1 Mathematical constants

1.1 zeta2, $\zeta(2)$

$$\zeta(2) = \frac{2}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{(m-1)^4}{2m-1} \right) \tag{No label}$$

S-fraction

$$\zeta(2) = 1 + \frac{1}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\left(\frac{m}{2}\right)^2}{1} \right) \tag{10.4.8}$$

$$\zeta(2) = \sum_{n=1}^{k} \left(\frac{1}{n^2} \right) + \frac{2}{2k+1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{(m-1)^4}{(2m-1)(2k+1)} \right) \tag{10.4.9}$$

$$k \in \mathbb{Z} \backslash \mathbb{Z}^-$$

$$\zeta(2) = 2 \sum_{n=1}^{k} \left(\frac{(-1)^{n-1}}{n^2} \right) + \frac{(-1)^k}{k^2+k+1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-(m-1)^4}{(m-1)^2 + m^2 + k^2 + k} \right) \tag{10.4.10}$$

$$k \in \mathbb{Z} \backslash \mathbb{Z}^-$$

$$\zeta(2) = \frac{5}{3} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{(m-1)^4}{11(m-1)^2 + 11(m-1) + 3} \right) \tag{10.4.11}$$

$$\zeta(2) = \frac{1}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-\frac{\left(\left((m-2)\frac{1}{2}+1\right)\left((m-2)\frac{1}{2}+1\right)\right)\left((m-2)\frac{1}{2}+1\right)}{(m-1)(1+(m-1))}}{1} \right) \tag{15.6.8}$$

continued fraction

1.2 Archimedes' constant, symbol π

$$\pi = 4\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \tag{10.2.1}$$

power series

$$\pi = 3 + \frac{1}{7} + \frac{1}{15} + \frac{1}{1} + \frac{1}{292} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{1} + \frac{1}{3} + \cdots \tag{10.2.4}$$

regular continued fraction

$$\pi = 2 + \frac{1}{1} + 2\operatorname*{K}_{m=2}^{\infty}\left(\frac{(m-1)\,m}{1}\right) \tag{10.2.5}$$

$$\pi = \frac{4}{1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{(m-1)^2}{2m-1}\right) \tag{10.2.6}$$

S-fraction

$$\pi = \frac{4}{4k+1} + 2^{4k}\left(\frac{(k!)^2}{(2k)!}\right)^2 \operatorname*{K}_{m=2}^{\infty}\left(\frac{(2m-3)^2}{8k+2}\right) \tag{10.2.7}$$

S-fraction

$$\pi = \frac{4}{1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{(2m-3)^2}{2}\right) \tag{10.2.8}$$

S-fraction

$$\pi = \frac{4}{4+1} + 4\operatorname*{K}_{m=2}^{\infty}\left(\frac{(2m-3)^2}{8+2}\right) \tag{10.2.9}$$

S-fraction

$$\pi = 3 + \operatorname*{K}_{m=1}^{\infty}\left(\frac{(2m-1)^2}{6}\right) \tag{10.2.10}$$

S-fraction

1.3 Euler's number, base of the natural logarithm

$$e = \sum_{k=0}^{\infty} \frac{1}{(k)!} \tag{10.3.2}$$

power series

$$e = 2 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{1} \right) \tag{10.3.6}$$

regular continued fraction

$$e = \frac{1}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-1}{m-1} \right) \tag{10.3.7}$$

regular continued fraction

$$\frac{e-1}{e+1} = \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{4m-2} \right) \tag{10.3.8}$$

regular continued fraction

$$e = 2 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{m+1}{m+1} \right) \tag{10.3.9}$$

regular continued fraction

$$e = 1 + \frac{2}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{1}{6 + (m-2)\, 4} \right) \tag{10.3.10}$$

regular continued fraction

1.4 Regular continued fractions

$$\sqrt{e} = 1 + \frac{1}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{1}{1} \right) \tag{10.4.1}$$

regular continued fraction

$$e^{\frac{1}{n}} = 1 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{2\frac{m+2}{3} - 1n - 1} \right) \tag{10.4.2}$$

regular continued fraction

$$e^{\frac{1}{n}} = \frac{n+1}{n} + \frac{1}{n} \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{2n-1} \right) \tag{10.4.3}$$

regular continued fraction

$$e^{\frac{1}{n}} = \frac{1}{n-1} + \frac{1}{2n} + n \operatorname*{K}_{m=3}^{\infty} \left(\frac{1}{1} \right) \tag{10.4.4}$$

regular continued fraction

$$\sqrt{\pi} = 1 + \frac{1}{1} + \frac{1}{3} + \frac{1}{2} + \frac{1}{1} + \frac{1}{1} + \frac{1}{6} + \frac{1}{1} + \frac{1}{28} + \frac{1}{13} + \frac{1}{1} + \dots \tag{10.4.5}$$

regular continued fraction

$$ee = 7 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{\frac{3(m+4)}{5} - 1} \right) \tag{10.4.6}$$

regular continued fraction

$$\frac{\pi^2}{12} = \frac{1}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{(m-1)^4}{2m-1} \right) \tag{10.4.7}$$

S-fraction

$$\frac{e^{\frac{2n}{\beta}} - 1}{e^{\frac{2n}{\beta}} + 1} = \frac{n}{\beta} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{n^2}{(2m-1)\,\beta} \right) \tag{10.4.12}$$

S-fraction

1.5 The natural logarithm, $\ln(2)$

$$\ln(2) = \sum_{k=0}^{\infty} \frac{(-1)^k \, 1^{k+1}}{k+1} \tag{10.5.2}$$

power series

$$\ln(2) = \frac{1}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{(m-1)^2}{1} \right) \tag{10.5.3}$$

S-fraction

$$\ln(2) = \frac{1}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{m}{4m-4}}{1} \right) \tag{10.5.4}$$

S-fraction

1.6 Pythagoras' constant, the square root of two

$$1 + \sqrt{2} = 2 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{2} \right) \tag{10.6.3}$$

regular continued fraction

$$\left(1 + \sqrt{2}\right)^2 = 5 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{1} \right) \tag{10.7.1}$$

regular continued fraction

$$\left(1 + \sqrt{2}\right)^3 = 14 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{14} \right) \tag{10.7.2}$$

regular continued fraction

$$\left(1 + \sqrt{2}\right)^4 = 33 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{1} \right) \tag{10.7.3}$$

regular continued fraction

$$\left(1 + \sqrt{2}\right)^5 = 82 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{82} \right) \tag{10.7.4}$$

regular continued fraction

1.7 Delian

$$\sqrt[3]{2} = 1 + \frac{1}{3} + \frac{1}{1} + \frac{1}{5} + \frac{1}{1} + \frac{1}{1} + \frac{1}{4} + \frac{1}{1} + \frac{1}{1} + \frac{1}{8} + \frac{1}{1} + \cdots \tag{10.8.1}$$

regular continued fraction

$$\sqrt[3]{2} = 1 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{\frac{3(m+1)}{2} - 2}{3m} \right) \tag{10.8.2}$$

1.8 Theodorus

$$\sqrt{3} = 1 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{1} \right) \tag{10.9.1}$$

1.9 Euler's constant, symbol γ

$$\gamma = -\log(n) + \sum_{k=0}^{\infty} \frac{1}{k} \tag{10.10.1}$$

power series

$$\gamma = 0 + \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{1} + \frac{1}{2} + \frac{1}{1} + \frac{1}{4} + \frac{1}{3} + \frac{1}{13} + \frac{1}{5} + \cdots \tag{10.10.4}$$

regular continued fraction

1.10 Golden ratio, symbol ϕ

$$\phi = 1 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{1} \right) \tag{10.11.4}$$

regular continued fraction

1.11 The rabbit constant, symbol ρ

$$\rho = \sum_{k=0}^{\infty} 2^{-\left\lfloor (k+1)\frac{\sqrt{5}+1}{2} \right\rfloor} \qquad (10.12.2)$$

power series

$$\rho = \operatorname*{K}_{m=1}^{\infty} \left(\frac{1}{2^{:-F_{m-1}}} \right) \qquad (10.12.5)$$

regular continued fraction

1.12 Catalan's constant, symbol G

$$G = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2} \qquad (10.14.1)$$

power series

$$G = 0 + \frac{1}{1} + \frac{1}{10} + \frac{1}{1} + \frac{1}{8} + \frac{1}{1} + \frac{1}{88} + \frac{1}{4} + \frac{1}{1} + \frac{1}{1} + \frac{1}{7} + \cdots \qquad (10.14.3)$$

regular continued fraction

$$G = \frac{1}{2} + \frac{1}{\frac{1}{2}} + \frac{1}{2} \operatorname*{K}_{m=2}^{\infty} \left(\frac{\left(\frac{m}{2}\right)^2}{\frac{1}{2}} \right) \qquad (10.14.4)$$

