1 Bessel functions

1.1 Bessel functions

$$J_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{k=0}^\infty \frac{(-1)^k}{k!\,\Gamma(\nu+k+1)} \left(\frac{z}{2}\right)^{2k} \tag{17.1.2a}$$

power series

$$|\arg z| < \pi$$

$$j_n(z) = \sqrt{\frac{\pi}{2z}} \left(\frac{z}{2}\right)^{n+\frac{1}{2}} \sum_{k=0}^\infty \frac{(-1)^k}{k!\,\Gamma\big((n+\frac{1}{2})+k+1\big)} \left(\frac{z}{2}\right)^{2k} \tag{17.1.12a}$$

power series

$$n \in \mathbb{Z}$$

$$J_\nu(z) = \frac{e^{-iz}\left(\frac{z}{2}\right)^\nu}{\Gamma(\nu+1)} \sum_{k=0}^\infty \frac{\left(\nu+\frac{1}{2}\right)_k (2i)^k z^k}{(2\nu+1)_k k!} \tag{17.1.22}$$

power series

$$|\arg z| < \pi$$

$$j_n(z) = \frac{\sqrt{\pi}}{(2n+1)\,\Gamma\big(n+\frac{1}{2}\big)} \left(\frac{z}{2}\right)^n \sum_{k=0}^\infty \frac{\frac{1}{(n+\frac{3}{2})_k}\left(-\frac{z^2}{4}\right)^k}{k!} \tag{17.1.25}$$

power series

$$n \in \mathbb{Z}$$

$$j_n(z) = \frac{\sqrt{\pi}e^{-iz}}{(2n+1)\,\Gamma\big(n+\frac{1}{2}\big)} \left(\frac{z}{2}\right)^n \sum_{k=0}^\infty \frac{\frac{(n+1)_k}{(2n+2)_k}(2iz)^k}{k!} \tag{17.1.26}$$

power series

$$n \in \mathbb{Z}$$

$$J_\nu(z) = \sqrt{\frac{2}{\pi z}} \sum_{k=0}^\infty \left(\frac{(-1)^k\,(\nu,2k)}{(2z)^{2k}} \cos\left(z - \left(\frac{\nu}{2}+\frac{1}{4}\right)\pi \right) - \frac{(-1)^k\,(\nu,2k+1)}{(2z)^{2k+1}} \sin\left(z - \left(\frac{\nu}{2}+\frac{1}{4}\right)\pi \right) \right) \tag{17.1.28}$$

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asymptotic series

$$|\arg z| < \pi$$

$$Y_\nu(z) = \sqrt{\frac{2}{\pi z}} \sum_{k=0}^{\infty} \left(\frac{(-1)^k (\nu, 2k)}{(2z)^{2k}} \sin\left(z - \left(\frac{\nu}{2} + \frac{1}{4}\right)\pi\right) + \frac{(-1)^k (\nu, 2k+1)}{(2z)^{2k+1}} \cos\left(z - \left(\frac{\nu}{2} + \frac{1}{4}\right)\pi\right) \right)$$

$$(17.1.29)$$

asymptotic series

$$|\arg z| < \pi$$

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = \frac{\frac{z}{2\nu+2}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{(iz)^2}{4(\nu+m-1)(\nu+m)}}{1} \right) \qquad (17.1.38)$$

S-fraction

$$\nu \geq 0$$

$$\frac{\mathrm{j}_{n+1}(z)}{\mathrm{j}_n(z)} = \frac{\frac{z}{2n+3}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{(iz)^2}{4\left(n+\frac{1}{2}+m-1\right)\left(n+\frac{1}{2}+m\right)}}{1} \right) \qquad (17.1.39)$$

S-fraction

$$n \in \mathbb{Z} \backslash \mathbb{Z}^-$$

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = -\mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{-1}{\frac{2(\nu+m)}{z}} \right) \qquad (17.1.40)$$

S-fraction

$$\nu \in \mathbb{Z} \backslash \mathbb{Z}^-$$

$$\frac{H_{\nu+1}^{(1)}(z)}{H_\nu^{(1)}(z)} = \frac{-1}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{m-3-2\nu}{-2iz}}{1} \right) \qquad (17.1.44)$$

C-fraction

$$|\arg iz| < \pi$$

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = \frac{z}{2\nu+2-iz} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(2\nu+2m-1)\,iz}{2\nu+m+1+(-2i)\,z} \right) \qquad (17.1.48)$$

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$$\nu \in \mathbb{C} \backslash \mathbb{Z}^-$$

$$\frac{\mathsf{j}_{n+1}(z)}{\mathsf{j}_n(z)} = \frac{z}{2n+3-iz} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{2\,(n+m)\,iz}{2n+m+2+(-2i)\,z} \right) \tag{17.1.49}$$

$$n \in \mathbb{Z} \backslash \mathbb{Z}^-$$

$$\frac{H^{(1)}_{\nu+1}(z)}{H^{(1)}_\nu(z)} = \frac{2\nu+1-2iz}{2z} - \frac{1}{z} \underset{m=1}{\overset{\infty}{\mathrm{K}}} \left(\frac{\nu^2 - \frac{(2m-1)^2}{4}}{2\,(iz-m)} \right) \tag{17.1.51}$$

$$|\arg -iz| < \pi$$

1.2 Modified Bessel functions

$$I_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{2k}}{k!\,\Gamma(\nu+k+1)} \tag{17.2.20}$$

$$|\arg z| < \pi$$

$$I_\nu(z) = \frac{e^{-z}\left(\frac{z}{2}\right)^\nu}{\Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{\left(\nu+\frac{1}{2}\right)_k 2^k z^k}{(2\nu+1)_k k!} \tag{17.2.21}$$

$$|\arg z| < \pi$$

$$\mathsf{i}^{(1)}_n(z) = \frac{\sqrt{\pi}}{(2n+1)\,\Gamma\left(n+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(\frac{z^2}{4}\right)^k}{k!\left(n+\frac{3}{2}\right)_k} \tag{17.2.22}$$

$$n \in \mathbb{Z}$$

$$\mathsf{i}^{(1)}_n(z) = \frac{\sqrt{\pi}e^{-iz}}{(2n+1)\,\Gamma\left(n+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^n \sum_{k=0}^{\infty} \frac{(n+1)_k\,(2z)^k}{k!(2n+2)_k} \tag{17.2.23}$$

<div align="center">power series</div>

$$n \in \mathbb{Z}$$

$$I_\nu(z) = \sum_{k=0}^{\infty} \left(\frac{\left((-1)^k e^z + e^{-z + \frac{(2\nu+1)i\pi}{2}} \right)(\nu, k)}{\sqrt{2\pi z}\,(2z)^k} \right) \tag{17.2.24}$$

<div align="center">asymptotic series</div>

$$\arg z \in \left(\left(-\frac{\pi}{2} \right), \left(\frac{3\pi}{2} \right) \right)$$

$$I_\nu(z) = \sum_{k=0}^{\infty} \left(\frac{\left((-1)^k e^z + e^{-z - \frac{(2\nu+1)i\pi}{2}} \right)(\nu, k)}{\sqrt{2\pi z}\,(2z)^k} \right) \tag{17.2.25}$$

