Smart Mobile Platform
Decision Theory

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Outline

- Decision Theory Family
  - **Game Theory**
  - Stable Marriage
  - Decision Tree: ID3

- [Example] During the 8PM to 9PM, two TV broadcast service providers (e.g., MBC and KBS) compete for an audience of 100 viewers. The networks announce their schedule ahead of time and do not know of each other's decision until the show time. Based on that a certain number of people will tune to MBC while the rest will watch KBS. The market research revealed the following expected number of viewers of MBC.

MBC (KBS) Payoff Matrix

		KBS		
		Movie	Opera	Comedy
MBC	Movie	35 (65)	15 (85)	60 (40)
	Opera	45 (55)	58 (42)	50 (50)
	Comedy	38 (62)	14 (86)	70 (30)

- [Example] During the 8PM to 9PM, two TV broadcast service providers (e.g., MBC and KBS) compete for an audience of 100 viewers. The networks announce their schedule ahead of time and do not know of each other's decision until the show time. Based on that a certain number of people will tune to MBC while the rest will watch KBS. The market research revealed the following expected number of viewers of MBC.

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- [Example] Player 1 draws a card from a card deck (and hides it from the other player). Then he decides to either pass in which case he discards the card and pays $1 to Player 2, or he will bet in which case it is 2nd player's turn. Player 2 can either fold in which case she pays $1 to Player 1, or she will call and the card is revealed. If the revealed card is HIGH (10, Jack, Queen, King, Ace), then Player 2 pays $2 to Player 1. Otherwise, the card is LOW (2 through 9) and Player 1 pays $2 to Player 2. Now, let's analyze the possible strategies for Player 1. Given the card, the card can be either HIGH or LOW. Based on that the player can either pass or bet. So there are 4 possible strategies: pass on both HIGH and LOW (PP), pass on HIGH and bet on LOW (PB), bet on HIGH and pass on LOW (BP), and bet on both HIGH and LOW (BB). Player 2 can either Call or Fold.

Player 1 Payoff Matrix		Player 2	
		Call	Fold
Player 1	PP	-1	-1
	PB	-21/13	3/13
	BP	2/13	-3/13
	BB	-6/13	1

Player 1 Payoff Matrix		**Player 2**	
		Call	Fold
Player 1	PP	-1	-1
		-21/13	3/13
		2/13	-3/13
	BB	-6/13	1

- Player 1 is always passing: **getting -1 profit with the probability of 1 (HIGH and LOW) and no chance to Call or Fold in Player 2**
- Finally, (-1)*1=-1

Player 1 Payoff Matrix		Player 2	
		Call	Fold
Player 1	PP	-1	-1
	PB	-21/13	3/13
		2/13	-3/13
		-6/13	1

- Player 1 is passing with HIGH: **getting -1 profit with the probability of 5/13**
- Player 1 is betting with LOW and Player 2 is calling: **getting -2 profit with the probability of 8/13**
- Finally, (-1)*(5/13)+(-2)*(8/13) = -21/13

Player 1 Payoff Matrix		**Player 2**	
		Call	Fold
Player 1	PP	-1	-1
	PB	-21/13	3/13
		2/13	-3/13
		-6/13	1

- Player 1 is passing with HIGH: **getting -1 profit with the probability of 5/13**
- Player 1 is betting with LOW and Player 2 is folding: **getting 1 profit with the probability of 8/13**
- Finally, (-1)*(5/13)+(1)*(8/13) = 3/13

Player 1 Payoff Matrix		Player 2	
		Call	Fold
		-1	-1
		-21/13	3/13
	BP	2/13	-3/13
	BB	-6/13	1

- Player 1 is betting with HIGH and Player 2 is calling: **getting 2 profit with the probability of 5/13**
- Player 1 is folding with LOW: **getting -1 profit with the probability of 8/13**
- Finally, (2)*(5/13)+(-1)*(8/13) = 2/13

