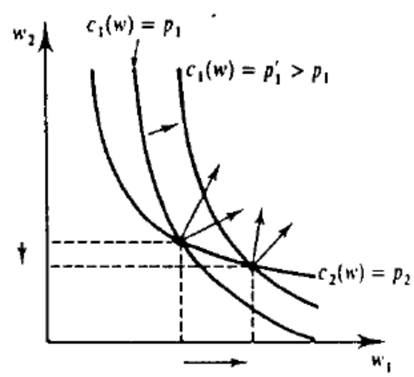
Definition. The production of good 1 is **relatively more intensive** in factor 1 than is the production of good 2 if

$$\frac{a_{11}(w)}{a_{21}(w)} > \frac{a_{12}(w)}{a_{22}(w)}$$

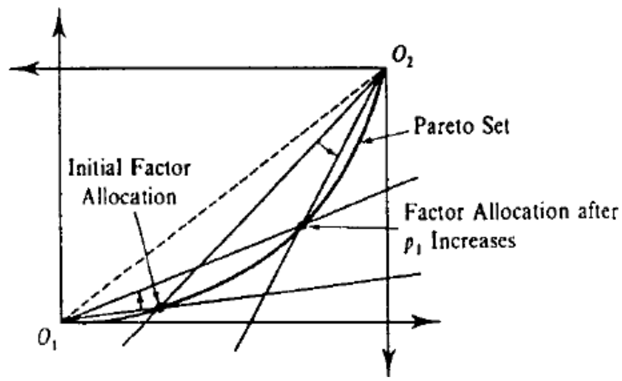
at all factor prices $w = (w_1, w_2)$.

Theorem. *In the 2 x 2 production model with the factor intensity assumption, if p_i increases, then the equilibrium price of the factor more intensively used in the production of good i increases, while the price of the other factor decreases (assuming interior equilibria both before and after the price change).*

How does a change in the price of one of the outputs, say p_1, affect the equilibrium factor prices and factor allocations? The increase in p_1 shifts industry 1's curve outward toward higher factor price levels; the point of intersection of the two curves moves out along industry 2's curve to a higher level of w_1 and a lower level of w_2.

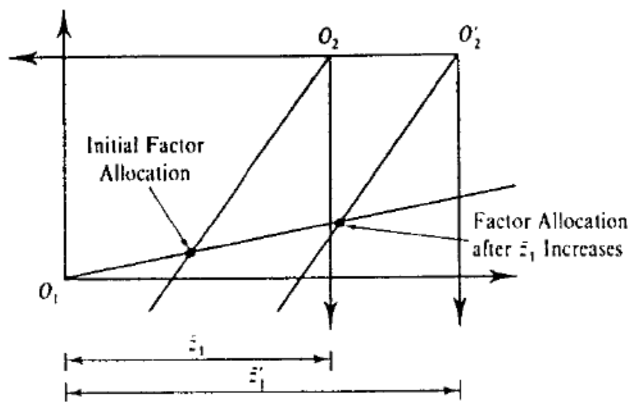


This figure below depicts the resulting change in the equilibrium allocation of factors from an increase in p_1. As can be seen, the factor allocation moves to a new point in the Pareto set at which the output of good I has risen and that of good 2 has fallen.



Theorem (Rybcszynski Theorem)**.** *In the 2 x 2 production model with the factor intensity assumption, if the endowment of a factor increases, then the production of the good that uses this factor relatively more intensively increases and the production of the other good decreases (assuming interior equilibria both before and after the change of endowment).*

What happens when the total availability of factor 1 increases from z_1 to z_1'? Because neither the output prices nor the technologies have changed, the factor input prices remain unaltered (as long as the economy does not specialize). As a result, factor intensities also do not change. The new input allocation is then easily determined in the superimposed Edgeworth boxes; we merely find the new intersection of the two rays associated with the unaltered factor intensity levels.


O_2
O_2'
Initial Factor Allocation
Factor Allocation after $\bar{z}_1$ Increases
O_1
$\bar{z}_1$
$\bar{z}_1'$