<div align="center">asymptotic series</div>

$$\arg z \in \left(\left(-\frac{3\pi}{2} \right), \left(\frac{\pi}{2} \right) \right)$$

$$K_\nu(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \sum_{k=0}^{\infty} \left(\frac{(\nu, k)}{(-2z)^k} \right) \tag{17.2.27}$$

<div align="center">asymptotic series</div>

$$|\arg z| < \frac{3\pi}{2}$$

$$\frac{I_{\nu+1}(z)}{I_\nu(z)} = \frac{\frac{z}{2(\nu+1)}}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{1}{4(\nu+m-1)(\nu+m)} z^2}{1} \right) \tag{17.2.32}$$

<div align="center">S-fraction</div>

$$\nu \geq 0$$

$$\frac{\mathrm{i}_{n+1}^{(1)}(z)}{\mathrm{i}_n^{(1)}(z)} = \frac{\frac{z}{2n+3}}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{1}{4\left(\left(n+\frac{1}{2}\right)+m-1\right)\left(\left(n+\frac{1}{2}\right)+m\right)} z^2}{1} \right) \tag{17.2.33}$$

<div align="center">S-fraction</div>

$$n \in \mathbb{Z} \backslash \mathbb{Z}^-$$

$$\frac{\nu}{z} - \frac{K_{\nu+1}(z)}{K_\nu(z)} = \frac{\nu}{z} + \frac{1}{1} + \frac{\frac{-2\nu-1}{2z}}{1} + - \overset{\infty}{\underset{m=3}{\mathrm{K}}} \left(\frac{\frac{\frac{m}{2}+\nu}{2z}}{1} \right) \tag{17.2.34}$$

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$$\frac{I_{\nu+1}(z)}{I_\nu(z)} = \frac{z}{2\nu + 2 + z} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{-(2\nu + 2m - 1)\,z}{2\nu + m + 1 + 2z}\right) \tag{17.2.38}$$

T-fraction

$\nu \in \mathbb{C}\backslash\mathbb{Z}^-$

$$\frac{\mathrm{i}_{n+1}^{(1)}(z)}{\mathrm{i}_n^{(1)}(z)} = \frac{z}{2n + 3 + z} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{-2\,(n + m)\,z}{2n + m + 2 + 2z}\right) \tag{17.2.39}$$

T-fraction

$n \in \mathbb{Z}\backslash\mathbb{Z}^-$

$$\frac{\nu}{z} - \frac{K_{\nu+1}(z)}{K_\nu(z)} = \frac{\nu}{z} - \frac{2\nu + 1 + 2z}{2z} - \frac{1}{z}\mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{\nu^2 - \frac{(2m-1)^2}{4}}{2\,(z + m)}\right) \tag{17.2.40}$$

J-fraction

$|\arg z| < \pi$

2 Confluent hypergeometric functions

2.1 Confluent hypergeometric series $_2F_0$

$$\frac{_1F_0(a;;z)}{_1F_0(a;;z)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{-\left(b + \frac{m}{2}\right)z}{1}\right) \tag{16.2.4}$$

C-fraction

$z < 0$

2.2 Confluent hypergeometric limit function

$$_0F_1(;b;z) = \sum_{k=0}^{\infty}\frac{z^k}{(b)_k\,k!} \tag{16.3.1}$$

power series

$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$

$$\frac{{}_0F_1(;b;z)}{{}_0F_1(;b+1;z)} = 1 + \underset{m=1}{\overset{\infty}{\mathrm{K}}}\left(\frac{\frac{1}{(b+m-1)(b+m)}z}{1}\right) \qquad (16.3.4)$$

C-fraction

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

$$\frac{{}_0F_1(;b;z)}{{}_0F_1(;b+1;z)} = 1 + \frac{\sqrt{z}}{b} + \frac{1}{2b}\underset{m=1}{\overset{\infty}{\mathrm{K}}}\left(\frac{-2\left(2b+2m-1\right)\sqrt{z}}{2b+m+4\sqrt{z}}\right) \qquad (16.3.6)$$

T-fraction

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

2.3   Kummer functions

$$\,{}_1F_1(a;b;z) = \sum_{k=0}^{\infty}\frac{\frac{(a)_k}{(b)_k}z^k}{k!} \qquad (16.1.2)$$

power series

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

$$\,{}_1F_0(a;;z) = \sum_{k=0}^{\infty}\frac{(a)_k(b)_k z^k}{k!} \qquad (16.1.12)$$

power series

$$\frac{{}_1F_1(a;b;z)}{{}_1F_1(a+1;b+1;z)} = 1 + \underset{m=1}{\overset{\infty}{\mathrm{K}}}\left(\frac{\frac{z\left(a+\frac{m}{2}\right)}{(b+m-1)(b+m)}}{1}\right) \qquad (16.1.13)$$

C-fraction

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

$$z\,{}_1F_1(1;b+1;z) = \frac{z}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{-\frac{z\left(b+\frac{m}{2}-1\right)}{(b+m-2)(b+m-1)}}{1}\right) \qquad (16.1.14)$$

C-fraction

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

$$\frac{{}_1F_1(a;b;z)}{{}_1F_1(a+1;b+1;z)} = \frac{b-z}{b} + \frac{1}{b}\operatorname*{K}_{m=1}^{\infty}\left(\frac{(a+m)\,z}{b+m-z}\right) \tag{16.1.16}$$

T-fraction

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

$$_1F_1(1;b+1;z) = \frac{b}{b-z} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{(m-1)\,z}{b+m-1-z}\right) \tag{16.1.17}$$

M-fraction

$$b \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

$$\frac{U(a,b,z)}{U(a+1,b,z)} = 2a-b+2+z - \operatorname*{K}_{m=1}^{\infty}\left(\frac{(a+m)\,(b-a-m-1)}{b-2a-2m-2-z}\right) \tag{16.1.20}$$

J-fraction

$$b \in \mathbb{C}\backslash\mathbb{Z}$$

2.4   Parabolic cylinder functions

$$\frac{U(a,x)}{U(a-1,x)} = \frac{1}{x} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{a+(m-1)-\frac{1}{2}}{x}\right) \tag{16.5.7}$$

C-fraction

$$x > 0, (2a) \in \mathbb{Z}^+$$

2.5   Whittaker functions

$$W_{\kappa,\mu}(z) = e^{-z^2}z^{\kappa}\sum_{k=0}^{\infty}\left(\frac{\left(-\kappa-\mu+\frac{1}{2}\right)_k\left(-\kappa+\mu+\frac{1}{2}\right)_k(-z)^{-k}}{k!}\right) \tag{16.4.7}$$

asymptotic series

$$|\arg z| < \frac{3\pi}{2}$$

$$\Psi_{n,\beta}(z) = \sum_{k=0}^{\infty}\left(\frac{\left(n+\frac{1}{2}\right)_k\left(\beta+\frac{1}{2},k\right)z^{-k-1}}{k!}\right) \tag{16.4.12}$$

asymptotic series

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3 Mathematical constants

3.1 Apéry's constant, $\zeta(3)$

$$\zeta(z) = \sum_{k=0}^{\infty} \frac{1}{(k+1)^z} \tag{10.11.1}$$

power series

$$\zeta(3) = \sum_{k=0}^{\infty} (-1)^k \left(\frac{(k!)^{10} \left(205k^2 + 250k + 77\right)}{64 \left((2k+1)!\right)^5} \right) \tag{10.11.3}$$