S-fraction

$$G = 1 + \frac{-1}{3} + \frac{1}{2} \operatorname*{K}_{m=2}^{\infty} \left(\frac{m^2}{1} \right) \qquad (10.14.5)$$

continued fraction

$$G = \frac{\frac{13}{2}}{7} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{(2m-3)^4 (2m-2)^4 \left(20(m-2)^2 - 8(m-2) + 1\right) \left(20(m)^2 - 8(m) + 1\right)}{3520(m-1)^6 + 5632(m-1)^5 + 2064(m-1)^4 - 384(m-1)^3 - 156(m-1)^2 + 16(m-1) + 7} \right) \qquad (10.14.6)$$

continued fraction

1.13 Gompertz' constant, symbol G

$$G = \frac{1}{2} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-(m-1)^2}{2m} \right) \tag{10.15.1}$$

continued fraction

$$G = \frac{1}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{m}{2}}{1} \right) \tag{10.15.2}$$

continued fraction

1.14 zeta4, $\zeta(4)$

$$\zeta(4) = \frac{13}{12} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(m-1)^7 (3m-2)(3m-3)(3m-4)}{3(2(m-1)+1)\left(3(m-1)^2+3(m-1)+1\right)\left(15(m-1)^2+15(m-1)+4\right)} \right) \tag{22.1.18}$$

$$\zeta(4) = 1 + \frac{1}{12} + \frac{1}{6} + \frac{1}{1} + \frac{1}{3} + \frac{1}{1} + \frac{1}{4} + \frac{1}{183} + \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \ldots \tag{22.1.19}$$

regular continued fraction

2 Elementary functions

2.1 The exponential function

$$e^z = \frac{1}{1} + \frac{-z}{1} + \mathop{\mathrm{K}}_{m=3}^{\infty} \left(\frac{-\frac{1}{2(m-1)} z}{1} \right) \tag{11.1.x}$$

C-fraction

$$e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!} \tag{11.1.1}$$

power series

$$e^z = 1 + \frac{2z}{2-z} + \frac{\frac{z^2}{6}}{1} + \underset{m=3}{\overset{\infty}{\mathrm{K}}} \left(\frac{\frac{1}{4(2m-3)(2m-1)} z^2}{1} \right) \tag{11.1.2}$$

S-fraction

$$e^z = 1 + \frac{z}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{-\frac{1}{2(m-1)} z}{1} \right) \tag{11.1.3}$$

C-fraction

$$e^z = 1 + \frac{z}{1-z} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{(m-1)\, z}{m-z} \right) \tag{11.1.4}$$

T-fraction

2.2 The natural logarithm

$$\ln(1+z) = z + \frac{-\frac{z^2}{2}}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\frac{\left(\frac{m}{2}+1\right)^2 z}{m(m+1)}}{1} \right) \tag{6.8.8}$$

Thiele interpolating fraction

$$\ln(1+z) = \sum_{k=0}^{\infty} \frac{(-1)^{k+2}}{k+1} z^{k+1} \tag{11.2.1}$$

$|z| < 1$

power series

$$\ln(1+z) = \frac{z}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\frac{m}{4(m-1)} z}{1} \right) \tag{11.2.2}$$

$|\arg 1 + z| < \pi$

S-fraction

$$\ln\left(1+z\right) = \frac{2z}{2+z} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{-\left(m-1\right)^{2}z^{2}}{\left(2m-1\right)\left(2+z\right)}\right) \tag{11.2.3}$$

$$\left|\arg 1 - \frac{z^{2}}{\left(2+z\right)^{2}}\right| < \pi$$

Euler even contraction

$$\ln\left(\frac{1+z}{1-z}\right) = \frac{2z}{1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{-\frac{\left(m-1\right)^{2}}{\left(2m-3\right)\left(2m-1\right)}z^{2}}{1}\right) \tag{11.2.4}$$

$$\left|\arg 1 - z^{2}\right| < \pi$$

S-fraction

2.3 Trigonometric functions

$$\sin\left(z\right) = \sum_{k=0}^{\infty}\frac{\left(-1\right)^{k}}{\left(2k+1\right)!}z^{2k+1} \tag{11.3.1}$$

power series

2.4 Trigonometric functions

$$\cos\left(z\right) = \sum_{k=0}^{\infty}\frac{\left(-1\right)^{k}}{\left(2k\right)!}z^{2k} \tag{11.3.2}$$

power series

2.5 Trigonometric functions

$$\tan\left(z\right) = \sum_{k=0}^{\infty}\frac{4^{k+1}\left(4^{k+1}-1\right)\left|B_{2(k+1)}\right|}{\left(2\left(k+1\right)\right)!}z^{2(k+1)-1} \tag{11.3.3}$$

$$|z| < \frac{\pi}{2}$$

power series

$$\tan\left(z\right) = \frac{z}{1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{-\frac{z^{2}}{\left(2m-1\right)\left(2m-3\right)}}{1}\right) \tag{11.3.7}$$

S-fraction

$$\tan\left(\frac{\pi z}{4}\right) = \frac{z}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{(2m-3)^2 - z^2}{2}\right) \tag{11.3.8}$$

Thiele interpolating fraction

$$\tan(z) = \frac{z}{1} + \frac{-4\pi^{-2}z^2}{1} + \underset{m=3}{\overset{\infty}{\mathrm{K}}}\left(\frac{(m-2)^4 - 4\pi^{-2}(m-2)^2 z^2}{2m-3}\right) \tag{11.3.9}$$

Thiele interpolating fraction

2.6 Inverse trigonometric functions

$$\mathrm{Arcsin}(z) = z + \sum_{k=0}^{\infty} \frac{(2k-1)!!}{(2k)!!\,(2k+1)} z^{2k+1} \tag{11.4.1}$$

$$|z| < 1$$

power series

$$\mathrm{Arcsin}(z) = \frac{\frac{z}{\sqrt{1-z^2}}}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{\frac{\frac{(m-1)^2}{(2m-3)(2m-1)}z^2}{1-z^2}}{1}\right) \tag{11.4.4}$$

$$\left|\arg 1 - z^2\right| < \pi$$

S-fraction

$$\mathrm{Arcsin}(z) = \frac{z\sqrt{1-z^2}}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{-\frac{m(m-1)}{(2m-1)(2m-3)}z^2}{1}\right) \tag{11.4.5}$$

$$\left|\arg 1 - z^2\right| < \pi$$

S-fraction

2.7 Inverse trigonometric functions

$$\operatorname{Arccos}(z) = \frac{\pi}{2} - z + \sum_{k=0}^{\infty} -\frac{(2k-1)!!}{(2k)!!\,(2k+1)} z^{2k+1} \tag{11.4.2}$$

$$|z| < 1$$

power series

$$\operatorname{Arccos}(z) = \frac{\frac{\sqrt{1-z^2}}{z}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{\frac{(m-1)^2}{(2m-3)(2m-1)}\left(1-z^2\right)}{z^2}}{1} \right) \tag{11.4.6}$$

$$\operatorname{Re}(z) > 0, \frac{1-z^2}{z^2} > -1$$

S-fraction

$$\operatorname{Arccos}(z) = \frac{z\sqrt{1-z^2}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\frac{m(m-1)}{(2m-1)(2m-3)}\left(1-z^2\right)}{1} \right) \tag{11.4.7}$$

