1 Elementary functions

1.1 The exponential function

$$e^z = \frac{1}{1} + \frac{-z}{1} + \operatorname*{K}_{m=3}^{\infty}\left(\frac{-\frac{1}{2(m-1)}z}{1}\right) \tag{No label}$$

$$e^z = \sum_{k=0}^{\infty}\frac{z^k}{k!} \tag{11.1.1}$$

$$e^z = 1 + \frac{2z}{2-z} + \frac{\frac{z^2}{6}}{1} + \operatorname*{K}_{m=3}^{\infty}\left(\frac{\frac{1}{4(2m-3)(2m-1)}z^2}{1}\right) \tag{11.1.2}$$

$$e^z = 1 + \frac{z}{1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{-\frac{1}{2(m-1)}z}{1}\right) \tag{11.1.3}$$

$$e^z = 1 + \frac{z}{1-z} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{(m-1)\,z}{m-z}\right) \tag{11.1.4}$$

1.2 The natural logarithm

$$\ln(1+z) = z + \frac{-\frac{z^2}{2}}{1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{\frac{\left(\frac{m}{2}+1\right)^2 z}{m(m+1)}}{1}\right) \tag{6.8.8}$$

$$\ln(1+z) = \sum_{k=0}^{\infty}\frac{(-1)^{k+2}}{k+1}z^{k+1} \tag{11.2.1}$$

$$\ln(1+z) = \frac{z}{1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{\frac{m}{4(m-1)}z}{1}\right) \tag{11.2.2}$$

$$\ln(1+z) = \frac{2z}{2+z} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{-(m-1)^2\,z^2}{(2m-1)\,(2+z)}\right) \tag{11.2.3}$$

$$\ln\left(\frac{1+z}{1-z}\right) = \frac{2z}{1} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{-\frac{(m-1)^2}{(2m-3)(2m-1)}z^2}{1}\right) \tag{11.2.4}$$

1.3 Trigonometric functions

$$\sin(z) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}z^{2k+1} \tag{11.3.1}$$

1.4 Trigonometric functions

$$\cos(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} z^{2k} \tag{11.3.2}$$

1.5 Trigonometric functions

$$\tan(z) = \sum_{k=0}^{\infty} \frac{4^{k+1}\left(4^{k+1}-1\right)\left|B_{2(k+1)}\right|}{(2(k+1))!} z^{2(k+1)-1} \tag{11.3.3}$$

$$\tan(z) = \frac{z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{-\frac{z^2}{(2m-1)(2m-3)}}{1}\right) \tag{11.3.7}$$

$$\tan\left(\frac{\pi z}{4}\right) = \frac{z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{(2m-3)^2 - z^2}{2}\right) \tag{11.3.8}$$

$$\tan(z) = \frac{z}{1} + \frac{-4\pi^{-2}z^2}{1} + \mathop{\mathrm{K}}_{m=3}^{\infty}\left(\frac{(m-2)^4 - 4\pi^{-2}(m-2)^2 z^2}{2m-3}\right) \tag{11.3.9}$$

1.6 Inverse trigonometric functions

$$\mathrm{Arcsin}(z) = z + \sum_{k=0}^{\infty} \frac{(2k-1)!!}{(2k)!!\,(2k+1)} z^{2k+1} \tag{11.4.1}$$

$$\mathrm{Arcsin}(z) = \frac{\frac{z}{\sqrt{1-z^2}}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{\frac{\frac{(m-1)^2}{(2m-3)(2m-1)}z^2}{1-z^2}}{1}\right) \tag{11.4.4}$$

$$\mathrm{Arcsin}(z) = \frac{z\sqrt{1-z^2}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{-\frac{m(m-1)}{(2m-1)(2m-3)}z^2}{1}\right) \tag{11.4.5}$$

1.7 Inverse trigonometric functions

$$\mathrm{Arccos}(z) = \frac{\pi}{2} - z + \sum_{k=0}^{\infty} -\frac{(2k-1)!!}{(2k)!!\,(2k+1)} z^{2k+1} \tag{11.4.2}$$

$$\mathrm{Arccos}(z) = \frac{\frac{\sqrt{1-z^2}}{z}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{\frac{\frac{(m-1)^2}{(2m-3)(2m-1)}\left(1-z^2\right)}{z^2}}{1}\right) \tag{11.4.6}$$

$$\mathrm{Arccos}(z) = \frac{z\sqrt{1-z^2}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{-\frac{m(m-1)}{(2m-1)(2m-3)}\left(1-z^2\right)}{1}\right) \tag{11.4.7}$$

1.8 Inverse trigonometric functions

$$\operatorname{Arctan}(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} z^{2k+1} \tag{11.4.3}$$

$$\operatorname{Arctan}(z) = \frac{z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(m-1)^2 z^2}{2m-1} \right) \tag{11.4.8}$$

$$\operatorname{Arctan}(z) = \frac{\frac{z}{1+z^2}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\frac{m(m-1)}{(2m-3)(2m-1)} \frac{z^2}{1+z^2}}{1} \right) \tag{11.4.9}$$

1.9 Hyperbolic functions

$$\sinh(z) = \sum_{k=0}^{\infty} \frac{1}{(2k+1)!} z^{2k+1} \tag{11.5.1}$$

1.10 Hyperbolic functions

$$\cosh(z) = \sum_{k=0}^{\infty} \frac{1}{(2k)!} z^{2k} \tag{11.5.2}$$

1.11 Hyperbolic functions

$$\tanh(z) = \sum_{k=0}^{\infty} \left(4^{k+1} \left(4^{k+1} - 1 \right) \frac{B_{2(k+1)}}{(2(k+1))!} \right) z^{2(k+1)-1} \tag{11.5.3}$$

$$\tanh(z) = \frac{z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{1}{(2m-3)(2m-1)} z^2}{1} \right) \tag{11.5.5}$$