power series

$$\zeta(3) = \frac{6}{5} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{-(m-1)^6}{34(m-1)^3 + 51(m-1)^2 + 27(m-1) + 5} \right) \tag{10.11.5}$$

$$\zeta(3) = 1 + \frac{1}{22} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\left(\frac{m}{2}\right)^3}{1} \right) \tag{No label}$$

$$\zeta(3) = \sum_{n=1}^{k} \left(\frac{1}{n^3} \right) + \frac{1}{2k^2 + 2k + 1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{-(m-1)^6}{(m-1)^3 + m^3 + (2m-1)(2k^2 + 2k)} \right) \tag{No label}$$

$$k \in \mathbb{Z} \backslash \mathbb{Z}^-$$

$$\zeta(3) = 1 + \frac{1}{5} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{-(m-1)^6}{2(m-1)^3 + 3(m-1)^2 + 11(m-1) + 5} \right) \tag{No label}$$

$$\zeta(3) = 1 + \frac{1}{4} + \frac{1}{1} + \frac{1}{18} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{4} + \frac{1}{1} + \frac{1}{9} + \frac{1}{9} + \cdots \tag{10.11.4}$$

regular continued fraction

3.2 Archimedes' constant, symbol π

$$\pi = 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \tag{10.2.1}$$

power series

$$\pi = 3 + \frac{1}{7} + \frac{1}{15} + \frac{1}{1} + \frac{1}{292} + \frac{1}{1} + \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{1} + \frac{1}{3} + \cdots \tag{10.2.4}$$

regular continued fraction

$$\pi = \frac{4}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{(m-1)^2}{2m-1} \right) \tag{10.2.5}$$

S-fraction

$$\pi = 3 + \operatorname*{K}_{m=1}^{\infty} \left(\frac{(2m-1)^2}{6} \right) \tag{10.2.6}$$

S-fraction

$$P = 2 + \frac{1}{1} + 2 \operatorname*{K}_{m=2}^{\infty} \left(\frac{(m-1)\,m}{1} \right) \tag{No label}$$

$$\pi = \frac{4}{4k+1} + 2^{4k} \left(\frac{(k!)^2}{(2k)!} \right)^2 \operatorname*{K}_{m=2}^{\infty} \left(\frac{(2m-3)^2}{8k+2} \right) \tag{No label}$$

S-fraction

$$\pi = \frac{4}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{(2m-3)^2}{2} \right) \tag{No label}$$

$$\pi = \frac{4}{4+1} + 4 \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{(2m-3)^2}{8+2} \right)$$

(No label)

S-fraction

3.3 Catalan's constant, symbol G

$$G = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)^2}$$

(10.12.1)

power series

$$G = 0 + \frac{1}{1} + \frac{1}{10} + \frac{1}{1} + \frac{1}{8} + \frac{1}{1} + \frac{1}{88} + \frac{1}{4} + \frac{1}{1} + \frac{1}{1} + \frac{1}{7} + \cdots$$

(10.12.2)

regular continued fraction

$$G = \frac{1}{2} + \frac{1}{\frac{1}{2}} + \frac{1}{2} \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\left(\frac{m}{2} \right)^2}{\frac{1}{2}} \right)$$

(10.12.3)

S-fraction

$$G = \frac{\frac{13}{2}}{7} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{(2m-3)^4 (2m-2)^4 \left(20 (m-2)^2 - 8 (m-2) + 1 \right) \left(20 (m)^2 - 8 (m) + 1 \right)}{3520 (m-1)^6 + 5632 (m-1)^5 + 2064 (m-1)^4 - 384 (m-1)^3 - 156 (m-1)^2 + 16 (m-1) + 7} \right)$$

(10.12.5)

continued fraction

$$G = 1 + \frac{-1}{3} + \frac{1}{2} \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{m^2}{1} \right)$$

(No label)

continued fraction

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3.4 Euler's constant, symbol γ

$$\gamma = -\log(n) + \sum_{k=0}^{\infty} \frac{1}{k} \tag{10.8.1}$$

power series

$$\gamma = 0 + \frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{1} + \frac{1}{2} + \frac{1}{1} + \frac{1}{4} + \frac{1}{3} + \frac{1}{13} + \frac{1}{5} + \cdots \tag{No label}$$

regular continued fraction

3.5 Euler's number, base of the natural logarithm

$$e = \sum_{k=0}^{\infty} \frac{1}{(k)!} \tag{10.3.1b}$$

power series

$$e = 2 + \mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{1}{1}\right) \tag{10.3.5}$$

regular continued fraction

$$\frac{e-1}{e+1} = \mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{1}{(4m-2)}\right) \tag{10.3.6}$$

regular continued fraction

$$e = 2 + \mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{m+1}{m+1}\right) \tag{No label}$$

regular continued fraction

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$$e = 1 + \frac{2}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{1}{6 + (m-2)\,4} \right) \qquad \text{(No label)}$$

regular continued fraction

$$e = \frac{1}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{-1}{m-1} \right) \qquad \text{(No label)}$$

regular continued fraction

3.6 Golden ratio, symbol ϕ

$$\phi = 1 + \underset{m=1}{\overset{\infty}{\mathrm{K}}} \left(\frac{1}{1} \right) \qquad (10.9.4)$$

regular continued fraction

3.7 Gompertz' constant, symbol G

$$G = \frac{1}{2} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{-(m-1)^2}{2m} \right) \qquad (10.13.1)$$

continued fraction

$$G = \frac{1}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\frac{m}{2}}{1} \right) \qquad \text{(No label)}$$

continued fraction

3.8 The natural logarithm, $\ln(2)$

$$\ln(2) = \sum_{k=0}^{\infty} \frac{(-1)^k \, 1^{k+1}}{k+1} \qquad (10.5.2)$$

power series

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$$\ln(2) = \cfrac{1}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{(m-1)^2}{1}\right) \tag{10.5.3}$$

S-fraction

$$\ln(2) = \cfrac{1}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{\frac{m}{4m-4}}{1}\right) \tag{10.5.4}$$

S-fraction

3.9   Regular continued fractions

$$\sqrt{e} = 1 + \cfrac{1}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{1}{1}\right) \tag{10.4.1}$$

regular continued fraction

$$e^{\frac{1}{n}} = 1 + \underset{m=1}{\overset{\infty}{\mathrm{K}}}\left(\frac{1}{\left(2\left(\frac{m+2}{3}\right)-1\right)n-1}\right) \tag{10.4.2}$$

regular continued fraction

$$e^{\frac{1}{n}} = \frac{n+1}{n} + \cfrac{1}{n}\underset{m=1}{\overset{\infty}{\mathrm{K}}}\left(\frac{1}{2n-1}\right) \tag{10.4.3}$$

regular continued fraction

$$e^{\frac{1}{n}} = \cfrac{1}{n-1} + \cfrac{1}{2n} + n\underset{m=3}{\overset{\infty}{\mathrm{K}}}\left(\frac{1}{1}\right) \tag{10.4.4}$$

regular continued fraction

$$\sqrt{\pi} = 1 + \cfrac{1}{1} + \cfrac{1}{3} + \cfrac{1}{2} + \cfrac{1}{1} + \cfrac{1}{1} + \cfrac{1}{6} + \cfrac{1}{1} + \cfrac{1}{28} + \cfrac{1}{13} + \cfrac{1}{1} + \cdots \tag{No label}$$

regular continued fraction

$$ee = 7 + \overset{\infty}{\underset{m=1}{\mathrm{K}}} \left(\frac{1}{\frac{3(m+4)}{5} - 1} \right)$$