$$\operatorname{Re}(z) > 0$$

S-fraction

2.8 Inverse trigonometric functions

$$\operatorname{Arctan}(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} z^{2k+1} \tag{11.4.3}$$

$$|z| < 1$$

power series

$$\operatorname{Arctan}(z) = \frac{z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(m-1)^2 z^2}{2m-1} \right) \tag{11.4.8}$$

$$\mathit{not}\,(iz < -1)\,, \mathit{not}\,(iz > 1)$$

S-fraction

$$\operatorname{Arctan}(z) = \frac{\frac{z}{1+z^2}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\frac{m(m-1)}{(2m-3)(2m-1)}\frac{z^2}{1+z^2}}{1} \right) \tag{11.4.9}$$

$$\mathit{not}\,(iz < -1)\,, \mathit{not}\,(iz > 1)$$

S-fraction

2.9 Hyperbolic functions

$$\sinh(z) = \sum_{k=0}^{\infty} \frac{1}{(2k+1)!} z^{2k+1} \tag{11.5.1}$$

power series

2.10 Hyperbolic functions

$$\cosh(z) = \sum_{k=0}^{\infty} \frac{1}{(2k)!} z^{2k} \tag{11.5.2}$$

power series

2.11 Hyperbolic functions

$$\tanh(z) = \sum_{k=0}^{\infty} \left(4^{k+1}\left(4^{k+1}-1\right)\frac{B_{2(k+1)}}{(2(k+1))!}\right) z^{2(k+1)-1} \tag{11.5.3}$$

$$|z| < \frac{\pi}{2}$$

power series

$$\tanh(z) = \frac{z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{\frac{1}{(2m-3)(2m-1)}z^2}{1}\right) \tag{11.5.5}$$

S-fraction

2.12 Hyperbolic functions

$$\coth(z) = \sum_{k=0}^{\infty} \frac{4^k B_{2k}}{(2k)!} z^{2k-1} \tag{11.5.4}$$

$$|z| < \pi$$

power series

$$\coth(z) = \frac{1}{z} + \frac{4\pi^{-2}z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{(m-1)^2\left((m-1)^2 + 4\pi^{-2}z^2\right)}{2m-1}\right) \tag{11.5.6}$$

Thiele interpolating fraction

2.13 Inverse hyperbolic functions

$$\operatorname{Arcsinh}(z) = z + \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1)!!}{(2k)!! (2k+1)} z^{2k+1} \tag{11.6.1}$$

$$|z| < 1$$

power series

$$\operatorname{Arcsinh}(z) = \frac{z\sqrt{1+z^2}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{m(m-1)}{(2m-3)(2m-1)} z^2}{1} \right) \tag{11.6.4}$$

$$\mathit{not}\,(iz < -1)\,, \mathit{not}\,(iz > 1)$$

S-fraction

$$\operatorname{Arcsinh}(z) = \frac{\frac{z}{\sqrt{1+z^2}}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-\frac{\frac{(m-1)^2}{(2m-3)(2m-1)} z^2}{1+z^2}}{1} \right) \tag{11.6.5}$$

$$\mathit{not}\,(iz < -1)\,, \mathit{not}\,(iz > 1)$$

S-fraction

2.14 Inverse hyperbolic functions

$$\operatorname{Arccosh}\left(\frac{1}{z}\right) = \ln\left(\frac{2}{z}\right) + \sum_{k=0}^{\infty} -\frac{(2k-1)!!}{(2k)!! (2k)} z^{2k} \tag{11.6.2}$$

$$|z| < 1$$

power series

$$\operatorname{Arccosh}(z) = \frac{z\sqrt{z^2-1}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{m(m-1)}{(2m-3)(2m-1)} \left(z^2-1\right)}{1} \right) \tag{11.6.6}$$

$$\operatorname{Re}(z) > 0$$

S-fraction

$$\operatorname{Arccosh}(z) = \frac{\frac{\sqrt{z^2-1}}{z}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-\frac{\frac{(m-1)^2}{(2m-3)(2m-1)} \left(z^2-1\right)}{z^2}}{1} \right) \tag{11.6.7}$$

$$\left|\arg \frac{1}{z^2}\right| < \pi$$

S-fraction

2.15 Inverse hyperbolic functions

$$\operatorname{Arctanh}(z) = \sum_{k=0}^{\infty} \frac{1}{2k+1} z^{2k+1} \tag{11.6.3}$$

$$|z| < 1$$

power series

$$\operatorname{Arctanh}(z) = \frac{\frac{z}{1-z^2}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{\frac{m(m-1)}{(2m-3)(2m-1)} z^2}{1-z^2}}{1} \right) \tag{11.6.8}$$

$$\left|\arg 1 - z^2\right| < \pi$$

S-fraction

$$\operatorname{Arctanh}(z) = \frac{z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\frac{(m-1)^2 z^2}{4(m-1)^2-1}}{1} \right) \tag{11.6.9}$$

$$\left|\arg 1 - z^2\right| < \pi$$

S-fraction

2.16 The power function

$$(1+z)^n = 1 + \frac{nz}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{\frac{m}{2} - nz}{2(m-1)}}{1} \right) \tag{11.7.1}$$

$$|\arg z + 1| < \pi$$

C-fraction

$$(1+z)^n = \frac{1}{1} + \frac{-nz}{1} + \mathop{\mathrm{K}}_{m=3}^{\infty} \left(\frac{\frac{\frac{m-1}{2} + nz}{2(m-2)}}{1} \right) \tag{11.7.2}$$

$$|\arg z + 1| < \pi$$

C-fraction

$$(1+z)^n = \frac{1}{1} + \frac{-nz}{1+z} + \frac{\frac{(n-1)z}{2}}{1} + \mathop{\mathrm{K}}_{m=4}^{\infty} \left(\frac{\frac{\left(-n - \frac{m-2}{2}\right)z}{2(m-1)(1+z)}}{1} \right) \tag{11.7.3}$$

$$|\arg z + 1| < \pi$$

C-fraction

$$\left(\frac{z+1}{z-1}\right)^n = 1 + \frac{\frac{2n}{z}}{1-\frac{n}{z}} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{\frac{n^2-(m-1)^2}{(2(m-1)-1)(2(m-1)+1)z^2}}{1}\right) \tag{11.7.4}$$

$(\mathrm{Re}\,(z) < -1)$ *or* $(\mathrm{Re}\,(z) > 1)$

C-fraction

3 Gamma function and related functions

3.1 Binet function

$$Binet\,(z) = \sum_{k=0}^{\infty}\left(\frac{B_{2(k+1)}}{(2k+1)\,(2k+2)}z^{-2k-1}\right) \tag{12.2.6}$$

$|\arg z| < \pi$

asymptotic series

3.2 Polygamma functions

$$\psi(z) = -\gamma + \sum_{k=0}^{\infty}\frac{1}{k} - \frac{1}{z+k-1} \tag{12.3.2a}$$

$z \in \mathbb{C}\backslash\mathbb{Z}_0^-$

power series

$$\psi^{(n)}(z) = (-1)^{n+1}\,n!\sum_{k=0}^{\infty}\frac{1}{(z+k)^{n+1}} \tag{12.3.2b}$$

$z \in \mathbb{C}\backslash\mathbb{Z}_0^-, n \in \mathbb{Z}^+$

power series

$$\psi(z) = \ln(z) - \frac{1}{2z} + \sum_{k=0}^{\infty}\left(-\frac{B_{2k}}{2k}z^{-2k}\right) \tag{12.3.7}$$

$|\arg z| < \pi$

asymptotic series

$$\psi^{(n)}(z) = (-1)^{n-1}\sum_{k=0}^{\infty}\left(\frac{B_{2k}\,(2k+n-1)!}{(2k)!}z^{-2k-n}\right) \tag{12.3.8}$$