1.12 Hyperbolic functions

$$\coth(z) = \sum_{k=0}^{\infty} \frac{4^k B_{2k}}{(2k)!} z^{2k-1} \tag{11.5.4}$$

$$\coth(z) = \frac{1}{z} + \frac{4\pi^{-2} z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(m-1)^2 \left((m-1)^2 + 4\pi^{-2} z^2 \right)}{2m-1} \right) \tag{11.5.6}$$

1.13 Inverse hyperbolic functions

$$\operatorname{Arcsinh}(z) = z + \sum_{k=0}^{\infty} \frac{(-1)^k (2k-1)!!}{(2k)!!\,(2k+1)} z^{2k+1} \tag{11.6.1}$$

$$\operatorname{Arcsinh}(z) = \frac{z\sqrt{1+z^2}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{m(m-1)}{(2m-3)(2m-1)} z^2}{1} \right) \tag{11.6.4}$$

$$\operatorname{Arcsinh}(z) = \frac{\frac{z}{\sqrt{1+z^2}}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-\frac{\frac{(m-1)^2}{(2m-3)(2m-1)} z^2}{1+z^2}}{1} \right) \tag{11.6.5}$$

1.14 Inverse hyperbolic functions

$$\operatorname{Arccosh}\left(\frac{1}{z}\right) = \ln\left(\frac{2}{z}\right) + \sum_{k=0}^{\infty} -\frac{(2k-1)!!}{(2k)!!\,(2k)} z^{2k} \tag{11.6.2}$$

$$\operatorname{Arccosh}(z) = \frac{z\sqrt{z^2-1}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{m(m-1)}{(2m-3)(2m-1)} \left(z^2-1\right)}{1} \right) \tag{11.6.6}$$

$$\operatorname{Arccosh}(z) = \frac{\frac{\sqrt{z^2-1}}{z}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-\frac{\frac{(m-1)^2}{(2m-3)(2m-1)} \left(z^2-1\right)}{z^2}}{1} \right) \tag{11.6.7}$$

1.15 Inverse hyperbolic functions

$$\operatorname{Arctanh}(z) = \sum_{k=0}^{\infty} \frac{1}{2k+1} z^{2k+1} \tag{11.6.3}$$

$$\operatorname{Arctanh}(z) = \frac{\frac{z}{1-z^2}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{\frac{m(m-1)}{(2m-3)(2m-1)} z^2}{1-z^2}}{1} \right) \tag{11.6.8}$$

$$\operatorname{Arctanh}(z) = \frac{z}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-\frac{(m-1)^2 z^2}{4(m-1)^2-1}}{1} \right) \tag{11.6.9}$$

1.16 The power function

$$(1+z)^n = 1 + \frac{nz}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{\frac{\frac{m}{2}-nz}{2(m-1)}}{1}\right) \tag{11.7.1}$$

$$(1+z)^n = \frac{1}{1} + \frac{-nz}{1} + \mathop{\mathrm{K}}_{m=3}^{\infty}\left(\frac{\frac{\frac{m-1}{2}+nz}{2(m-2)}}{1}\right) \tag{11.7.2}$$

$$(1+z)^n = \frac{1}{1} + \frac{-nz}{1+z} + \frac{\frac{(n-1)z}{2}}{1} + \mathop{\mathrm{K}}_{m=4}^{\infty}\left(\frac{\frac{\left(-n-\frac{m-2}{2}\right)z}{2(m-1)(1+z)}}{1}\right) \tag{11.7.3}$$

$$\left(\frac{z+1}{z-1}\right)^n = 1 + \frac{\frac{2n}{z}}{1-\frac{n}{z}} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{\frac{n^2-(m-1)^2}{(2(m-1)-1)(2(m-1)+1)z^2}}{1}\right) \tag{11.7.4}$$

2 Gamma function and related functions

2.1 Binet function

$$Binet\,(z) = \sum_{k=0}^{\infty}\left(\frac{B_{2(k+1)}}{(2k+1)\,(2k+2)}z^{-2k-1}\right) \tag{12.2.6}$$

2.2 Polygamma functions

$$\psi(z) = -\gamma + \sum_{k=0}^{\infty}\frac{1}{k} - \frac{1}{z+k-1} \tag{12.3.2a}$$

$$\psi^{(n)}(z) = (-1)^{n+1}\,n!\sum_{k=0}^{\infty}\frac{1}{(z+k)^{n+1}} \tag{12.3.2b}$$

$$\psi(z) = \ln(z) - \frac{1}{2z} + \sum_{k=0}^{\infty}\left(-\frac{B_{2k}}{2k}z^{-2k}\right) \tag{12.3.7}$$

$$\psi^{(n)}(z) = (-1)^{n-1}\sum_{k=0}^{\infty}\left(\frac{B_{2k}\,(2k+n-1)!}{(2k)!}z^{-2k-n}\right) \tag{12.3.8}$$

2.3 Trigamma function

$$\psi^{(1)}(z) = \frac{1}{z} + \frac{1}{2z^2} + \frac{\frac{1}{12\pi}}{z^2} + \frac{2\pi}{z}\mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{\frac{m^2\left(m^2-1\right)}{4(4m^2-1)}}{1}\right) \tag{12.4.1}$$

$$\psi^{(1)}(z) = \frac{z^{-1}}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{-\frac{\left(\frac{m}{2}\right)^2}{2m-2} z^{-1}}{1} \right) \tag{12.4.2}$$

$$\psi^{(1)}(z) = \frac{1}{-\frac{1}{2}+z} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{(m-1)^4}{4(2m-3)(2m-1)}}{-\frac{1}{2}+z} \right) \tag{12.4.3}$$