(10.4.5)

regular continued fraction

$$\frac{\pi^2}{12} = \frac{1}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{(m-1)^4}{2m-1} \right)$$

(10.4.6)

S-fraction

$$\frac{e^{\frac{2n}{\beta}} - 1}{e^{\frac{2n}{\beta}} + 1} = \frac{n}{\beta} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{n^2}{(2m-1)\,\beta} \right)$$

(10.4.7)

S-fraction

3.10 Pythagoras' constant, the square root of two

$$1 + \sqrt{2} = 2 + \overset{\infty}{\underset{m=1}{\mathrm{K}}} \left(\frac{1}{2} \right)$$

(10.6.3)

regular continued fraction

$$\left(1 + \sqrt{2} \right)^2 = 5 + \overset{\infty}{\underset{m=1}{\mathrm{K}}} \left(\frac{1}{1} \right)$$

(No label)

regular continued fraction

$$\left(1 + \sqrt{2} \right)^3 = 14 + \overset{\infty}{\underset{m=1}{\mathrm{K}}} \left(\frac{1}{14} \right)$$

(No label)

regular continued fraction

$$\left(1 + \sqrt{2}\right)^4 = 33 + \overset{\infty}{\underset{m=1}{\mathrm{K}}} \left(\frac{1}{1}\right)$$

(No label)

regular continued fraction

$$\left(1 + \sqrt{2}\right)^5 = 82 + \overset{\infty}{\underset{m=1}{\mathrm{K}}} \left(\frac{1}{82}\right)$$

(No label)

regular continued fraction

3.11 The rabbit constant, symbol ρ

$$\rho = \sum_{k=0}^{\infty} 2^{-\left\lfloor (k+1)\left(\frac{\sqrt{5}+1}{2}\right)\right\rfloor}$$

(10.10.2)

power series

$$\rho = \overset{\infty}{\underset{m=1}{\mathrm{K}}} \left(\frac{1}{2^{F_{m-1}}}\right)$$

(10.10.5)

regular continued fraction

3.12 zeta2, $\zeta(2)$

$$\zeta(2) = \frac{2}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{(m-1)^4}{2m-1}\right)$$

(No label)

S-fraction

$$\zeta(2) = \frac{1}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{-\frac{\left(\left((m-2)\left(\frac{1}{2}\right)+1\right)\left((m-2)\left(\frac{1}{2}\right)+1\right)\right)\left((m-2)\left(\frac{1}{2}\right)+1\right)}{(m-1)(1+(m-1))}}{1}\right)$$

(No label)

continued fraction

$$\zeta(2) = 1 + \cfrac{1}{1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{\left(\frac{m}{2}\right)^2}{1}\right) \qquad \text{(No label)}$$

$$\zeta(2) = \sum_{n=1}^{k}\left(\frac{1}{n^2}\right) + \cfrac{2}{2k+1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{(m-1)^4}{(2m-1)(2k+1)}\right) \qquad \text{(No label)}$$

$$k \in \mathbb{Z}\backslash\mathbb{Z}^-$$

$$\zeta(2) = 2\sum_{n=1}^{k}\left(\frac{(-1)^{n-1}}{n^2}\right) + \cfrac{(-1)^k}{k^2+k+1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{-(m-1)^4}{(m-1)^2+m^2+k^2+k}\right) \qquad \text{(No label)}$$

$$k \in \mathbb{Z}\backslash\mathbb{Z}^-$$

$$\zeta(2) = \frac{5}{3} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{(m-1)^4}{11(m-1)^2+11(m-1)+3}\right) \qquad \text{(No label)}$$

3.13 zeta4, $\zeta(4)$

$$\zeta(4) = \frac{13}{12} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{(m-1)^7(3m-2)(3m-3)(3m-4)}{3(2(m-1)+1)\left(3(m-1)^2+3(m-1)+1\right)\left(15(m-1)^2+15(m-1)+4\right)}\right) \qquad \text{(No label)}$$

$$\zeta(4) = 1 + \cfrac{1}{12} + \cfrac{1}{6} + \cfrac{1}{1} + \cfrac{1}{3} + \cfrac{1}{1} + \cfrac{1}{4} + \cfrac{1}{183} + \cfrac{1}{1} + \cfrac{1}{1} + \cfrac{1}{2} + \cdots \qquad \text{(No label)}$$

regular continued fraction

4 Elementary functions

4.1 Inverse trigonometric functions

$$\text{Arccos}\,(z) = \frac{\pi}{2} - z + \sum_{k=0}^{\infty} - \left(\frac{(2k-1)!!}{(2k)!!\,(2k+1)}\right) z^{2k+1} \tag{11.4.2}$$

power series

$$|z| < 1$$

$$\text{Arccos}\,(z) = \frac{\frac{\sqrt{1-z^2}}{z}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\left(\frac{(m-1)^2}{(2m-3)(2m-1)}\right)\left(1-z^2\right)}{z^2}\right) \tag{11.4.6}$$

S-fraction

$$\text{Re}\,(z) > 0, \frac{1-z^2}{z^2} > -1$$

$$\text{Arccos}\,(z) = \frac{z\sqrt{1-z^2}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\left(-\frac{m(m-1)}{(2m-1)(2m-3)}\right)\left(1-z^2\right)}{1}\right) \tag{11.4.7}$$

S-fraction

$$\text{Re}\,(z) > 0$$

4.2 Inverse hyperbolic functions

$$\text{Arccosh}\left(\frac{1}{z}\right) = \ln\left(\frac{2}{z}\right) + \sum_{k=0}^{\infty} - \left(\frac{(2k-1)!!}{(2k)!!\,(2k)}\right) z^{2k} \tag{11.6.2}$$

power series

$$|z| < 1$$

$$\text{Arccosh}\,(z) = \frac{z\sqrt{z^2-1}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\left(\frac{m(m-1)}{(2m-3)(2m-1)}\right)\left(z^2-1\right)}{1}\right) \tag{11.6.6}$$

S-fraction

$$\text{Re}\,(z) > 0$$

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$$\text{Arccosh}\,(z) = \frac{\frac{\sqrt{z^2-1}}{z}}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{-\frac{\left(\frac{(m-1)^2}{(2m-3)(2m-1)}\right)(z^2-1)}{z^2}}{1} \right) \tag{11.6.7}$$

S-fraction

$$\left| \arg \frac{1}{z^2} \right| < \pi$$

4.3 Inverse trigonometric functions

$$\text{Arcsin}\,(z) = z + \sum_{k=0}^{\infty} \left(\frac{(2k-1)!!}{(2k)!!\,(2k+1)} \right) z^{2k+1} \tag{11.4.1}$$

power series

$$|z| < 1$$

$$\text{Arcsin}\,(z) = \frac{\frac{z}{\sqrt{1-z^2}}}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{\left(\frac{(m-1)^2}{(2m-3)(2m-1)}\right)z^2}{1-z^2}}{1} \right) \tag{11.4.4}$$