$|\arg z| < \pi, n \in \mathbb{Z}^+$

asymptotic series

3.3 Trigamma function

$$\psi^{(1)}(z) = \frac{1}{z} + \frac{1}{2z^2} + \frac{\frac{1}{12\pi}}{z^2} + \frac{2\pi}{z} \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{m^2\left(m^2-1\right)}{4(4m^2-1)}}{1} \right) \tag{12.4.1}$$

$$|\arg z| < \frac{\pi}{2}$$

S-fraction

$$\psi^{(1)}(z) = \frac{z^{-1}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-\frac{\left(\frac{m}{2}\right)^2}{2m-2} z^{-1}}{1} \right) \tag{12.4.2}$$

$$\operatorname{Re}(z) > \frac{1}{2}$$

C-fraction

$$\psi^{(1)}(z) = \frac{1}{-\frac{1}{2} + z} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{(m-1)^4}{4(2m-3)(2m-1)}}{-\frac{1}{2} + z} \right) \tag{12.4.3}$$

$$\operatorname{Re}(z) > \frac{1}{2}$$

J-fraction

3.4 Tetragamma function

$$\psi^{(2)}(z) = -\frac{1}{z^2} - \frac{1}{z^3} + \frac{\frac{1}{8\pi^2}}{z^2} + -\left(\frac{2\pi}{z}\right)^2 \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{\left(\frac{m}{2}\right)^2 \frac{m}{2}+1}{2(m+1)}}{1} \right) \tag{12.5.1}$$

$$|\arg z| < \frac{\pi}{2}$$

S-fraction

$$\psi^{(2)}(z) = \frac{\frac{1}{z(z-1)}}{1} + -\operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{\left(\frac{m}{2}\right)^4}{z(z-1)}}{m} \right) \tag{12.5.2}$$

$$\operatorname{Re}(z) > \frac{1}{2}, z \notin \frac{1}{2}, 1$$

S-fraction

$$\psi^{(2)}(z) = -\frac{1}{z} + \frac{\frac{1}{z}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{\frac{\left(\frac{m+2}{4}\right)^2-2\frac{m+2}{4}+2}{2\frac{m+2}{4}-1}z^{-1}}{1}\right) \tag{12.5.3}$$

$\mathrm{Re}\,(z) > 1$

C-fraction

3.5 Incomplete gamma functions

$$\gamma(a,z) = z^a \sum_{k=0}^{\infty} \frac{(-z)^k}{(a+k)\,k!} \tag{12.6.7}$$

$\mathrm{Re}\,(a) > 0$

power series

$$\gamma(a,z) = \frac{z^a e^{-z}}{a} \sum_{k=0}^{\infty} \frac{z^k}{(1+a)_k} \tag{12.6.8}$$

$\mathrm{Re}\,(a) > 0$

power series

$$\Gamma(a,z) = z^a e^{-z} \sum_{k=0}^{\infty} \left((-1)^k (1-a)_k z^{-k-1}\right) \tag{12.6.10}$$

$|\arg z| < \pi$

asymptotic series

$$\Gamma(a,z) = \frac{1}{z} + z^a e^{-z} \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{\frac{m}{2}-a}{1}\right) \tag{12.6.15}$$

$a \in (-\infty, (1))\,, |\arg z| < \pi$

S-fraction

$$\Gamma(a,z) = \frac{\frac{1}{z}}{1} + z^a e^{-z} \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{\frac{m}{2}-az^{-1}}{1}\right) \tag{12.6.17}$$

$|\arg z| < \pi$

C-fraction

$$\gamma(a,z) = \frac{\frac{z}{a}}{1} + z^{a-1}e^{-z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\frac{a+\frac{m}{2}-1}{(a+m-2)(a+m-1)} z}{1} \right) \tag{12.6.23}$$

$\mathrm{Re}\,(a) > 0$

C-fraction

$$\Gamma(a,z) = \Gamma(a) + \frac{\frac{z}{a}}{1} + -\frac{z^a e^{-z}}{z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\frac{a+\frac{m}{2}-1}{(a+m-2)(a+m-1)} z}{1} \right) \tag{12.6.24}$$

$|\arg z| < \pi, a \in \mathbb{C}\backslash\mathbb{Z}_0^-$

C-fraction

$$\gamma(a,z) = \frac{1}{a-z} + z^a e^{-z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(m-1)\,z}{a+(m-1)-z} \right) \tag{12.6.30}$$

$a \in \mathbb{C}\backslash\mathbb{Z}_0^-$

M-fraction

$$\Gamma(a,z) = \Gamma(a) + \frac{-1}{a-z} + z^a e^{-z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(m-1)\,z}{a+(m-1)-z} \right) \tag{12.6.31}$$

$|\arg z| < \pi, \mathrm{Re}\,(a) > 0$

M-fraction

$$\Gamma(a,z) = \frac{1}{1-a+z} + z^a e^{-z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(1-m)\,(m-1-a)}{((2m)-1)-a+z} \right) \tag{12.6.34}$$

$|\arg z| < \pi$

J-fraction

$$\Gamma(a,z) = z^{a-1}e^{-z} + \frac{a-1}{2+z-a} + z^{a-1}e^{-z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(1-m)\,(m-a)}{2m+z-a} \right) \tag{12.6.35}$$

$|\arg z| < \pi$

J-fraction

4 Error function and related integrals

4.1 Error function and Dawson's integral

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k+1}}{(2k+1)\,k!} \tag{13.1.7}$$

power series

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} e^{-z^2} \sum_{k=0}^{\infty} \frac{z^{2k+1}}{\left(\frac{3}{2}\right)_k} \tag{13.1.8}$$

power series

$$F(z) = e^{-z^2} \sum_{k=0}^{\infty} \frac{z^{2k+1}}{(2k+1)\,k!} \tag{13.1.9}$$

power series

$$F(z) = \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k+1}}{\left(\frac{3}{2}\right)_k} \tag{13.1.10}$$

power series

$$\operatorname{erf}(z) = \frac{2z^2}{1} + \frac{1}{\sqrt{\pi} z e^{z^2}} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{-2(m-1)}{(2m-3)(2m-1)} z^2}{1} \right) \tag{13.1.11a}$$

C-fraction

$$F(z) = \frac{-2z^2}{1} + -\frac{1}{2z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{2(m-1)}{(2m-3)(2m-1)} z^2}{1} \right) \tag{13.1.11b}$$

C-fraction

$$\operatorname{erf}(z) = \frac{2z^2}{1-2z^2} + \frac{1}{\sqrt{\pi} z e^{z^2}} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{4(m-1)}{(2m-3)(2m-1)} z^2}{1 - \frac{2}{2m-1} z^2} \right) \tag{13.1.13a}$$

T-fraction

$$F(z) = \frac{-2z^2}{1+2z^2} + -\frac{1}{2z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\frac{4(m-1)}{(2m-3)(2m-1)} z^2}{1 + \frac{2}{2m-1} z^2} \right) \tag{13.1.13b}$$

T-fraction

4.2 Complementary and complex error function

$$\operatorname{erfc}(z) = e^{-z^2} \sum_{k=0}^{\infty} \frac{(-z)^k}{\Gamma\left(\frac{k}{2}+1\right)} \tag{13.2.9}$$

power series

$$w(z) = \sum_{k=0}^{\infty} \frac{(iz)^k}{\Gamma\left(\frac{k}{2}+1\right)} \tag{13.2.10}$$

power series

$$\operatorname{erfc}(z) = \frac{1}{\sqrt{\pi} z e^{z^2}} \sum_{k=0}^{\infty} \left((-1)^k \left(\frac{1}{2}\right)_k z^{-2k} \right) \tag{13.2.11}$$

$$|\arg z| < \frac{3\pi}{4}$$

asymptotic series

$$w(z) = \frac{i}{\pi z} \sum_{k=0}^{\infty} \left(\left(\frac{1}{2}\right)_k z^{-2k} \right) \tag{13.2.12}$$

$$|\arg -iz| < \frac{3\pi}{4}$$

asymptotic series

$$\operatorname{erfc}(z) = \frac{1}{z^2} + \frac{z}{\sqrt{\pi}} e^{-z^2} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{m-1}{2}}{1} \right) \tag{13.2.20a}$$

$$\operatorname{Re}(z) > 0$$

S-fraction

$$w(z) = \frac{1}{-z^2} + -\frac{iz}{\sqrt{\pi}} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{m-1}{2}}{1} \right) \tag{13.2.20b}$$