- Player 1 is betting with HIGH and Player 2 is folding: **getting 1 profit with the probability of 5/13**
- Player 1 is folding with LOW: **getting -1 profit with the probability of 8/13**
- Finally, (1)*(5/13)+(-1)*(8/13) = -3/13

Player 1 Payoff Matrix		**Player 2**	
		Call	Fold
		-1	-1
		-21/13	3/13
	BP	2/13	-3/13
	BB	-6/13	1

Game Theory: Mixed Strategy Game

Player 1 Payoff Matrix		**Player 2**	
		Call	Fold
Player	PP	-1	-1
		-21/13	3/13
		2/13	-3/13
	BB	-6/13	1

- Player 1 is always betting and Player 2 is calling: **getting 2 profit with the probability of 5/13 (HIGH) and -2 profit with the probability of 8/13 (LOW)**
- Finally, (2)*(5/13)+(-2)*(8/13) = -6/13

Player 1 Payoff Matrix		Player 2	
		Call	Fold
Player	PP	-1	-1
		-21/13	3/13
		2/13	-3/13
	BB	-6/13	1

- Player 1 is always betting and Player 2 is folding: **getting 1 profit with the probability of 1 (HIGH and LOW)**
- Finally, 1*1=1

Player 1 Payoff Matrix		Player 2: Call	Player 2: Fold
Player 1	PP	-1	-1
	PB	-21/13	3/13
	BP	2/13	-3/13
	BB	-6/13	1

Player 1 Payoff Matrix (Reduced Form)		**Player 2**	
		Call	Fold
Player 1	BP	2/13	-3/13
	BB	-6/13	1

Player 1 **Payoff Matrix (Reduced Form)**		Player 2 is doing Call and Fold with the probabilities of y_1 and y_2, respectively	
		Call	Fold
Player 1 is doing BP and BB with the probabilities of x_1 and x_2, respectively	BP	2/13	-3/13
	BB	-6/13	1

- Linear Programming Formulation
  - **Maximizing the profit of Player 1**

		Player 2	
		Call	Fold
Player 1	BP	2/13	-3/13
	BB	-6/13	1

$$\begin{aligned} \text{maximize} \quad & z \\ \text{subject to} \quad & \frac{2}{13}x_1 - \frac{6}{13}x_2 \geq z \\ & -\frac{3}{13}x_1 + x_2 \geq z \\ & \boxed{\begin{aligned} & x_1 + x_2 = 1 \\ & x_1 \geq 0 \\ & x_2 \geq 0 \end{aligned}} \end{aligned}$$

Expected Profit if Player 2 is doing Call

Expected Profit if Player 2 is doing Fold

- Linear Programming Formulation
  - **Minimizing the loss of Player 2**

		Player 2	
		Call	Fold
Player 1	BP	2/13	-3/13
	BB	-6/13	1

$$
\begin{aligned}
\text{minimize} \quad & w \\
\text{subject to} \quad & \frac{2}{13}y_1 - \frac{3}{13}y_2 \leq w \\
& -\frac{6}{13}y_1 + y_2 \leq w \\
& \boxed{\begin{aligned} & y_1 + y_2 = 1 \\ & y_1 \geq 0 \\ & y_2 \geq 0 \end{aligned}}
\end{aligned}
$$