S-fraction

$$\left| \arg 1 - z^2 \right| < \pi$$

$$\text{Arcsin}\,(z) = \frac{z\sqrt{1-z^2}}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\left(-\frac{m(m-1)}{(2m-1)(2m-3)}\right)z^2}{1} \right) \tag{11.4.5}$$

S-fraction

$$\left| \arg 1 - z^2 \right| < \pi$$

4.4 Inverse hyperbolic functions

$$\text{Arcsinh}\,(z) = z + \sum_{k=0}^{\infty} \left(\frac{(-1)^k\,(2k-1)!!}{(2k)!!\,(2k+1)} \right) z^{2k+1} \tag{11.6.1}$$

power series

$$|z| < 1$$

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$$\text{Arcsinh}\,(z) = \frac{z\sqrt{1+z^2}}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{\left(\frac{m(m-1)}{(2m-3)(2m-1)}\right)z^2}{1}\right) \tag{11.6.4}$$

S-fraction

$$not\,(iz < -1)\,, not\,(iz > 1)$$

$$\text{Arcsinh}\,(z) = \frac{\frac{z}{\sqrt{1+z^2}}}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{-\frac{\left(\frac{(m-1)^2}{(2m-3)(2m-1)}\right)z^2}{1+z^2}}{1}\right) \tag{11.6.5}$$

S-fraction

$$not\,(iz < -1)\,, not\,(iz > 1)$$

4.5   Inverse trigonometric functions

$$\text{Arctan}\,(z) = \sum_{k=0}^{\infty}\left(\frac{(-1)^k}{2k+1}\right)z^{2k+1} \tag{11.4.3}$$

power series

$$|z| < 1$$

$$\text{Arctan}\,(z) = \frac{z}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{(m-1)^2\,z^2}{2m-1}\right) \tag{11.4.8}$$

S-fraction

$$not\,(iz < -1)\,, not\,(iz > 1)$$

$$\text{Arctan}\,(z) = \frac{\frac{z}{1+z^2}}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}}\left(\frac{-\left(\frac{m(m-1)}{(2m-3)(2m-1)}\right)\left(\frac{z^2}{1+z^2}\right)}{1}\right) \tag{11.4.9}$$

S-fraction

$$not\,(iz < -1)\,, not\,(iz > 1)$$

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4.6 Inverse hyperbolic functions

$$\text{Arctanh}\,(z) = \sum_{k=0}^{\infty} \left(\frac{1}{2k+1} \right) z^{2k+1} \tag{11.6.3}$$

power series

$$|z| < 1$$

$$\text{Arctanh}\,(z) = \frac{\frac{z}{1-z^2}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\left(\frac{m(m-1)}{(2m-3)(2m-1)} \right) z^2}{\frac{1-z^2}{1}} \right) \tag{11.6.8}$$

S-fraction

$$\left| \arg 1 - z^2 \right| < \pi$$

$$\text{Arctanh}\,(z) = \frac{z}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\left(-\frac{(m-1)^2 z^2}{4(m-1)^2 - 1} \right)}{1} \right) \tag{11.6.9}$$

S-fraction

$$\left| \arg 1 - z^2 \right| < \pi$$

4.7 Trigonometric functions

$$\cos\,(z) = \sum_{k=0}^{\infty} \left(\frac{(-1)^k}{(2k)!} \right) z^{2k} \tag{11.3.2}$$

power series

4.8 Hyperbolic functions

$$\cosh\,(z) = \sum_{k=0}^{\infty} \left(\frac{1}{(2k)!} \right) z^{2k} \tag{11.5.2}$$

power series

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4.9 Hyperbolic functions

$$\coth(z) = \sum_{k=0}^{\infty} \left(\frac{4^k B_{2k}}{(2k)!} \right) z^{2k-1} \tag{11.5.4}$$

power series

$$|z| < \pi$$

$$\coth(z) = \frac{1}{z} + \frac{4\pi^{-2}z}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{(m-1)^2 \left((m-1)^2 + 4\pi^{-2}z^2 \right)}{2m-1} \right) \tag{11.5.6}$$

Thiele interpolating fraction

4.10 The exponential function

$$e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!} \tag{11.1.1}$$

power series

$$e^z = 1 + \frac{2z}{2-z} + \frac{\frac{z^2}{6}}{1} + \underset{m=3}{\overset{\infty}{\mathrm{K}}} \left(\frac{\left(\frac{1}{4(2m-3)(2m-1)} \right) z^2}{1} \right) \tag{11.1.2}$$

S-fraction

$$e^z = 1 + \frac{z}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\left(-\frac{1}{2(m-1)} \right) z}{1} \right) \tag{11.1.3}$$

C-fraction

$$e^z = \frac{1}{1} + \frac{-z}{1} + \underset{m=3}{\overset{\infty}{\mathrm{K}}} \left(\frac{\left(-\frac{1}{2(m-1)} \right) z}{1} \right) \tag{No label}$$

$$e^z = 1 + \frac{z}{1-z} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(m-1)\,z}{m-z} \right) \tag{11.1.4}$$

T-fraction

4.11   The natural logarithm

$$\ln(1+z) = \sum_{k=0}^{\infty} \left(\frac{(-1)^{k+2}}{k+1} \right) z^{k+1} \tag{11.2.1}$$

power series

$$|z| < 1$$

$$\ln(1+z) = \frac{z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\left(\frac{m}{4(m-1)} \right) z}{1} \right) \tag{11.2.2}$$

S-fraction

$$|\arg 1 + z| < \pi$$

$$\ln(1+z) = \frac{2z}{2+z} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-(m-1)^2\,z^2}{(2m-1)(2+z)} \right) \tag{11.2.3}$$

Euler even contraction

$$\left| \arg 1 - \frac{z^2}{(2+z)^2} \right| < \pi$$

$$\ln\left(\frac{1+z}{1-z} \right) = \frac{2z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\left(-\left(\frac{(m-1)^2}{(2m-3)(2m-1)} \right) z^2 \right)}{1} \right) \tag{11.2.4}$$

S-fraction

$$\left| \arg 1 - z^2 \right| < \pi$$

$$\ln(1+z) = z + \frac{-\frac{z^2}{2}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{\left(\left(\frac{m}{2} \right) + 1 \right)^2 z}{m(m+1)}}{1} \right) \tag{6.8.8}$$

Thiele interpolating fraction

4.12 The power function

$$(1+z)^n = 1 + \frac{nz}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{\left(\left(\frac{m}{2}\right)-n\right)z}{2(m-1)}}{1} \right) \tag{11.7.1}$$

C-fraction

$$|\arg z + 1| < \pi$$

$$(1+z)^n = \frac{1}{1} + \frac{-nz}{1} + \overset{\infty}{\underset{m=3}{\mathrm{K}}} \left(\frac{\frac{\left(\frac{m-1}{2}+n\right)z}{2(m-2)}}{1} \right) \tag{11.7.2}$$