$\mathrm{Im}(z) > 0$

S-fraction

$$\mathrm{erfc}(z) = \frac{2z}{2z^2+1} + \frac{e^{-z^2}}{\sqrt{\pi}} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-(2m-3)(2m-2)}{4m-3+2z^2} \right) \tag{13.2.23a}$$

$\mathrm{Re}(z) > 0$

J-fraction

$$w(z) = \frac{-\frac{iz}{\sqrt{\pi}}}{\frac{1}{2} - z^2} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{- - \frac{3}{2} + m(m-1)}{2m - \frac{3}{2} - z} \right) \tag{13.2.23b}$$

$\mathrm{Im}(z) > 0$

J-fraction

4.3 Repeated integrals

$$I^n \,\mathrm{erfc}(z) = \frac{\frac{2}{\sqrt{\pi}} e^{-z^2}}{(2z)^{n+1}} \sum_{k=0}^{\infty} \left(\frac{(-1)^k (2k+n)!}{n!k!(2z)^{2k}} \right) \tag{13.3.2}$$

$|\arg z| < \frac{3\pi}{4}$

asymptotic series

$$\frac{I^n \,\mathrm{erfc}(z)}{I^{n-1} \,\mathrm{erfc}(z)} = \frac{\frac{1}{2}}{z} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{n+m-1}{2}}{z} \right) \tag{13.3.5}$$

$\mathrm{Re}(z) > 0, n \in \mathbb{Z} \backslash \mathbb{Z}^-$

S-fraction

4.4 Fresnel integrals

$$C(z) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{2}\right)^{2k}}{(2k)!\,(4k+1)} z^{4k+1} \tag{13.4.6a}$$

power series

$$S(z) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{2}\right)^{2k+1}}{(2k+1)!\,(4k+3)} z^{4k+3} \tag{13.4.6b}$$

power series

$$C(z) + i\,S(z) = \frac{z^2}{1} + \frac{e^{\frac{i\pi z^2}{2}}}{z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{i\pi(m-1)}{(2m-3)(2m-1)} z^2}{1} \right) \tag{13.4.9}$$

C-fraction

$$C(z) + i\,S(z) = \frac{z^2}{1 + i\pi z^2} + \frac{e^{\frac{i\pi z^2}{2}}}{z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\frac{2i\pi(m-1)}{(2m-3)(2m-1)} z^2}{1 + \frac{i\pi}{2m-1} z^2} \right) \tag{13.4.10}$$

T-fraction

5 Hypergeometric functions

5.1 Hypergeometric functions

$$\frac{{}_2F_1\left({a,b \atop c}; z\right)}{{}_2F_1\left({a,b+1 \atop c+1}; z\right)} = \frac{c + (b-a+1)\,z}{c} + \frac{1}{c} \mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{-\,(c-a+m)\,(b+m)\,z}{c + m + (b-a+m+1)\,z} \right) \tag{15.3.8}$$

$$|z| < 1, c \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

T-fraction

$${}_2F_1\left({a, 1 \atop c+1}; z\right) = \frac{c}{c + (1-a)\,z} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\,(m-1)\,z\,(c-a+m-1)}{c + m - 1 + (m-a)\,z} \right) \tag{15.3.9a}$$

$$|z| < 1, c \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

M-fraction

$$
{}_2F_1\left(\begin{matrix}1-c,1\\2-a\end{matrix};\frac{1}{z}\right)=\frac{c}{c+(1-a)\,z}+\frac{(1-a)\,z}{c}\mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{-(m-1)\,z\,(c-a+m-1)}{c+m-1+(m-a)\,z}\right) \quad (15.3.9b)
$$

$$
|z|<1, a\in\mathbb{C}\backslash\mathbb{Z}_0^-, a<>1, a<>2
$$

M-fraction

$$
{}_2F_1\left(\begin{matrix}\frac{1}{2},1\\\frac{3}{2}\end{matrix};z\right)=\frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}z}+\mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{-(m-1)\,z\,(m-1)}{\frac{1}{2}+m-1+\left(m-\frac{1}{2}\right)z}\right) \quad (15.3.12)
$$

$$
|z|<1
$$

M-fraction

$$
\frac{{}_2F_1\left(\begin{smallmatrix}a,b\\c\end{smallmatrix};z\right)}{{}_2F_1\left(\begin{smallmatrix}a+1,b+1\\c+1\end{smallmatrix};z\right)}=1-\frac{a+b+1}{c}z+\mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{\frac{(a+m)(b+m)}{(c+m-1)(c+m)}\left(z-z^2\right)}{1-\frac{a+b+2m+1}{c+m}z}\right) \quad (15.3.13)
$$

$$
\mathrm{Re}\,(z)<\frac{1}{2}, c\in\mathbb{C}\backslash\mathbb{Z}_0^-
$$

T-fraction

$$
{}_2F_1\left(\begin{matrix}a+1,1\\c+1\end{matrix};z\right)=\frac{c}{c-(a+1)\,z}+\mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{(a+m-1)\,(m-1)\left(z-z^2\right)}{c+m-1-(a+2\,(m-1)+1)\,z}\right) \quad (15.3.14)
$$

M-fraction

$$
{}_2F_1\left(\begin{matrix}\frac{1}{2},1\\\frac{3}{2}\end{matrix};z\right)=\frac{1}{1-z}+\mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{\frac{m-1}{m-1+\frac{1}{2}}\left(z-z^2\right)}{1-\frac{2(m-1)+\frac{1}{2}}{m-1+\frac{1}{2}}z}\right) \quad (15.3.17)
$$

$$
\mathrm{Re}\,(z)<\frac{1}{2}
$$

Norlund fraction

$$
\frac{{}_3F_2\left(\begin{smallmatrix}a,b,c\\d,e\end{smallmatrix};1\right)}{{}_3F_2\left(\begin{smallmatrix}a+1,b,c\\d,e\end{smallmatrix};1\right)}=1+\frac{-\frac{bc}{d}}{e-a-1}+\mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{\frac{\left(a+\frac{m}{2}\right)\left(d-b+\frac{m}{2}-1\right)\left(d-c+\frac{m}{2}-1\right)}{(d+m-2)(d+m-1)}}{1}\right) \quad (15.6.4)
$$

continued fraction

$$\frac{{}_3F_2\left({a,b,c \atop d,e};1\right)}{{}_3F_2\left({a,b,c \atop d+1,e};1\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{-\frac{\frac{m}{2}+d-a\,\frac{m}{2}+d-b\,\frac{m}{2}+d-c}{(d+m-1)(d+m)}}{1}\right) \tag{15.6.5}$$

continued fraction

$$\frac{{}_3F_2\left({a,b,c \atop d,e};1\right)}{{}_3F_2\left({a+1,b,c \atop d+1,e};1\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{\frac{\left(a+\frac{m}{2}\right)\left(d-b+\frac{m}{2}\right)\left(d-c+\frac{m}{2}\right)}{(d+m-1)(d+m)}}{1}\right) \tag{15.6.6}$$

continued fraction

$$\frac{{}_3F_2\left({a,b,c \atop d,e};1\right)}{{}_3F_2\left({a,b+1,c+1 \atop d+1,e+1};1\right)} = \frac{e-(a-1)-1}{e} + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{-\frac{\left((d-1)-(a-1)+\frac{m}{2}\right)\left(b+\frac{m}{2}\right)\left(c+\frac{m}{2}\right)}{\left(e+\frac{m}{2}\right)((d-1)+m)((d-1)+m+1)}}{\frac{e-(a-1)-1}{e+\frac{m}{2}}}\right) \tag{15.6.7}$$

continued fraction

6 Confluent hypergeometric functions

6.1 Kummer functions

$$ {}_1F_1\left({a \atop b};z\right) = \sum_{k=0}^{\infty} \frac{\frac{(a)_k}{(b)_k} z^k}{k!} \tag{17.1.2}$$