2.4 Tetragamma function

$$\psi^{(2)}(z) = -\frac{1}{z^2} - \frac{1}{z^3} + \frac{\frac{1}{8\pi^2}}{z^2} + -\left(\frac{2\pi}{z}\right)^2 \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{\left(\frac{m}{2}\right)^2 \frac{m}{2}+1}{2(m+1)}}{1} \right) \tag{12.5.1}$$

$$\psi^{(2)}(z) = \frac{\frac{1}{z(z-1)}}{1} + -\overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{\left(\frac{m}{2}\right)^4}{z(z-1)}}{m} \right) \tag{12.5.2}$$

$$\psi^{(2)}(z) = -\frac{1}{z} + \frac{\frac{1}{z}}{1} + \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{\left(\frac{m+2}{4}\right)^2 - 2\frac{m+2}{4}+2}{2\frac{m+2}{4}-1} z^{-1}}{1} \right) \tag{12.5.3}$$

2.5 Incomplete gamma functions

$$\gamma(a,z) = z^a \sum_{k=0}^{\infty} \frac{(-z)^k}{(a+k)\,k!} \tag{12.6.7}$$

$$\gamma(a,z) = \frac{z^a e^{-z}}{a} \sum_{k=0}^{\infty} \frac{z^k}{(1+a)_k} \tag{12.6.8}$$

$$\Gamma(a,z) = z^a e^{-z} \sum_{k=0}^{\infty} \left((-1)^k (1-a)_k z^{-k-1} \right) \tag{12.6.10}$$

$$\Gamma(a,z) = \frac{1}{z} + z^a e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{m}{2} - a}{1} \right) \tag{12.6.15}$$

$$\Gamma(a,z) = \frac{\frac{1}{z}}{1} + z^a e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{\frac{m}{2} - a z^{-1}}{1} \right) \tag{12.6.17}$$

$$\gamma(a,z) = \frac{\frac{z}{a}}{1} + z^{a-1} e^{-z} \overset{\infty}{\underset{m=2}{\mathrm{K}}} \left(\frac{-\frac{a+\frac{m}{2}-1}{(a+m-2)(a+m-1)} z}{1} \right) \tag{12.6.23}$$

$$\Gamma(a,z) = \Gamma(a) + \frac{\frac{z}{a}}{1} + -\frac{z^a e^{-z}}{z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\frac{a+\frac{m}{2}-1}{(a+m-2)(a+m-1)} z}{1} \right) \tag{12.6.24}$$

$$\gamma(a,z) = \frac{1}{a-z} + z^a e^{-z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(m-1)\, z}{a+(m-1)-z} \right) \tag{12.6.30}$$

$$\Gamma(a,z) = \Gamma(a) + \frac{-1}{a-z} + z^a e^{-z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(m-1)\, z}{a+(m-1)-z} \right) \tag{12.6.31}$$

$$\Gamma(a,z) = \frac{1}{1-a+z} + z^a e^{-z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(1-m)\,(m-1-a)}{((2m)-1)-a+z} \right) \tag{12.6.34}$$

$$\Gamma(a,z) = z^{a-1} e^{-z} + \frac{a-1}{2+z-a} + z^{a-1} e^{-z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(1-m)\,(m-a)}{2m+z-a} \right) \tag{12.6.35}$$

3 Error function and related integrals

3.1 Error function and Dawson's integral

$$\mathrm{erf}\,(z) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k\, z^{2k+1}}{(2k+1)\, k!} \tag{13.1.7}$$

$$\mathrm{erf}\,(z) = \frac{2}{\sqrt{\pi}} e^{-z^2} \sum_{k=0}^{\infty} \frac{z^{2k+1}}{\left(\frac{3}{2}\right)_k} \tag{13.1.8}$$

$$F(z) = e^{-z^2} \sum_{k=0}^{\infty} \frac{z^{2k+1}}{(2k+1)\, k!} \tag{13.1.9}$$

$$F(z) = \sum_{k=0}^{\infty} \frac{(-1)^k\, z^{2k+1}}{\left(\frac{3}{2}\right)_k} \tag{13.1.10}$$

$$\mathrm{erf}\,(z) = \frac{2z^2}{1} + \frac{1}{\sqrt{\pi} z e^{z^2}} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{-2(m-1)}{(2m-3)(2m-1)} z^2}{1} \right) \tag{13.1.11a}$$

$$F(z) = \frac{-2z^2}{1} + -\frac{1}{2z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{2(m-1)}{(2m-3)(2m-1)} z^2}{1} \right) \tag{13.1.11b}$$

$$\mathrm{erf}\,(z) = \frac{2z^2}{1-2z^2} + \frac{1}{\sqrt{\pi} z e^{z^2}} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{4(m-1)}{(2m-3)(2m-1)} z^2}{1-\frac{2}{2m-1} z^2} \right) \tag{13.1.13a}$$

$$F(z) = \frac{-2z^2}{1+2z^2} + -\frac{1}{2z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\frac{4(m-1)}{(2m-3)(2m-1)} z^2}{1+\frac{2}{2m-1} z^2} \right) \tag{13.1.13b}$$