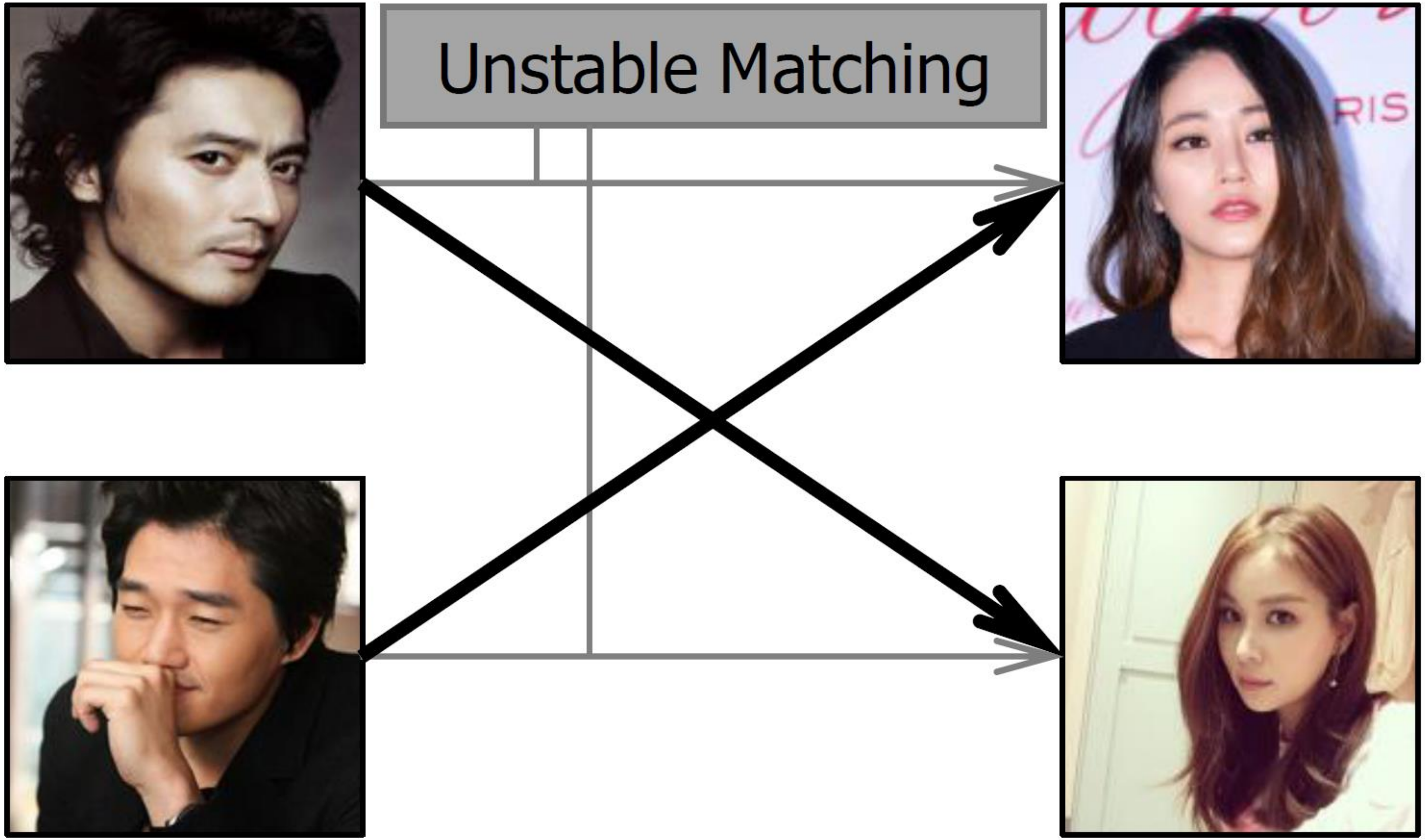
Expected Loss if Player 1 is doing BP

Expected Loss if Player 1 is doing BB

Outline

- Decision Theory Family
    - Game Theory
    - **Stable Marriage**
    - Decision Tree: ID3

Stable Marriage Example



Stable Marriage Problem: Setting and Definitions

- General Setting
  - *N* men and *N* women; Each man (woman) has a preference list (ordering woman (man))
- Definition
  - **Matching:** A pairing of each of the *N* men with a woman $(i, p(i))$ denotes man i paired with woman $p(i)$.
  - **Unstable Matching:** There exist pairs $(i, p(i))$ and $(j, p(j))$; i prefers $p(j)$ to $p(i)$; and $p(j)$ prefer i to j.
  - **Stable Matching:** A matching that is not unstable.