C-fraction

$$|\arg z + 1| < \pi$$

$$(1+z)^n = \frac{1}{1} + \frac{-nz}{1+z} + \frac{\frac{(n-1)z}{2}}{1} + \overset{\infty}{\underset{m=4}{\mathrm{K}}} \left(\frac{\frac{\left(-n-\left(\frac{m-2}{2}\right)\right)z}{2(m-1)(1+z)}}{1} \right) \tag{11.7.3}$$

C-fraction

$$|\arg z + 1| < \pi$$

$$\left(\frac{z+1}{z-1}\right)^n = 1 + \frac{\frac{2n}{z}}{1-\frac{n}{z}} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{n^2-(m-1)^2}{(2(m-1)-1)(2(m-1)+1)z^2}}{1} \right) \tag{11.7.4}$$

C-fraction

$$(\mathrm{Re}\,(z) < -1)\,or\,(\mathrm{Re}\,(z) > 1)$$

4.13 Trigonometric functions

$$\sin(z) = \sum_{k=0}^{\infty} \left(\frac{(-1)^k}{(2k+1)!} \right) z^{2k+1} \tag{11.3.1}$$

power series

4.14 Hyperbolic functions

$$\sinh(z) = \sum_{k=0}^{\infty} \left(\frac{1}{(2k+1)!} \right) z^{2k+1} \tag{11.5.1}$$

power series

4.15 Trigonometric functions

$$\tan\left(z\right) = \sum_{k=0}^{\infty} \left(\frac{4^{k+1}\left(4^{k+1}-1\right)\left|B_{2(k+1)}\right|}{\left(2\left(k+1\right)\right)!} \right) z^{2(k+1)-1} \tag{11.3.3}$$

power series

$$\left|z\right| < \frac{\pi}{2}$$

$$\tan\left(z\right) = \frac{z}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{-\frac{z^2}{(2m-1)(2m-3)}}{1} \right) \tag{11.3.7}$$

S-fraction

$$\tan\left(\frac{\pi z}{4}\right) = \frac{z}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{(2m-3)^2 - z^2}{2} \right) \tag{11.3.8}$$

Thiele interpolating fraction

$$\tan\left(z\right) = \frac{z}{1} + \frac{-4\pi^{-2}z^2}{1} + \underset{m=3}{\overset{\infty}{\mathrm{K}}} \left(\frac{(m-2)^4 - 4\pi^{-2}\left(m-2\right)^2 z^2}{2m-3} \right) \tag{11.3.9}$$

Thiele interpolating fraction

4.16 Hyperbolic functions

$$\tanh\left(z\right) = \sum_{k=0}^{\infty} \left(4^{k+1}\left(4^{k+1}-1\right)\left(\frac{B_{2(k+1)}}{\left(2\left(k+1\right)\right)!} \right) \right) z^{2(k+1)-1} \tag{11.5.3}$$

power series

$$\left|z\right| < \frac{\pi}{2}$$

$$\tanh\left(z\right) = \frac{z}{1} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\left(\frac{1}{(2m-3)(2m-1)} \right) z^2}{1} \right) \tag{11.5.5}$$

S-fraction

5 Error function and related integrals

5.1 Complementary and complex error function

$$\operatorname{erfc}(z) = e^{-z^2} \sum_{k=0}^{\infty} \frac{(-z)^k}{\Gamma(\frac{k}{2}+1)} \tag{13.2.9}$$

power series

$$w(z) = \sum_{k=0}^{\infty} \frac{(iz)^k}{\Gamma(\frac{k}{2}+1)} \tag{13.2.10}$$

power series

$$\operatorname{erfc}(z) = \frac{1}{\sqrt{\pi}ze^{z^2}} \sum_{k=0}^{\infty} \left((-1)^k \left(\frac{1}{2}\right)_k z^{-2k} \right) \tag{13.2.11}$$

asymptotic series

$$|\arg z| < \frac{3\pi}{4}$$

$$w(z) = \frac{i}{\pi z} \sum_{k=0}^{\infty} \left(\left(\frac{1}{2}\right)_k z^{-2k} \right) \tag{13.2.12}$$

asymptotic series

$$|\arg -iz| < \frac{3\pi}{4}$$

$$\operatorname{erfc}(z) = \frac{1}{z^2} + \frac{z}{\sqrt{\pi}}e^{-z^2} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{m-1}{2}}{1} \right) \tag{13.2.20a}$$

S-fraction

$$\operatorname{Re}(z) > 0$$

$$w(z) = \frac{1}{-z^2} + -\frac{iz}{\sqrt{\pi}} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{m-1}{2}}{1} \right) \tag{13.2.20b}$$

S-fraction

$$\operatorname{Im}(z) > 0$$

$$\mathrm{erfc}\,(z) = \frac{2z}{2z^2 + 1} + \frac{e^{-z^2}}{\sqrt{\pi}} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{-(2m-3)(2m-2)}{4m-3+2z^2} \right) \tag{13.2.23a}$$

<div align="center">J-fraction</div>

$$\mathrm{Re}\,(z) > 0$$

$$w\,(z) = \frac{-\frac{iz}{\sqrt{\pi}}}{\frac{1}{2} - z^2} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{-\left(-\frac{3}{2} + m\right)(m-1)}{2m - \frac{3}{2} - z} \right) \tag{13.2.23b}$$

<div align="center">J-fraction</div>

$$\mathrm{Im}\,(z) > 0$$

5.2   Error function and Dawson's integral

$$\mathrm{erf}\,(z) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k\, z^{2k+1}}{(2k+1)\, k!} \tag{13.1.7}$$

<div align="center">power series</div>

$$\mathrm{erf}\,(z) = \frac{2}{\sqrt{\pi}} e^{-z^2} \sum_{k=0}^{\infty} \frac{z^{2k+1}}{\left(\frac{3}{2}\right)_k} \tag{13.1.8}$$

<div align="center">power series</div>

$$F(z) = e^{-z^2} \sum_{k=0}^{\infty} \frac{z^{2k+1}}{(2k+1)\, k!} \tag{13.1.9}$$

<div align="center">power series</div>

$$F(z) = 1 \sum_{k=0}^{\infty} \frac{(-1)^k\, z^{2k+1}}{\left(\frac{3}{2}\right)_k} \tag{13.1.10}$$

<div align="center">power series</div>

$$\operatorname{erf}(z) = \frac{2z^2}{1} \; + \; \frac{1}{\sqrt{\pi}ze^{z^2}} \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\frac{-2(m-1)}{(2m-3)(2m-1)}z^2}{1} \right) \tag{13.1.11a}$$

<div align="center">C-fraction</div>

$$F(z) = \frac{-2z^2}{1} \; + \; -\frac{1}{2z} \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\frac{2(m-1)}{(2m-3)(2m-1)}z^2}{1} \right) \tag{13.1.11b}$$

<div align="center">C-fraction</div>

$$\operatorname{erf}(z) = \frac{2z^2}{1 - 2z^2} \; + \; \frac{1}{\sqrt{\pi}ze^{z^2}} \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\frac{4(m-1)}{(2m-3)(2m-1)}z^2}{1 - \frac{2}{2m-1}z^2} \right) \tag{13.1.13a}$$