3.2 Complementary and complex error function

$$\operatorname{erfc}(z) = e^{-z^2} \sum_{k=0}^{\infty} \frac{(-z)^k}{\Gamma\left(\frac{k}{2}+1\right)} \tag{13.2.9}$$

$$w(z) = \sum_{k=0}^{\infty} \frac{(kz)^k}{\Gamma\left(\frac{k}{2}+1\right)} \tag{13.2.10}$$

$$\operatorname{erfc}(z) = \frac{1}{\sqrt{\pi} z e^{z^2}} \sum_{k=0}^{\infty} \left((-1)^k \left(\frac{1}{2}\right)_k z^{-2k} \right) \tag{13.2.11}$$

$$w(z) = \frac{k}{\pi z} \sum_{k=0}^{\infty} \left(\left(\frac{1}{2}\right)_k z^{-2k} \right) \tag{13.2.12}$$

$$\operatorname{erfc}(z) = \frac{1}{z^2} + \frac{z}{\sqrt{\pi}} e^{-z^2} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{m-1}{2}}{1} \right) \tag{13.2.20a}$$

$$w(z) = \frac{1}{-z^2} + -\frac{kz}{\sqrt{\pi}} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{m-1}{2}}{1} \right) \tag{13.2.20b}$$

$$\operatorname{erfc}(z) = \frac{2z}{2z^2+1} + \frac{e^{-z^2}}{\sqrt{\pi}} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-(2m-3)(2m-2)}{4m-3+2z^2} \right) \tag{13.2.23a}$$

$$w(z) = \frac{-\frac{kz}{\sqrt{\pi}}}{\frac{1}{2}-z^2} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{--\frac{3}{2}+m(m-1)}{2m-\frac{3}{2}-z} \right) \tag{13.2.23b}$$

3.3 Repeated integrals

$$I^n \operatorname{erfc}(z) = \frac{\frac{2}{\sqrt{\pi}} e^{-z^2}}{(2z)^{n+1}} \sum_{k=0}^{\infty} \left(\frac{(-1)^k (2k+n)!}{n!k!(2z)^{2k}} \right) \tag{13.3.2}$$

$$\frac{I^n \operatorname{erfc}(z)}{I^{n-1} \operatorname{erfc}(z)} = \frac{\frac{1}{2}}{z} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{n+m-1}{2}}{z} \right) \tag{13.3.5}$$

3.4 Fresnel integrals

$$C(z) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{2}\right)^{2k}}{(2k)!(4k+1)} z^{4k+1} \tag{13.4.6a}$$

$$S(z) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{\pi}{2}\right)^{2k+1}}{(2k+1)!(4k+3)} z^{4k+3} \tag{13.4.6b}$$

$$C(z) + k\,S(z) = \frac{z^2}{1} + \frac{e^{\frac{k\pi z^2}{2}}}{z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{k\pi(m-1)}{(2m-3)(2m-1)} z^2}{1} \right) \tag{13.4.9}$$

$$C(z) + k\,S(z) = \frac{z^2}{1 + k\pi z^2} + \frac{e^{\frac{k\pi z^2}{2}}}{z} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\frac{2k\pi(m-1)}{(2m-3)(2m-1)} z^2}{1 + \frac{k\pi}{2m-1} z^2} \right) \tag{13.4.10}$$

4 Hypergeometric functions

4.1 Hypergeometric functions

$$\frac{{}_2F_1\left({a,b \atop c}; z\right)}{{}_2F_1\left({a,b+1 \atop c+1}; z\right)} = \frac{c + (b-a+1)\,z}{c} + \frac{1}{c} \mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{-\,(c-a+m)\,(b+m)\,z}{c+m+(b-a+m+1)\,z} \right) \tag{15.3.8}$$

$${}_2F_1\left({a, 1 \atop c+1}; z\right) = \frac{c}{c + (1-a)\,z} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\,(m-1)\,z\,(c-a+m-1)}{c+m-1+(m-a)\,z} \right) \tag{15.3.9a}$$

$${}_2F_1\left({1-c, 1 \atop 2-a}; \frac{1}{z}\right) = \frac{c}{c + (1-a)\,z} + \frac{(1-a)\,z}{c} \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\,(m-1)\,z\,(c-a+m-1)}{c+m-1+(m-a)\,z} \right) \tag{15.3.9b}$$

$${}_2F_1\left({\frac{1}{2}, 1 \atop \frac{3}{2}}; z\right) = \frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{2} z} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\,(m-1)\,z\,(m-1)}{\frac{1}{2}+m-1+\left(m-\frac{1}{2}\right) z} \right) \tag{15.3.12}$$

$$\frac{{}_2F_1\left({a,b \atop c}; z\right)}{{}_2F_1\left({a+1,b+1 \atop c+1}; z\right)} = 1 - \frac{a+b+1}{c} z + \mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{\frac{(a+m)(b+m)}{(c+m-1)(c+m)} \left(z - z^2\right)}{1 - \frac{a+b+2m+1}{c+m} z} \right) \tag{15.3.13}$$

$${}_2F_1\left({a+1, 1 \atop c+1}; z\right) = \frac{c}{c - (a+1)\,z} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(a+m-1)\,(m-1)\,\left(z - z^2\right)}{c+m-1-(a+2\,(m-1)+1)\,z} \right) \tag{15.3.14}$$

$${}_2F_1\left({\frac{1}{2}, 1 \atop \frac{3}{2}}; z\right) = \frac{1}{1-z} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{m-1}{m-1+\frac{1}{2}} \left(z - z^2\right)}{1 - \frac{2(m-1)+\frac{1}{2}}{m-1+\frac{1}{2}} z} \right) \tag{15.3.17}$$

$$\frac{{}_3F_2\left({a,b,c \atop d,e}; 1\right)}{{}_3F_2\left({a+1,b,c \atop d,e}; 1\right)} = 1 + \frac{-\frac{bc}{d}}{e-a-1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{\left(a+\frac{m}{2}\right)\left(d-b+\frac{m}{2}-1\right)\left(d-c+\frac{m}{2}-1\right)}{(d+m-2)(d+m-1)}}{1} \right) \tag{15.6.4}$$

$$\frac{{}_3F_2\left({a,b,c \atop d,e}; 1\right)}{{}_3F_2\left({a,b,c \atop d+1,e}; 1\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{-\frac{\frac{m}{2}+d-a\,\frac{m}{2}+d-b\,\frac{m}{2}+d-c}{(d+m-1)(d+m)}}{1} \right) \tag{15.6.5}$$