Stable Marriage Problem: Example 1

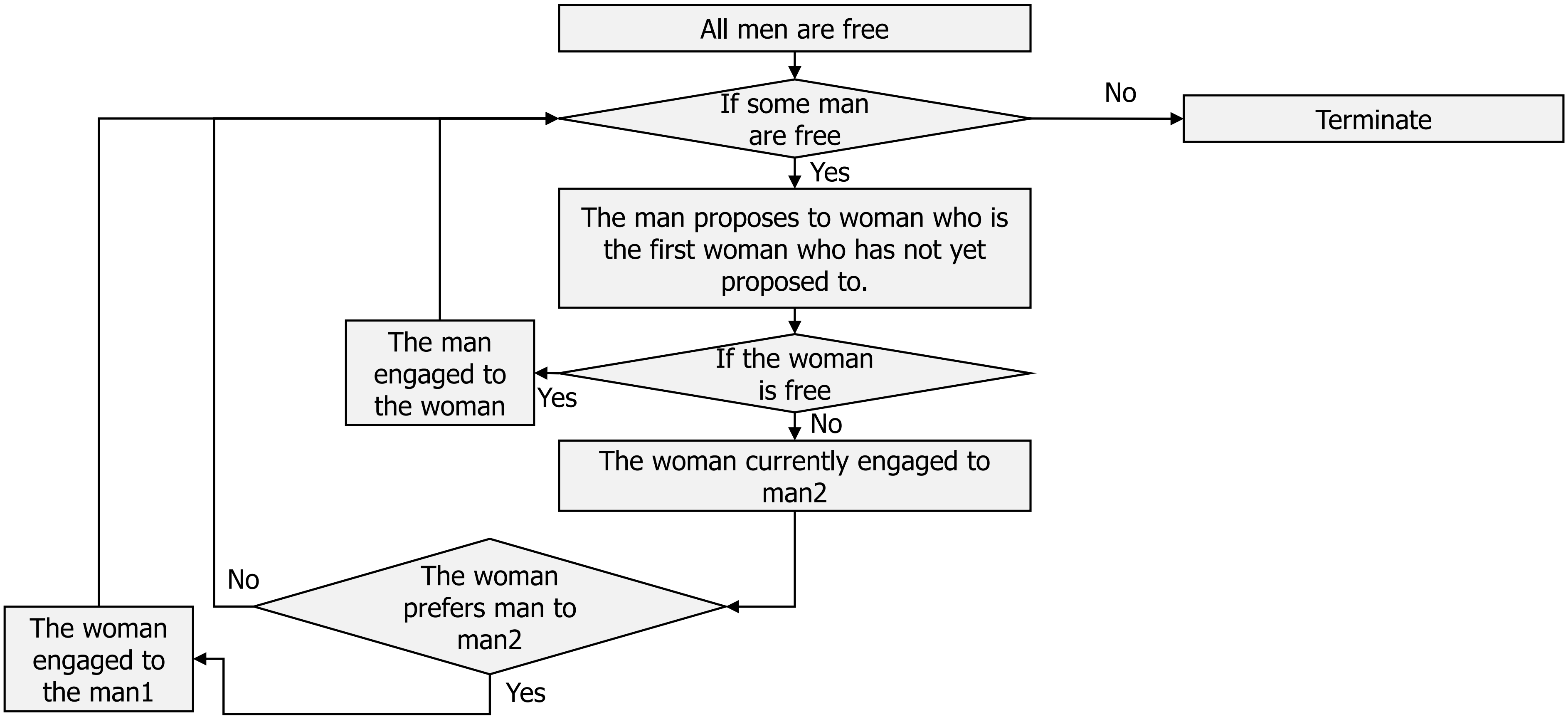
Man	Preference List	
 **Kim**	 **Jeon**	 **Jin**
 **Lee**	 **Jin**	 **Jeon**