<div align="center">T-fraction</div>

$$F(z) = \frac{-2z^2}{1 + 2z^2} \; + \; -\frac{1}{2z} \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{-\frac{4(m-1)}{(2m-3)(2m-1)}z^2}{1 + \frac{2}{2m-1}z^2} \right) \tag{13.1.13b}$$

<div align="center">T-fraction</div>

5.3   Fresnel integrals

$$C(z) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{2}\right)^{2k}}{(2k)!\,(4k+1)} z^{4k+1} \tag{13.4.6a}$$

<div align="center">power series</div>

$$S(z) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{2}\right)^{2k+1}}{(2k+1)!\,(4k+3)} z^{4k+3} \tag{13.4.6b}$$

<div align="center">power series</div>

<div align="center">27</div>

$$C(z) + i\,S(z) = \frac{z^2}{1} + \frac{e^{\frac{i\pi z^2}{2}}}{z} \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\frac{i\pi(m-1)}{(2m-3)(2m-1)}z^2}{1} \right) \tag{13.4.9}$$

<center>C-fraction</center>

$$C(z) + i\,S(z) = \frac{z^2}{1 + i\pi z^2} + \frac{e^{\frac{i\pi z^2}{2}}}{z} \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{-\frac{2i\pi(m-1)}{(2m-3)(2m-1)}z^2}{1 + \frac{i\pi}{2m-1}z^2} \right) \tag{13.4.10}$$

<center>T-fraction</center>

5.4 Repeated integrals

$$I^n \operatorname{erfc}(z) = \frac{\frac{2}{\sqrt{\pi}}e^{-z^2}}{(2z)^{n+1}} \sum_{k=0}^{\infty} \left(\frac{(-1)^k (2k+n)!}{n!k!(2z)^{2k}} \right) \tag{13.3.2}$$

<center>asymptotic series</center>

$$|\arg z| < \frac{3\pi}{4}$$

$$\frac{I^n \operatorname{erfc}(z)}{I^{n-1} \operatorname{erfc}(z)} = \frac{\frac{1}{2}}{z} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\frac{n+m-1}{2}}{z} \right) \tag{13.3.5}$$

<center>S-fraction</center>

$$\operatorname{Re}(z) > 0, n \in \mathbb{Z}\backslash\mathbb{Z}^-$$

6 Exponential integrals and related functions

6.1 Exponential integrals

$$E_1(z) = -1 \sum_{k=0}^{\infty} \frac{(-1)^k z^k}{kk!} \tag{14.1.10}$$

<center>power series</center>

$$|\arg z| < \pi$$

$$E_n(z) = \left(\frac{(-z)^{n-1}}{(n-1)!} \right) \left(-\gamma - \ln(z) + \sum_{i=1}^{n-1} \left(\frac{1}{i} \right) \right) - \sum_{i=0}^{n-2} \left(\frac{(-z)^i}{(i-n+1)\,i!} \right) + \sum_{k=0}^{\infty} - \left(\frac{(-z)^{k+n-1}}{(k)\,(k+n-1)!} \right) \tag{14.1.11}$$

<center>28</center>

power series

$$|\arg z| < \pi, n \in \mathbb{Z}^+$$

$$E_\nu(z) = \Gamma(1-\nu)z^{\nu-1} - \frac{1}{1-\nu} + \sum_{k=0}^{\infty} \frac{(-1)^{k+1} z^k}{(k!)\,(k+1-\nu)} \qquad (14.1.12)$$

power series

$$z <> 0, \nu \in \mathbb{C}\backslash\mathbb{Z}^+$$

$$E_\nu(z) = e^{-z} \sum_{k=0}^{\infty} \left(\frac{(-1)^k \, (\nu)_k}{z^{k+1}} \right) \qquad (14.1.13)$$

asymptotic series

$$|\arg z| < \pi$$

$$E_n(z) = \frac{1}{z} + \frac{n}{1} + e^{-z} \overset{\infty}{\underset{m=3}{\mathrm{K}}} \left(\frac{n + \frac{m-2}{2}}{1} \right) \qquad (14.1.16)$$

S-fraction

$$|\arg z| < \pi, n \in \mathbb{Z}^+$$

$$E_\nu(z) = \frac{\frac{e^{-z}}{z}}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{m}{2}+\nu-1}{1} \right) \qquad (14.1.19)$$

C-fraction

$$|\arg z| < \pi$$

$$E_\nu(z) = z^{\nu-1}\,\Gamma(1-\nu) + \frac{-e^{-z}}{1-\nu} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{(\nu - \frac{m}{2})\,z}{m - \nu} \right) \qquad (14.1.20)$$

C-fraction

$$z <> 0, \nu \in \mathbb{C}\backslash\mathbb{Z}^+$$

$$E_\nu(z) = z^{\nu-1}\,\Gamma(1-\nu) + \frac{1}{1-\nu-z} + -e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{(m-1)\,z}{m - \nu - z} \right) \qquad (14.1.22)$$

M-fraction

$$z <> 0, \nu \in \mathbb{C}\backslash\mathbb{Z}^+$$

$$E_\nu(z) = \frac{1}{\nu + z} + e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{-(m-1)(\nu + m - 2)}{\nu + (m-1)2 + z} \right) \qquad (14.1.23)$$

J-fraction

$$|\arg z| < \pi$$

$$E_\nu(z) = \frac{e^{-z}}{z} + \frac{-e^{-z}\nu}{z(1+z) + \nu z} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{-(m-1)(\nu + m - 1)z^2}{z(m+z) + (\nu + m - 1)z} \right) \qquad (14.1.24)$$

J-fraction

$$|\arg z| < \pi$$

6.2   Related functions

$$E_1(x) = \gamma + \ln(x) + \sum_{k=0}^{\infty} \frac{x^k}{kk!} \qquad (14.2.14)$$

power series

$$x > 0$$

$$\mathrm{Ein}(z) = -1 \sum_{k=0}^{\infty} \frac{(-1)^{k+1} z^{k+1}}{(k+1)(k+1)!} \qquad (14.2.16)$$

power series

$$E_1(x) = \frac{e^x}{x} \sum_{k=0}^{\infty} \left(k! x^{-k} \right) \qquad (14.2.19)$$

asymptotic series

$$E_1(z) = 2 \sum_{i=0}^{\infty} \left(\frac{z^{2i+1}}{(2i+1)(2i+1)!} \right) + \frac{\frac{1}{z}}{1} + -e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\left\lfloor \frac{m}{2} \right\rfloor}{z} \right) \qquad (14.2.21)$$

S-fraction

$$x > 0$$

$$\text{Ein}(z) = \gamma + \ln{(z)} + \cfrac{\frac{1}{z}}{1} + e^{-z} \operatornamewithlimits{K}_{m=2}^{\infty}\left(\cfrac{\frac{\left\lfloor \frac{m}{2}\right\rfloor}{z}}{1}\right) \tag{14.2.23}$$

S-fraction

$$|\arg z| < \pi$$

$$E_1(x) = \cfrac{\frac{e^x}{x}}{1} + \operatornamewithlimits{K}_{m=2}^{\infty}\left(\cfrac{-\frac{\left\lfloor \frac{m}{2}\right\rfloor}{x}}{1}\right) \tag{14.2.24}$$