$$\frac{{}_3F_2\left({a,b,c \atop d,e}; 1\right)}{{}_3F_2\left({a+1,b,c \atop d+1,e}; 1\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{\frac{\left(a+\frac{m}{2}\right)\left(d-b+\frac{m}{2}\right)\left(d-c+\frac{m}{2}\right)}{(d+m-1)(d+m)}}{1} \right) \tag{15.6.6}$$

$$\frac{{}_3F_2\left({a,b,c \atop d,e};1\right)}{{}_3F_2\left({a,b+1,c+1 \atop d+1,e+1};1\right)} = \frac{e-(a-1)-1}{e} + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{-\frac{\left((d-1)-(a-1)+\frac{m}{2}\right)\left(b+\frac{m}{2}\right)\left(c+\frac{m}{2}\right)}{\left(e+\frac{m}{2}\right)((d-1)+m)((d-1)+m+1)}}{\frac{e-(a-1)-1}{e+\frac{m}{2}}}\right) \tag{15.6.7}$$

5 Confluent hypergeometric functions

5.1 Kummer functions

$${}_1F_1\left({a \atop b};z\right) = \sum_{k=0}^{\infty}\frac{\frac{(a)_k}{(b)_k}z^k}{k!} \tag{16.1.2}$$

$${}_2F_0\left({a,b \atop };z\right) = \sum_{k=0}^{\infty}\frac{(a)_k(b)_k z^k}{k!} \tag{16.1.12}$$

$$\frac{{}_1F_1\left({a \atop b};z\right)}{{}_1F_1\left({a+1 \atop b+1};z\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{\frac{z\left(a+\frac{m}{2}\right)}{(b+m-1)(b+m)}}{1}\right) \tag{16.1.13}$$

$$z\,{}_1F_1\left({1 \atop b+1};z\right) = \frac{z}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{-\frac{z\left(b+\frac{m}{2}-1\right)}{(b+m-2)(b+m-1)}}{1}\right) \tag{16.1.14}$$

$$\frac{{}_1F_1\left({a \atop b};z\right)}{{}_1F_1\left({a+1 \atop b+1};z\right)} = \frac{b-z}{b} + \frac{1}{b}\mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{(a+m)\,z}{b+m-z}\right) \tag{16.1.16}$$

$${}_1F_1\left({1 \atop b+1};z\right) = \frac{b}{b-z} + \mathop{\mathrm{K}}_{m=2}^{\infty}\left(\frac{(m-1)\,z}{b+m-1-z}\right) \tag{16.1.17}$$

$$\frac{U(a,b,z)}{U(a+1,b,z)} = 2a-b+2+z - \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{(a+m)\,(b-a-m-1)}{b-2a-2m-2-z}\right) \tag{16.1.20}$$

5.2 Confluent hypergeometric series ${}_2F_0$

$$\frac{{}_2F_0\left({a,b \atop };z\right)}{{}_2F_0\left({a,b+1 \atop };z\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty}\left(\frac{-\left(b+\frac{m}{2}\right)z}{1}\right) \tag{16.2.4}$$

5.3 Confluent hypergeometric limit function

$$ {}_0F_1\left({}_{b};z\right)=\sum_{k=0}^{\infty}\frac{z^k}{(b)_k k!} \tag{16.3.1}$$

$$ \frac{{}_0F_1\left({}_{b};z\right)}{{}_0F_1\left({}_{b+1};z\right)}=1+\operatorname*{K}_{m=1}^{\infty}\left(\frac{\frac{1}{(b+m-1)(b+m)}z}{1}\right) \tag{16.3.4}$$

$$ \frac{{}_0F_1\left({}_{b};z\right)}{{}_0F_1\left({}_{b+1};z\right)}=1+\frac{\sqrt{z}}{b}+\frac{1}{2b}\operatorname*{K}_{m=1}^{\infty}\left(\frac{-2\left(2b+2m-1\right)\sqrt{z}}{2b+m+4\sqrt{z}}\right) \tag{16.3.6}$$

5.4 Whittaker functions

$$ W_{\kappa,\mu}(z)=e^{-z^2}z^{\kappa}\sum_{k=0}^{\infty}\left(\frac{\left(-\kappa-\mu+\frac{1}{2}\right)_k\left(-\kappa+\mu+\frac{1}{2}\right)_k(-z)^{-k}}{k!}\right) \tag{16.4.7}$$

$$ \Psi_{n,\beta}(z)=\sum_{k=0}^{\infty}\left(\frac{\left(n+\frac{1}{2}\right)_k\left(\beta+\frac{1}{2},k\right)z^{-k-1}}{k!}\right) \tag{16.4.12}$$

5.5 Parabolic cylinder functions

$$ \frac{U(a,x)}{U(a-1,x)}=\frac{1}{x}+\operatorname*{K}_{m=2}^{\infty}\left(\frac{a+(m-1)-\frac{1}{2}}{x}\right) \tag{16.5.7}$$