Woman	Preference List	
 **Jeon**	 **Kim**	 **Lee**
 **Jin**	 **Kim**	 **Lee**

Two Possible Matching

 **Kim**  **Jeon**	 **Lee**  **Jin**	**Stable**
 **Lee**  **Jeon**	 **Kim**  **Jin**	**Unstable**

Stable Marriage Problem: Gale-Shapley Algorithm (GSA)



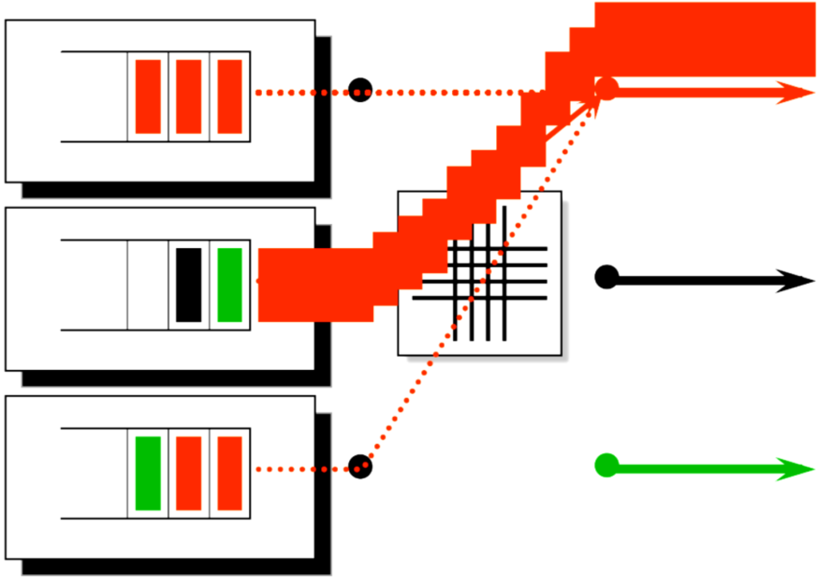
A	1 2 3 4
B	2 1 4 3
C	3 2 4 1
D	3 4 2 1

1	A C D B
2	C D A B
3	B A D C
4	A B C D

GSA Procedure

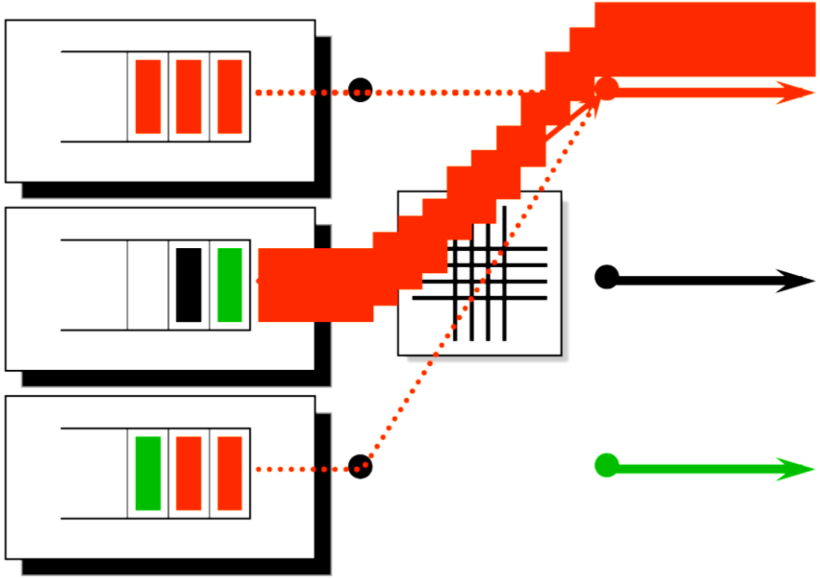
1. (A 1)
2. (A 1)(B 2)
3. (A 1)(B 2)(C 3)
4. 3 prefers D over C, i.e., (A 1)(B 2)(D 3)
5. C's next preference is "2".
   2 prefers C over B, i.e., (A 1)(C 2)(D 3)
6. B's next preference is "1".
   The a does not want to switch.
7. B's next preference is "4"; and the "4" is free., i.e.,
   (A 1)(C 2)(D 3)(B 4)