C-fraction

$$x > 0$$

7 Gamma function and related functions

7.1 Binet function

$$functions : Binet\,(z) = \sum_{k=0}^{\infty}\left(\left(\frac{B_{2(k+1)}}{(2k+1)\,(2k+2)}\right)z^{-2k-1}\right) \tag{12.2.6}$$

asymptotic series

$$|\arg z| < \pi$$

7.2 Incomplete gamma functions

$$\gamma(a,z) = z^a \sum_{k=0}^{\infty}\frac{(-z)^k}{(a+k)\,k!} \tag{12.6.7}$$

power series

$$\text{Re}\,(a) > 0$$

$$\gamma(a,z) = \frac{z^a e^{-z}}{a}\sum_{k=0}^{\infty}\frac{z^k}{(1+a)_k} \tag{12.6.8}$$

power series

$$\text{Re}\,(a) > 0$$

$$\Gamma(a,z) = z^a e^{-z}\sum_{k=0}^{\infty}\left((-1)^k\,(1-a)_k z^{-k-1}\right) \tag{12.6.10}$$

asymptotic series

$$|\arg z| < \pi$$

$$\Gamma(a,z) = \frac{1}{z} + z^a e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\left(\frac{m}{2}\right) - a}{1} \right) \tag{12.6.15}$$

S-fraction

$$a \in (-\infty,(1)), |\arg z| < \pi$$

$$\Gamma(a,z) = \frac{\frac{1}{z}}{1} + z^a e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\left(\left(\frac{m}{2}\right) - a\right) z^{-1}}{1} \right) \tag{12.6.17}$$

C-fraction

$$|\arg z| < \pi$$

$$\gamma(a,z) = \frac{\frac{z}{a}}{1} + z^{a-1} e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\left(-\frac{a+\left(\frac{m}{2}\right)-1}{(a+m-2)(a+m-1)}\right) z}{1} \right) \tag{12.6.23}$$

C-fraction

$$\mathrm{Re}\,(a) > 0$$

$$\Gamma(a,z) = \Gamma(a) + \frac{\frac{z}{a}}{1} + -\frac{z^a e^{-z}}{z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\left(-\frac{a+\left(\frac{m}{2}\right)-1}{(a+m-2)(a+m-1)}\right) z}{1} \right) \tag{12.6.24}$$

C-fraction

$$|\arg z| < \pi, a \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

$$\gamma(a,z) = \frac{1}{a-z} + z^a e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{(m-1)z}{a+(m-1)-z} \right) \tag{12.6.30}$$

M-fraction

$$a \in \mathbb{C}\backslash\mathbb{Z}_0^-$$

$$\Gamma(a,z) = \Gamma(a) + \frac{-1}{a-z} + z^a e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{(m-1)z}{a+(m-1)-z} \right) \tag{12.6.31}$$

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$$|\arg z| < \pi, \operatorname{Re}(a) > 0$$

$$\Gamma(a, z) = \frac{1}{1 - a + z} + z^a e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{(1 - m)(m - 1 - a)}{((2m) - 1) - a + z} \right) \tag{12.6.34}$$

J-fraction

$$|\arg z| < \pi$$

$$\Gamma(a, z) = z^{a-1} e^{-z} + \frac{a - 1}{2 + z - a} + z^{a-1} e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{(1 - m)(m - a)}{2m + z - a} \right) \tag{12.6.35}$$

J-fraction

$$|\arg z| < \pi$$

7.3 Polygamma functions

$$\psi(z) = -\gamma + \sum_{k=0}^{\infty} \left(\frac{1}{k} \right) - \left(\frac{1}{z + k - 1} \right) \tag{12.3.2a}$$

power series

$$z \in \mathbb{C} \backslash \mathbb{Z}_0^-$$

$$\psi^{(n)}(z) = (-1)^{n+1} n! \sum_{k=0}^{\infty} \frac{1}{(z + k)^{n+1}} \tag{12.3.2b}$$

power series

$$z \in \mathbb{C} \backslash \mathbb{Z}_0^-, n \in \mathbb{Z}^+$$

$$\psi(z) = \ln(z) - \left(\frac{1}{2z} \right) + \sum_{k=0}^{\infty} \left(-\frac{B_{2k}}{2k} z^{-2k} \right) \tag{12.3.7}$$

asymptotic series

$$|\arg z| < \pi$$

$$\psi^{(n)}(z) = (-1)^{n-1} \sum_{k=0}^{\infty} \left(\left(\frac{B_{2k}(2k + n - 1)!}{(2k)!} \right) z^{-2k-n} \right) \tag{12.3.8}$$

asymptotic series

$$|\arg z| < \pi, n \in \mathbb{Z}^+$$

7.4 Tetragamma function

$$\psi^{(2)}(z) = -\frac{1}{z^2} - \frac{1}{z^3} + \frac{\left(\frac{1}{8\pi^2}\right)}{z^2} + -\left(\frac{2\pi}{z}\right)^2 \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{\left(\frac{m}{2}\right)^2\left(\left(\frac{m}{2}\right)+1\right)}{2(m+1)}}{1}\right) \tag{12.5.1}$$

S-fraction

$$|\arg z| < \frac{\pi}{2}$$

$$\psi^{(2)}(z) = \frac{\frac{1}{z(z-1)}}{1} + -\overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{\left(\frac{m}{2}\right)^4}{z(z-1)}}{m}\right) \tag{12.5.2}$$

S-fraction

$$\mathrm{Re}\,(z) > \frac{1}{2},\, z \notin \left(\left(\left(\frac{1}{2}\right),1\right)\right)$$

$$\psi^{(2)}(z) = -\frac{1}{z} + \frac{\frac{1}{z}}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{\left(\frac{m+2}{4}\right)^2 - 2\left(\frac{m+2}{4}\right)+2}{2\left(\frac{m+2}{4}\right)-1}z^{-1}}{1}\right) \tag{12.5.3}$$

C-fraction

$$\mathrm{Re}\,(z) > 1$$

7.5 Trigamma function

$$\psi^{(1)}(z) = \frac{1}{z} + \frac{1}{2z^2} + \frac{\left(\frac{1}{12\pi}\right)}{z^2} + \frac{2\pi}{z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{m^2\left(m^2-1\right)}{4(4m^2-1)}}{1}\right) \tag{12.4.1}$$

S-fraction

$$|\arg z| < \frac{\pi}{2}$$

$$\psi^{(1)}(z) = \frac{z^{-1}}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\left(-\frac{\left(\frac{m}{2}\right)^2}{2m-2}\right)z^{-1}}{1}\right) \tag{12.4.2}$$

C-fraction

$$\mathrm{Re}\,(z) > \frac{1}{2}$$

$$\psi^{(1)}(z) = \frac{1}{-\frac{1}{2} + z} + \underset{m=2}{\overset{\infty}{\mathrm{K}}} \left(\frac{\frac{(m-1)^4}{4(2m-3)(2m-1)}}{-\frac{1}{2} + z} \right) \tag{12.4.3}$$

J-fraction

$$\mathrm{Re}\,(z) > \frac{1}{2}$$

8 Hypergeometric functions

9 q-Hypergeometric function

10 Probability functions