6 Bessel functions

6.1 Bessel functions

$$ J_{\nu}(z)=\left(\frac{z}{2}\right)^{\nu}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!\,\Gamma(\nu+k+1)}\left(\frac{z}{2}\right)^{2k} \tag{17.1.2a}$$

$$ \mathrm{j}_n(z)=\sqrt{\frac{\pi}{2z}}\left(\frac{z}{2}\right)^{n+\frac{1}{2}}\sum_{k=0}^{\infty}\frac{(-1)^k}{k!\,\Gamma\left(\left(n+\frac{1}{2}\right)+k+1\right)}\left(\frac{z}{2}\right)^{2k} \tag{17.1.12a}$$

$$ J_{\nu}(z)=\frac{e^{-kz}\left(\frac{z}{2}\right)^{\nu}}{\Gamma(\nu+1)}\sum_{k=0}^{\infty}\frac{\left(\nu+\frac{1}{2}\right)_k(2k)^k z^k}{(2\nu+1)_k k!} \tag{17.1.22}$$

$$ \mathrm{j}_n(z)=\frac{\sqrt{\pi}}{(2n+1)\,\Gamma\left(n+\frac{1}{2}\right)}\left(\frac{z}{2}\right)^{n}\sum_{k=0}^{\infty}\frac{\frac{1}{\left(n+\frac{3}{2}\right)_k}\left(-\frac{z^2}{4}\right)^k}{k!} \tag{17.1.25}$$

$$ \mathrm{j}_n(z)=\frac{\sqrt{\pi}e^{-kz}}{(2n+1)\,\Gamma\left(n+\frac{1}{2}\right)}\left(\frac{z}{2}\right)^{n}\sum_{k=0}^{\infty}\frac{\frac{(n+1)_k}{(2n+2)_k}(2kz)^k}{k!} \tag{17.1.26}$$

$$J_\nu(z) = \sqrt{\frac{2}{\pi z}} \sum_{k=0}^{\infty} \left(\frac{(-1)^k (\nu, 2k)}{(2z)^{2k}} \cos\left(z - \frac{\nu}{2} + \frac{1}{4}\pi\right) - \frac{(-1)^k (\nu, 2k+1)}{(2z)^{2k+1}} \sin\left(z - \frac{\nu}{2} + \frac{1}{4}\pi\right) \right) \tag{17.1.28}$$

$$Y_\nu(z) = \sqrt{\frac{2}{\pi z}} \sum_{k=0}^{\infty} \left(\frac{(-1)^k (\nu, 2k)}{(2z)^{2k}} \sin\left(z - \frac{\nu}{2} + \frac{1}{4}\pi\right) + \frac{(-1)^k (\nu, 2k+1)}{(2z)^{2k+1}} \cos\left(z - \frac{\nu}{2} + \frac{1}{4}\pi\right) \right) \tag{17.1.29}$$

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = \frac{\frac{z}{2\nu+2}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{(kz)^2}{4(\nu+m-1)(\nu+m)}}{1} \right) \tag{17.1.38}$$

$$\frac{\mathrm{j}_{n+1}(z)}{\mathrm{j}_n(z)} = \frac{\frac{z}{2n+3}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{(kz)^2}{4\left(n+\frac{1}{2}+m-1\right)\left(n+\frac{1}{2}+m\right)}}{1} \right) \tag{17.1.39}$$

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = -\mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{-1}{\frac{2(\nu+m)}{z}} \right) \tag{17.1.40}$$

$$\frac{H^{(1)}_{\nu+1}(z)}{H^{(1)}_\nu(z)} = \frac{-1}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{m-3-2\nu}{-2kz}}{1} \right) \tag{17.1.44}$$

$$\frac{J_{\nu+1}(z)}{J_\nu(z)} = \frac{z}{2\nu+2-kz} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{(2\nu+2m-1)\,kz}{2\nu+m+1+(-2k)\,z} \right) \tag{17.1.48}$$

$$\frac{\mathrm{j}_{n+1}(z)}{\mathrm{j}_n(z)} = \frac{z}{2n+3-kz} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{2\,(n+m)\,kz}{2n+m+2+(-2k)\,z} \right) \tag{17.1.49}$$

$$\frac{H^{(1)}_{\nu+1}(z)}{H^{(1)}_\nu(z)} = \frac{2\nu+1-2kz}{2z} - \frac{1}{z} \mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{\nu^2 - \frac{(2m-1)^2}{4}}{2\,(kz-m)} \right) \tag{17.1.51}$$

6.2 Modified Bessel functions

$$I_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{2k}}{k!\,\Gamma(\nu+k+1)} \tag{17.2.20}$$

$$I_\nu(z) = \frac{e^{-z}\left(\frac{z}{2}\right)^\nu}{\Gamma(\nu+1)} \sum_{k=0}^{\infty} \frac{\left(\nu+\frac{1}{2}\right)_k 2^k z^k}{(2\nu+1)_k k!} \tag{17.2.21}$$

$$\mathrm{i}^{(1)}_n(z) = \frac{\sqrt{\pi}}{(2n+1)\,\Gamma\left(n+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^n \sum_{k=0}^{\infty} \frac{\left(\frac{z^2}{4}\right)^k}{k!\left(n+\frac{3}{2}\right)_k} \tag{17.2.22}$$

$$\mathsf{i}_n^{(1)}(z) = \frac{\sqrt{\pi}e^{-kz}}{(2n+1)\,\Gamma\left(n+\frac{1}{2}\right)} \left(\frac{z}{2}\right)^n \sum_{k=0}^{\infty} \frac{(n+1)_k\,(2z)^k}{k!(2n+2)_k} \tag{17.2.23}$$

$$I_\nu(z) = \sum_{k=0}^{\infty} \left(\frac{\left((-1)^k\, e^z + e^{-z+\frac{(2\nu+1)k\pi}{2}}\right)(\nu,k)}{\sqrt{2\pi z}\,(2z)^k} \right) \tag{17.2.24}$$

$$I_\nu(z) = \sum_{k=0}^{\infty} \left(\frac{\left((-1)^k\, e^z + e^{-z-\frac{(2\nu+1)k\pi}{2}}\right)(\nu,k)}{\sqrt{2\pi z}\,(2z)^k} \right) \tag{17.2.25}$$

$$K_\nu(z) = \sqrt{\frac{\pi}{2z}} e^{-z} \sum_{k=0}^{\infty} \left(\frac{(\nu,k)}{(-2z)^k} \right) \tag{17.2.27}$$