- Head of Line Blocking

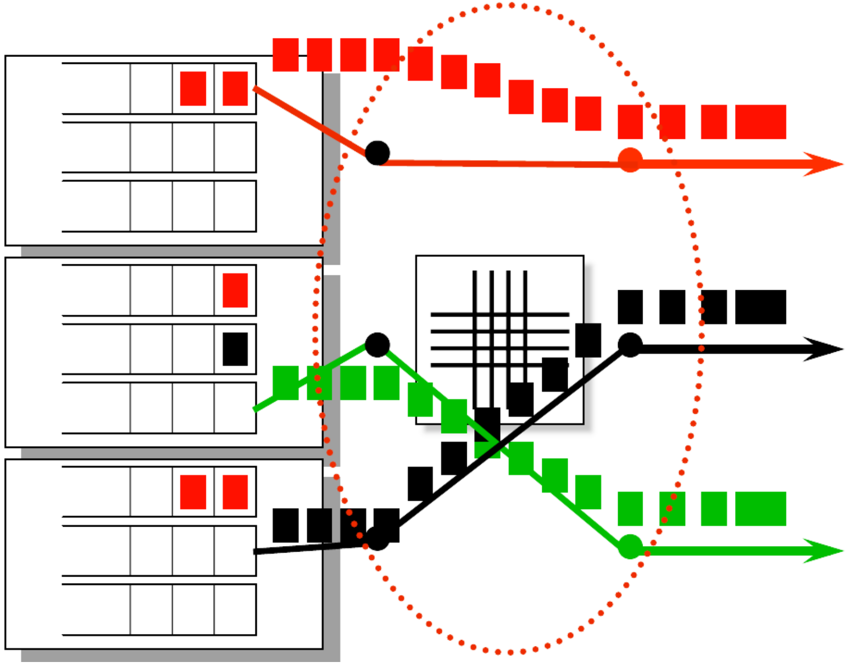


Stable Marriage Problem: Internet Router Design with Virtual Queues

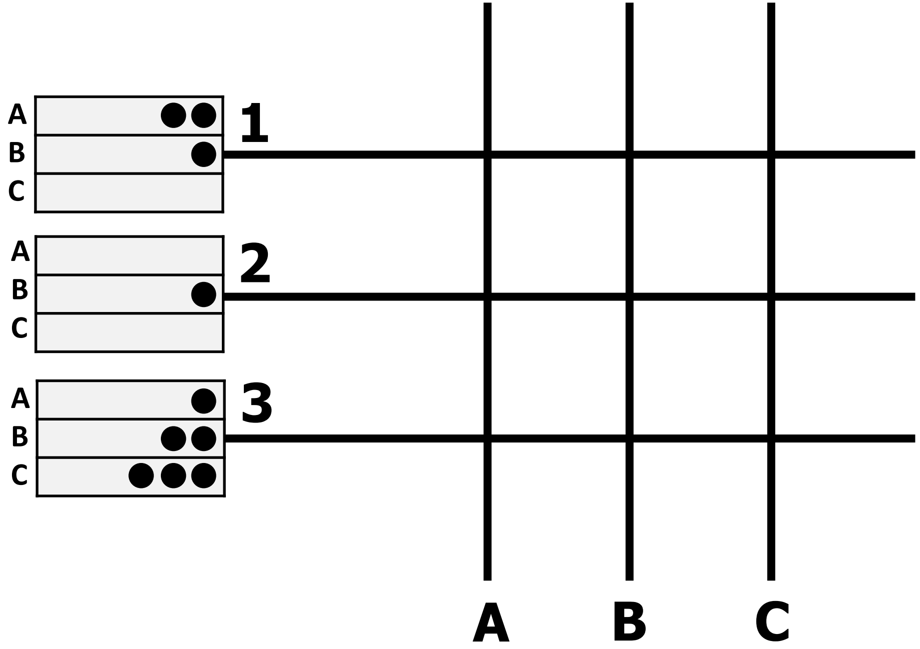
- Head of Line Blocking



- Virtual Queues



Stable Marriage Problem: Internet Router Design with Virtual Queues



A	1 3 2
B	3 1 2
C	3 1 2

1	A B C
2	B A C
3	C B A

Outline

- Decision Theory Family
  - Game Theory
  - Stable Marriage
  - **Decision Tree: ID3**

- How to construct decision trees?
- Example-based Explanation

Day	Outlook	Temperature	Humidity	Wind	Playball
D1	sunny	hot	high	weak	no
D2	sunny	hot	high	strong	no
D3	overcast	hot	high	weak	yes
D4	rain	mild	high	weak	yes
D5	rain	cool	normal	weak	yes
D6	rain	cool	normal	strong	no
D7	overcast	cool	normal	weak	yes
D8	sunny	mild	high	weak	no
D9	sunny	cool	normal	weak	yes
D10	rain	mild	normal	strong	yes
D11	sunny	mild	normal	strong	yes
D12	overcast	mild	high	strong	yes
D13	overcast	hot	normal	weak	yes
D14	rain	mild	high	strong	no

- Example)

Day	Outlook	Temperature	Humidity	Wind	Playball
D1	sunny	hot	high	weak	no
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D7	overcast	cool	normal	weak	yes
D8	sunny	mild	high	weak	no