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

power series

$$ {}_2F_0\left({a,b \atop };z\right) = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k z^k}{k!} \tag{17.1.12}$$

power series

$$\frac{{}_1F_1\left({a \atop b};z\right)}{{}_1F_1\left({a+1 \atop b+1};z\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{\frac{z\left(a+\frac{m}{2}\right)}{(b+m-1)(b+m)}}{1}\right) \tag{17.1.13}$$

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

C-fraction

$$z\,{}_1F_1\left({1 \atop b+1};z\right) = \frac{z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{-\frac{z\left(b+\frac{m}{2}-1\right)}{(b+m-2)(b+m-1)}}{1}\right) \tag{17.1.14}$$

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

C-fraction

$$\frac{{}_1F_1\left({a \atop b};z\right)}{{}_1F_1\left({a+1 \atop b+1};z\right)} = \frac{b-z}{b} + \frac{1}{b}\mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{(a+m)\,z}{b+m-z}\right) \tag{17.1.16}$$

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

T-fraction

$${}_1F_1\left({1 \atop b+1};z\right) = \frac{b}{b-z} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{(m-1)\,z}{b+m-1-z}\right) \tag{17.1.17}$$

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

M-fraction

$$\frac{U(a,b,z)}{U(a+1,b,z)} = 2a-b+2+z-\mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{(a+m)\,(b-a-m-1)}{b-2a-2m-2-z}\right) \tag{17.1.20}$$

$$b \in \mathbb{C}\backslash\mathbb{Z}$$

J-fraction

6.2 Confluent hypergeometric series ${}_2F_0$

$$\frac{{}_2F_0\left({a,b \atop };z\right)}{{}_2F_0\left({a,b+1 \atop };z\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{-\left(b+\frac{m}{2}\right)z}{1}\right) \tag{17.2.4}$$

$$z < 0$$

C-fraction

6.3 Confluent hypergeometric limit function

$$ {}_0F_1\left({\atop b};z\right) = \sum_{k=0}^{\infty} \frac{z^k}{(b)_k k!} \tag{17.3.1}$$

$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$

power series

$$ \frac{{}_0F_1\left({\atop b};z\right)}{{}_0F_1\left({\atop b+1};z\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{\frac{1}{(b+m-1)(b+m)}z}{1}\right) \tag{17.3.4}$$

$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$

C-fraction

$$ \frac{{}_0F_1\left({\atop b};z\right)}{{}_0F_1\left({\atop b+1};z\right)} = 1 + \frac{\sqrt{z}}{b} + \frac{1}{2b}\mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{-2\left(2b+2m-1\right)\sqrt{z}}{2b+m+4\sqrt{z}}\right) \tag{17.3.6}$$

$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$

T-fraction

6.4 Whittaker functions

$$ W_{\kappa,\mu}(z) = e^{-z^2} z^{\kappa} \sum_{k=0}^{\infty}\left(\frac{\left(-\kappa-\mu+\frac{1}{2}\right)_k\left(-\kappa+\mu+\frac{1}{2}\right)_k(-z)^{-k}}{k!}\right) \tag{17.4.7}$$

$|\arg z| < \frac{3\pi}{2}$

asymptotic series

$$ \Psi_{n,\beta}(z) = \sum_{k=0}^{\infty}\left(\frac{\left(n+\frac{1}{2}\right)_k\left(\beta+\frac{1}{2},k\right)z^{-k-1}}{k!}\right) \tag{17.4.12}$$

asymptotic series

6.5 Parabolic cylinder functions

$$ \frac{U(a,x)}{U(a-1,x)} = \frac{1}{x} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{a+(m-1)-\frac{1}{2}}{x}\right) \tag{17.5.7}$$

$x > 0, (2a) \in \mathbb{Z}^+$

C-fraction

7 Bessel functions

7.1 Bessel functions

$$J_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,\Gamma(\nu+k+1)} \left(\frac{z}{2}\right)^{2k} \tag{19.1.2a}$$

$|\arg z| < \pi$

power series

$$\mathrm{j}_n(z) = \sqrt{\frac{\pi}{2z}} \left(\frac{z}{2}\right)^{n+\frac{1}{2}} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,\Gamma\left(\left(n+\frac{1}{2}\right)+k+1\right)} \left(\frac{z}{2}\right)^{2k} \tag{19.1.12a}$$

$n \in \mathbb{Z}$

power series

$$J_\nu(z) = \frac{e^{-iz}\left(\frac{z}{2}\right)^\nu}{\Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{\left(\nu+\frac{1}{2}\right)_k (2i)^k z^k}{(2\nu+1)_k k!} \tag{19.1.22}$$

$|\arg z| < \pi$

power series

$$\mathrm{j}_n(z) = \frac{\sqrt{\pi}}{(2n+1)\,\Gamma\left(n+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^n \sum_{k=0}^{\infty} \frac{\frac{1}{\left(n+\frac{3}{2}\right)_k}\left(-\frac{z^2}{4}\right)^k}{k!} \tag{19.1.25}$$

$n \in \mathbb{Z}$

power series

$$\mathrm{j}_n(z) = \frac{\sqrt{\pi}e^{-iz}}{(2n+1)\,\Gamma\left(n+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^n \sum_{k=0}^{\infty} \frac{\frac{(n+1)_k}{(2n+2)_k}(2iz)^k}{k!} \tag{19.1.26}$$

$n \in \mathbb{Z}$

power series

$$J_\nu(z) = \sqrt{\frac{2}{\pi z}} \sum_{k=0}^{\infty} \left(\frac{(-1)^k (\nu, 2k)}{(2z)^{2k}} \cos\left(z - \frac{\nu}{2} + \frac{1}{4}\pi\right) - \frac{(-1)^k (\nu, 2k+1)}{(2z)^{2k+1}} \sin\left(z - \frac{\nu}{2} + \frac{1}{4}\pi\right) \right) \tag{19.1.28}$$

$|\arg z| < \pi$

asymptotic series

$$Y_\nu(z) = \sqrt{\frac{2}{\pi z}} \sum_{k=0}^{\infty} \left(\frac{(-1)^k (\nu, 2k)}{(2z)^{2k}} \sin\left(z - \frac{\nu}{2} + \frac{1}{4}\pi\right) + \frac{(-1)^k (\nu, 2k+1)}{(2z)^{2k+1}} \cos\left(z - \frac{\nu}{2} + \frac{1}{4}\pi\right)\right) \tag{19.1.29}$$

$|\arg z| < \pi$

asymptotic series

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = \frac{\frac{z}{2\nu+2}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{(iz)^2}{4(\nu+m-1)(\nu+m)}}{1} \right) \tag{19.1.38}$$

$\nu \geq 0$

S-fraction

$$\frac{\mathrm{j}_{n+1}(z)}{\mathrm{j}_n(z)} = \frac{\frac{z}{2n+3}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{(iz)^2}{4\left(n+\frac{1}{2}+m-1\right)\left(n+\frac{1}{2}+m\right)}}{1} \right) \tag{19.1.39}$$

$n \in \mathbb{Z}\backslash\mathbb{Z}^-$

S-fraction

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = -\mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{-1}{\frac{2(\nu+m)}{z}} \right) \tag{19.1.40}$$

$\nu \in \mathbb{Z}\backslash\mathbb{Z}^-$

S-fraction

$$\frac{H^{(1)}_{\nu+1}(z)}{H^{(1)}_\nu(z)} = \frac{-1}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{m-3-2\nu}{-2iz}}{1} \right) \tag{19.1.44}$$

$|\arg iz| < \pi$

C-fraction

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = \frac{z}{2\nu+2-iz} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(2\nu+2m-1)\,iz}{2\nu+m+1+(-2i)\,z} \right) \tag{19.1.48}$$

$\nu \in \mathbb{C}\backslash\mathbb{Z}^-$

T-fraction

$$\frac{\mathsf{j}_{n+1}(z)}{\mathsf{j}_n(z)} = \frac{z}{2n+3-iz} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{2\,(n+m)\,iz}{2n+m+2+(-2i)\,z}\right) \tag{19.1.49}$$