$$\frac{I_{\nu+1}(z)}{I_\nu(z)} = \frac{\frac{z}{2(\nu+1)}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{1}{4(\nu+m-1)(\nu+m)} z^2}{1} \right) \tag{17.2.32}$$

$$\frac{\mathsf{i}_{n+1}^{(1)}(z)}{\mathsf{i}_n^{(1)}(z)} = \frac{\frac{z}{2n+3}}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{1}{4\left(\left(n+\frac{1}{2}\right)+m-1\right)\left(\left(n+\frac{1}{2}\right)+m\right)} z^2}{1} \right) \tag{17.2.33}$$

$$\frac{\nu}{z} - \frac{K_{\nu+1}(z)}{K_\nu(z)} = \frac{\nu}{z} + \frac{1}{1} + \frac{\frac{-2\nu-1}{2z}}{1} + -\mathop{\mathrm{K}}_{m=3}^{\infty} \left(\frac{\frac{\frac{m}{2}+\nu}{2z}}{1} \right) \tag{17.2.34}$$

$$\frac{I_{\nu+1}(z)}{I_\nu(z)} = \frac{z}{2\nu+2+z} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-\left(2\nu+2m-1\right)z}{2\nu+m+1+2z} \right) \tag{17.2.38}$$

$$\frac{\mathsf{i}_{n+1}^{(1)}(z)}{\mathsf{i}_n^{(1)}(z)} = \frac{z}{2n+3+z} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{-2\left(n+m\right)z}{2n+m+2+2z} \right) \tag{17.2.39}$$

$$\frac{\nu}{z} - \frac{K_{\nu+1}(z)}{K_\nu(z)} = \frac{\nu}{z} - \frac{2\nu+1+2z}{2z} - \frac{1}{z} \mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{\nu^2 - \frac{(2m-1)^2}{4}}{2\left(z+m\right)} \right) \tag{17.2.40}$$

7 q-Hypergeometric function

7.1 q-Hypergeometric function

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({a,bq \atop cq};q,z\right)} = 1 + \mathop{\mathrm{K}}_{m=1}^{\infty} \left(\frac{\frac{\left(1-bq^{\frac{m}{2}}\right)\left(cq^{\frac{m}{2}}-a\right)q^{\frac{m}{2}-1}}{(1-cq^{m-1})(1-cq^m)} z}{1} \right) \tag{19.2.1}$$

$${}_2\phi_1\left({a,q \atop cq};q,z\right) = \frac{1}{1} + \mathop{\mathrm{K}}_{m=2}^{\infty} \left(\frac{\frac{\left(1-aq^{\frac{m-2}{2}}\right)\left(cq^{\frac{m-2}{2}}-1\right)q^{\frac{m-2}{2}}}{(1-cq^{m-2})(1-cq^{m-1})} z}{1} \right) \tag{19.2.2}$$

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({a,bq \atop cq};q,z\right)} = \frac{q\left(1-c\right)+\left(a-bq\right)z}{q\left(1-c\right)} + \frac{1}{q\left(1-c\right)} \operatorname*{K}_{m=1}^{\infty}\left(\frac{q\left(1-bq^{m}\right)\left(cq^{m}-a\right)z}{q\left(1-cq^{m}\right)+\left(a-bq^{m+1}\right)z}\right) \tag{19.2.5a}$$

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({a,bq \atop cq};q,z\right)} = 1 + \frac{q\left(1-c\right)}{\left(a-bq\right)z} + \frac{1}{\left(a-bq\right)z} \operatorname*{K}_{m=1}^{\infty}\left(\frac{q\left(1-bq^{m}\right)\left(cq^{m}-a\right)z}{q\left(1-cq^{m}\right)+\left(a-bq^{m+1}\right)z}\right) \tag{19.2.5b}$$

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({aq,bq \atop cq};q,z\right)} = \frac{1-c-\left(a+b-ab-abq\right)z}{1-c} + \frac{1}{1-c} \operatorname*{K}_{m=1}^{\infty}\left(\frac{\left(1-aq^{m}\right)\left(1-bq^{m}\right)\left(cz-abq^{m}z^{2}\right)q^{m-1}}{\left(1-cq^{m}\right)-\left(a+b-abq^{m}-abq^{m+1}\right)q^{m}z}\right) \tag{19.2.7}$$

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({aq,b \atop c};q,z\right)} = 1 + \operatorname*{K}_{m=1}^{\infty}\left(\frac{a}{-1}\right) \tag{19.2.10}$$

$$\frac{{}_2\phi_1\left({a,b \atop c};q,qz\right)}{{}_2\phi_1\left({a,b \atop c};q,z\right)} = \frac{\left(1-z\right)}{\left(1+cq^{-1}-\left(a+b\right)z\right)} + \operatorname*{K}_{m=2}^{\infty}\left(\frac{-\left(cq^{-1}-abq^{m-2}z\right)\left(1-q^{m-1}z\right)}{1+cq^{-1}-\left(a+b\right)q^{m-1}z}\right) \tag{19.2.11}$$

$$\frac{{}_2\phi_1\left({a,b \atop c};q,z\right)}{{}_2\phi_1\left({a,b \atop c};q,qz\right)} = 1 + \operatorname*{K}_{m=1}^{\infty}\left(\frac{\left(1-aq^{\frac{m-1}{2}}\right)\left(1-bq^{\frac{m-1}{2}}\right)z}{1-z}\right) \tag{19.2.12}$$

$$\frac{{}_2\phi_1\left({aq,b \atop c};q,z\right)}{{}_2\phi_1\left({a,b \atop c};q,qz\right)} = 1 + \operatorname*{K}_{m=1}^{\infty}\left(\frac{\left(1-bq^{\frac{m-1}{2}}\right)z}{\left(1-a\right)\left(1-z\right)}\right) \tag{19.2.13}$$

$$\frac{{}_3\phi_2\left({a,b,c \atop e,f};q,\frac{ef}{abc}\right)}{{}_3\phi_2\left({a,b,c \atop eq,f};q,\frac{efq}{abc}\right)} = 1 + \operatorname*{K}_{m=1}^{\infty}\left(\frac{-\frac{\left(1-\frac{eq^{\frac{m}{2}-1+1}}{a}\right)\left(1-\frac{eq^{\frac{m}{2}-1+1}}{b}\right)\left(1-\frac{eq^{\frac{m}{2}-1+1}}{c}\right)fq^{\frac{m}{2}-1}}{\left(1-eq^{2\frac{m}{2}-1+1}\right)\left(1-eq^{2\frac{m}{2}-1+2}\right)\left(1-fq^{\frac{m}{2}-1}\right)}}{1}\right) \tag{19.3.1}$$

$$\frac{{}_3\phi_2\left({aq,b,c \atop e,f};q,\frac{ef}{abcq}\right)}{{}_3\phi_2\left({a,b,c \atop e,f};q,\frac{ef}{abc}\right)} = 1 + \operatorname*{K}_{m=1}^{\infty}\left(\frac{a\left(1-\frac{eq^{\frac{m}{2}-1}}{a}\right)\left(1-\frac{fq^{\frac{m}{2}-1}}{a}\right)}{1}\right) \tag{19.3.2}$$

8 Exponential integrals and related functions

8.1 Exponential integrals

$$E_1(z) = -1 \sum_{k=0}^{\infty} \frac{(-1)^k z^k}{k k!} \tag{14.1.10}$$

$$E_n(z) = \frac{(-z)^{n-1}}{(n-1)!} \left(-\gamma - \ln(z) + \sum_{k=1}^{n-1} \left(\frac{1}{k}\right)\right) - \sum_{k=0}^{n-2} \left(\frac{(-z)^k}{(k-n+1)\,k!}\right) + \sum_{k=0}^{\infty} -\frac{(-z)^{k+n-1}}{(k)\,(k+n-1)!} \tag{14.1.11}$$

$$E_\nu(z) = \Gamma(1-\nu) z^{\nu-1} - \frac{1}{1-\nu} + \sum_{k=0}^{\infty} \frac{(-1)^{k+1} z^k}{(k!)\,(k+1-\nu)} \tag{14.1.12}$$

$$E_\nu(z) = e^{-z} \sum_{k=0}^{\infty} \left(\frac{(-1)^k (\nu)_k}{z^{k+1}}\right) \tag{14.1.13}$$

$$E_n(z) = \frac{1}{z} + \frac{n}{1} + e^{-z} \operatorname*{K}_{m=3}^{\infty} \left(\frac{n + \frac{m-2}{2}}{1}\right) \tag{14.1.16}$$

$$E_\nu(z) = \frac{\frac{e^{-z}}{z}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{\frac{m}{2}+\nu-1}{z}}{1}\right) \tag{14.1.19}$$

$$E_\nu(z) = z^{\nu-1}\,\Gamma(1-\nu) + \frac{-e^{-z}}{1-\nu} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{\left(\nu - \frac{m}{2}\right) z}{m-\nu}\right) \tag{14.1.20}$$

$$E_\nu(z) = z^{\nu-1}\,\Gamma(1-\nu) + \frac{1}{1-\nu-z} + -e^{-z} \operatorname*{K}_{m=2}^{\infty} \left(\frac{(m-1)\,z}{m-\nu-z}\right) \tag{14.1.22}$$

$$E_\nu(z) = \frac{1}{\nu+z} + e^{-z} \operatorname*{K}_{m=2}^{\infty} \left(\frac{-(m-1)\,(\nu+m-2)}{\nu+(m-1)\,2+z}\right) \tag{14.1.23}$$

$$E_\nu(z) = \frac{e^{-z}}{z} + \frac{-e^{-z}\nu}{z\,(1+z)+\nu z} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-(m-1)\,(\nu+m-1)\,z^2}{z\,(m+z)+(\nu+m-1)\,z}\right) \tag{14.1.24}$$

8.2 Related functions

$$E_1(x) = \gamma + \ln(x) + \sum_{k=0}^{\infty} \frac{x^k}{k k!} \tag{14.2.14}$$

$$\mathrm{Ein}(z) = -1 \sum_{k=0}^{\infty} \frac{(-1)^{k+1} z^{k+1}}{(k+1)\,(k+1)!} \tag{14.2.16}$$

$$E_1(x) = \frac{e^x}{x} \sum_{k=0}^{\infty} \left(k! x^{-k} \right) \tag{14.2.19}$$

$$E_1(z) = 2 \sum_{k=0}^{\infty} \left(\frac{z^{2k+1}}{(2k+1)\,(2k+1)!} \right) + \frac{\frac{1}{z}}{1} + -e^{-z} \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{\left\lfloor \frac{m}{2} \right\rfloor}{z}}{1} \right) \tag{14.2.21}$$

$$\mathrm{Ein}(z) = \gamma + \ln(z) + \frac{\frac{1}{z}}{1} + e^{-z} \operatorname*{K}_{m=2}^{\infty} \left(\frac{\frac{\left\lfloor \frac{m}{2} \right\rfloor}{z}}{1} \right) \tag{14.2.23}$$

$$E_1(x) = \frac{\frac{e^x}{x}}{1} + \operatorname*{K}_{m=2}^{\infty} \left(\frac{-\frac{\left\lfloor \frac{m}{2} \right\rfloor}{x}}{1} \right) \tag{14.2.24}$$