$n \in \mathbb{Z}\backslash\mathbb{Z}^-$

T-fraction

$$\frac{H^{(1)}_{\nu+1}(z)}{H^{(1)}_{\nu}(z)} = \frac{2\nu+1-2iz}{2z} - \frac{1}{z}\mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{\nu^2 - \frac{(2m-1)^2}{4}}{2\,(iz-m)}\right) \tag{19.1.51}$$

$|\arg -iz| < \pi$

J-fraction

7.2 Modified Bessel functions

$$I_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{2k}}{k!\,\Gamma(\nu+k+1)} \tag{19.2.20}$$

$|\arg z| < \pi$

power series

$$I_\nu(z) = \frac{e^{-z}\left(\frac{z}{2}\right)^\nu}{\Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{\left(\nu+\frac{1}{2}\right)_k 2^k z^k}{(2\nu+1)_k k!} \tag{19.2.21}$$

$|\arg z| < \pi$

power series

$$\mathsf{i}^{(1)}_n(z) = \frac{\sqrt{\pi}}{(2n+1)\,\Gamma\left(n+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(\frac{z^2}{4}\right)^k}{k!\left(n+\frac{3}{2}\right)_k} \tag{19.2.22}$$

$n \in \mathbb{Z}$

power series

$$\mathsf{i}^{(1)}_n(z) = \frac{\sqrt{\pi}e^{-iz}}{(2n+1)\,\Gamma\left(n+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^n \sum_{k=0}^{\infty} \frac{(n+1)_k\,(2z)^k}{k!(2n+2)_k} \tag{19.2.23}$$

$n \in \mathbb{Z}$

power series

$$I_\nu(z) = \sum_{k=0}^{\infty} \left(\frac{\left((-1)^k e^z + e^{-z+\frac{(2\nu+1)i\pi}{2}} \right) (\nu, k)}{\sqrt{2\pi z}\, (2z)^k} \right) \tag{19.2.24}$$

$$\arg z \in -\frac{\pi}{2}, \frac{3\pi}{2}$$

asymptotic series

$$I_\nu(z) = \sum_{k=0}^{\infty} \left(\frac{\left((-1)^k e^z + e^{-z-\frac{(2\nu+1)i\pi}{2}} \right) (\nu, k)}{\sqrt{2\pi z}\, (2z)^k} \right) \tag{19.2.25}$$

$$\arg z \in -\frac{3\pi}{2}, \frac{\pi}{2}$$

asymptotic series

$$K_\nu(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \sum_{k=0}^{\infty} \left(\frac{(\nu, k)}{(-2z)^k} \right) \tag{19.2.27}$$

$$|\arg z| < \frac{3\pi}{2}$$

asymptotic series

$$\frac{I_{\nu+1}(z)}{I_\nu(z)} = \frac{\frac{z}{2(\nu+1)}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{1}{4(\nu+m-1)(\nu+m)} z^2}{1} \right) \tag{19.2.32}$$

$$\nu \geq 0$$

S-fraction

$$\frac{\mathsf{i}_{n+1}^{(1)}(z)}{\mathsf{i}_n^{(1)}(z)} = \frac{\frac{z}{2n+3}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{1}{4\left(\left(n+\frac{1}{2}\right)+m-1\right)\left(\left(n+\frac{1}{2}\right)+m\right)} z^2}{1} \right) \tag{19.2.33}$$

$$n \in \mathbb{Z} \backslash \mathbb{Z}^-$$

S-fraction

$$\frac{\nu}{z} - \frac{K_{\nu+1}(z)}{K_\nu(z)} = \frac{\nu}{z} + \frac{1}{1} + \frac{\frac{-2\nu-1}{2z}}{1} + -\operatorname*{K}_{m=3}^{\infty} \left(\frac{\frac{\frac{m}{2}+\nu}{2z}}{1} \right) \tag{19.2.34}$$

$$|\arg z| < \pi$$

C-fraction

$$\frac{I_{\nu+1}(z)}{I_{\nu}(z)} = \frac{z}{2\nu+2+z} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{-(2\nu+2m-1)\,z}{2\nu+m+1+2z}\right) \tag{19.2.38}$$

$$\nu \in \mathbb{C}\backslash\mathbb{Z}^{-}$$

T-fraction

$$\frac{\mathsf{i}_{n+1}^{(1)}(z)}{\mathsf{i}_{n}^{(1)}(z)} = \frac{z}{2n+3+z} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{-2\,(n+m)\,z}{2n+m+2+2z}\right) \tag{19.2.39}$$

$$n \in \mathbb{Z}\backslash\mathbb{Z}^{-}$$

T-fraction

$$\frac{\nu}{z} - \frac{K_{\nu+1}(z)}{K_{\nu}(z)} = \frac{\nu}{z} - \frac{2\nu+1+2z}{2z} - \frac{1}{z}\operatorname*{K}_{m=1}^{\infty}\left(\frac{\nu^2-\frac{(2m-1)^2}{4}}{2\,(z+m)}\right) \tag{19.2.40}$$

$$|\arg z| < \pi$$

J-fraction

8 q-Hypergeometric function

8.1 q-Hypergeometric function

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({a,bq \atop cq};q,z\right)} = 1 + \operatorname*{K}_{m=1}^{\infty}\left(\frac{\frac{\left(1-bq^{\frac{m}{2}}\right)\left(cq^{\frac{m}{2}}-a\right)q^{\frac{m}{2}-1}}{(1-cq^{m-1})(1-cq^{m})}z}{1}\right) \tag{21.2.1}$$

$$|q|<1, \left(\log_q(c)\right) \notin \left(AndProp\left(\mathbb{Z},(-\infty,(0))\right)\right)$$

C-fraction

$${}_2\phi_1\left({a,q \atop cq};q,z\right) = \frac{1}{1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{\frac{\left(1-aq^{\frac{m-2}{2}}\right)\left(cq^{\frac{m-2}{2}}-1\right)q^{\frac{m-2}{2}}}{(1-cq^{m-2})(1-cq^{m-1})}z}{1}\right) \tag{21.2.2}$$

$$|q|<1, \left(\log_q(c)\right) \notin \left(AndProp\left(\mathbb{Z},(-\infty,(0))\right)\right)$$

C-fraction

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({a,bq \atop cq};q,z\right)} = \frac{q\,(1-c)+(a-bq)\,z}{q\,(1-c)} + \frac{1}{q\,(1-c)} \operatorname*{K}_{m=1}^{\infty} \left(\frac{q\,(1-bq^m)\,(cq^m-a)\,z}{q\,(1-cq^m)+(a-bq^{m+1})\,z}\right) \tag{21.2.5a}$$

$$|q|<1, \left(\log_q(c)\right) \notin (AndProp\,(\mathbb{Z},(-\infty,(0)))), |z| < \left|\frac{q}{a}\right|$$

T-fraction

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({a,bq \atop cq};q,z\right)} = 1 + \frac{q\,(1-c)}{(a-bq)\,z} + \frac{1}{(a-bq)\,z} \operatorname*{K}_{m=1}^{\infty} \left(\frac{q\,(1-bq^m)\,(cq^m-a)\,z}{q\,(1-cq^m)+(a-bq^{m+1})\,z}\right) \tag{21.2.5b}$$

$$|q|<1, \left(\log_q(c)\right) \notin (AndProp\,(\mathbb{Z},(-\infty,(0)))), |z| > \left|\frac{q}{a}\right|$$

T-fraction

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({aq,bq \atop cq};q,z\right)} = \frac{1-c-(a+b-ab-abq)\,z}{1-c} + \frac{1}{1-c} \operatorname*{K}_{m=1}^{\infty} \left(\frac{(1-aq^m)\,(1-bq^m)\left(cz-abq^m z^2\right)q^{m-1}}{(1-cq^m)-(a+b-abq^m-abq^{m+1})\,q^m z}\right) \tag{21.2.7}$$

$$|q|<1, \left(\log_q(c)\right) \notin (AndProp\,(\mathbb{Z},(-\infty,(0)))), |z| < \left|\frac{q}{a}\right|$$

Norlund fraction

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({aq,b \atop c};q,z\right)} = 1 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{a}{-1}\right) \tag{21.2.10}$$

$$|q|<1, \left(\log_q(c)\right) \notin (AndProp\,(\mathbb{Z},(-\infty,(0)))), |z| < 1$$

continued fraction

$$\frac{{}_2\phi_1\left({a,b \atop c};q,qz\right)}{{}_2\phi_1\left({a,b \atop c};q,z\right)} = \frac{(1-z)}{(1+cq^{-1}-(a+b)\,z)} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-\left(cq^{-1}-abq^{m-2}z\right)\left(1-q^{m-1}z\right)}{1+cq^{-1}-(a+b)\,q^{m-1}z}\right) \tag{21.2.11}$$

$$|q|<1, \left|\frac{c}{q}\right| <> 1$$

continued fraction

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({a,b \atop c};q,qz\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{\left(1-aq^{\frac{m-1}{2}}\right)\left(1-bq^{\frac{m-1}{2}}\right)z}{1-z}\right) \tag{21.2.12}$$

$$z <> 1, |q| < 1, \left(\log_q(c)\right) \notin \left(AndProp\left(\mathbb{Z}, (-\infty, (0))\right)\right)$$

continued fraction

$$\frac{{}_2\phi_1\left({aq,b \atop c};q,z\right)}{{}_2\phi_1\left({a,b \atop c};q,qz\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{\left(1-bq^{\frac{m-1}{2}}\right)z}{(1-a)(1-z)}\right) \tag{21.2.13}$$

$$\left|\frac{z}{(1-a)(1-z)}\right| < \frac{1}{4}, \left|\frac{a}{(1-a)(1-z)}\right| < \frac{1}{4}, |q| < 1, \left(\log_q(c)\right) \notin \left(AndProp\left(\mathbb{Z}, (-\infty, (0))\right)\right)$$

continued fraction

$$\frac{{}_3\phi_2\left({a,b,c \atop e,f};q,\frac{ef}{abc}\right)}{{}_3\phi_2\left({a,b,c \atop eq,f};q,\frac{efq}{abc}\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{-\frac{\left(1-\frac{eq^{\frac{m}{2}-1+1}}{a}\right)\left(1-\frac{eq^{\frac{m}{2}-1+1}}{b}\right)\left(1-\frac{eq^{\frac{m}{2}-1+1}}{c}\right)fq^{\frac{m}{2}-1}}{\left(1-eq^{2\frac{m}{2}-1+1}\right)\left(1-eq^{2\frac{m}{2}-1+2}\right)\left(1-fq^{\frac{m}{2}-1}\right)}}{1}\right) \tag{21.3.1}$$

continued fraction

$$\frac{{}_3\phi_2\left({aq,b,c \atop e,f};q,\frac{ef}{abcq}\right)}{{}_3\phi_2\left({a,b,c \atop e,f};q,\frac{ef}{abc}\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{a\left(1-\frac{eq^{\frac{m}{2}-1}}{a}\right)\left(1-\frac{fq^{\frac{m}{2}-1}}{a}\right)}{1}\right) \tag{21.3.2}$$

continued fraction

9 Exponential integrals and related functions

9.1 Exponential integrals

$$E_1(z) = -1\sum_{k=0}^{\infty}\frac{(-1)^k z^k}{kk!} \tag{14.1.10}$$

$$|\arg z| < \pi$$

power series

$$E_n(z) = \frac{(-z)^{n-1}}{(n-1)!}\left(-\gamma - \ln(z) + \sum_{k=1}^{n-1}\left(\frac{1}{k}\right)\right) - \sum_{k=0}^{n-2}\left(\frac{(-z)^k}{(k-n+1)\,k!}\right) + \sum_{k=0}^{\infty} - \frac{(-z)^{k+n-1}}{(k)\,(k+n-1)!} \tag{14.1.11}$$

$|\arg z| < \pi, n \in \mathbb{Z}^+$

power series

$$E_\nu(z) = \Gamma(1-\nu)z^{\nu-1} - \frac{1}{1-\nu} + \sum_{k=0}^{\infty}\frac{(-1)^{k+1}\,z^k}{(k!)\,(k+1-\nu)} \tag{14.1.12}$$

$z <> 0, \nu \in \mathbb{C}\backslash\mathbb{Z}^+$

power series

$$E_\nu(z) = e^{-z}\sum_{k=0}^{\infty}\left(\frac{(-1)^k\,(\nu)_k}{z^{k+1}}\right) \tag{14.1.13}$$

$|\arg z| < \pi$

asymptotic series

$$E_n(z) = \frac{1}{z} \, + \, \frac{n}{1} \, + \, e^{-z} \operatorname*{K}_{m=3}^{\infty}\left(\frac{n+\frac{m-2}{2}}{1}\right) \tag{14.1.16}$$

$|\arg z| < \pi, n \in \mathbb{Z}^+$

S-fraction

$$E_\nu(z) = \frac{\frac{e^{-z}}{z}}{1} \, + \operatorname*{K}_{m=2}^{\infty}\left(\frac{\frac{\frac{m}{2}+\nu-1}{z}}{1}\right) \tag{14.1.19}$$

$|\arg z| < \pi$

C-fraction

$$E_\nu(z) = z^{\nu-1}\,\Gamma(1-\nu) + \frac{-e^{-z}}{1-\nu} \, + \operatorname*{K}_{m=2}^{\infty}\left(\frac{\left(\nu-\frac{m}{2}\right)z}{m-\nu}\right) \tag{14.1.20}$$

$z <> 0, \nu \in \mathbb{C}\backslash\mathbb{Z}^+$

C-fraction

$$E_\nu(z) = z^{\nu-1}\,\Gamma(1-\nu) + \frac{1}{1-\nu-z} \, + \, -e^{-z} \operatorname*{K}_{m=2}^{\infty} \left(\frac{(m-1)\,z}{m-\nu-z} \right) \tag{14.1.22}$$

$z <> 0, \nu \in \mathbb{C} \backslash \mathbb{Z}^+$

M-fraction

$$E_\nu(z) = \frac{1}{\nu+z} \, + \, e^{-z} \operatorname*{K}_{m=2}^{\infty} \left(\frac{-(m-1)\,(\nu+m-2)}{\nu+(m-1)\,2+z} \right) \tag{14.1.23}$$

$|\arg z| < \pi$

J-fraction

$$E_\nu(z) = \frac{e^{-z}}{z} + \frac{-e^{-z}\nu}{z\,(1+z)+\nu z} \, + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-(m-1)\,(\nu+m-1)\,z^2}{z\,(m+z)+(\nu+m-1)\,z} \right) \tag{14.1.24}$$

$|\arg z| < \pi$

J-fraction

9.2 Related functions

$$E_1(x) = \gamma + \ln(x) + \sum_{k=0}^{\infty} \frac{x^k}{kk!} \tag{14.2.14}$$

$x > 0$

power series

$$\mathrm{Ein}(z) = -1 \sum_{k=0}^{\infty} \frac{(-1)^{k+1}\, z^{k+1}}{(k+1)\,(k+1)!} \tag{14.2.16}$$

power series

$$E_1(x) = \frac{e^x}{x} \sum_{k=0}^{\infty} \left(k! x^{-k} \right) \tag{14.2.19}$$

asymptotic series

$$E_1(z) = 2 \sum_{k=0}^{\infty} \left(\frac{z^{2k+1}}{(2k+1)\,(2k+1)!} \right) + \frac{\frac{1}{z}}{1} \, + \, -e^{-z} \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{\lfloor \frac{m}{2} \rfloor}{z}}{1} \right) \tag{14.2.21}$$

$x > 0$

S-fraction

$$\mathrm{Ein}(z) = \gamma + \ln(z) + \frac{\frac{1}{z}}{1} + e^{-z} \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{\lfloor \frac{m}{2} \rfloor}{z}}{1} \right) \tag{14.2.23}$$

$|\arg z| < \pi$

S-fraction

$$E_1(x) = \frac{\frac{e^x}{x}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-\frac{\lfloor \frac{m}{2} \rfloor}{x}}{1} \right) \tag{14.2.24}$$

$x > 0$

C-fraction