D9	sunny	cool	normal	weak	yes
D10	rain	mild	normal	strong	yes
D11	sunny	mild	normal	strong	yes
D12	overcast	mild	high	strong	yes
D13	overcast	hot	normal	weak	yes
D14	rain	mild	high	strong	no

Index Values for Individual Attributes

- Outlook = {sunny, overcast, rain}
- Temperature = {hot, mild, cool}
- Humidity = {high, normal}
- Wind = {weak, strong}

D = [9 Yes, 5 No], i.e.,
The overall Entropy can be calculated as follows:

Entropy(D) =$\left(-P_{yes}\, log_2\, P_{yes}\right) + \left(-P_{no}\, log_2\, P_{no}\right)$

where $P_{yes} = \frac{9}{14}$ and $P_{no} = \frac{5}{14}$

- Example)

Day	Outlook	Temperature	Humidity	Wind	Playball
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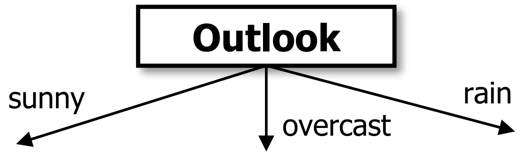
Find Entropy for Attribute "Wind"

- Set $D_{Wind,weak}$ = [6 Yes, 2 No]
- Set $D_{Wind,strong}$ = [3 Yes, 3 No]
- Calculate Entropies for $D_{Wind,weak}$ and $D_{Wind,strong}$
- Calculate **Gain(D,** Wind**)** as follows:

$$\textbf{Gain}(D,\text{Wind}) = \textbf{Entropy}(D) - \frac{|D_{Wind,weak}|}{|D|}\textbf{Entropy}(D_{Wind,weak}) - \frac{|D_{Wind,strong}|}{|D|}\textbf{Entropy}(D_{Wind,strong})$$

Find Entropies for the other Attributes

Pick the Max Gain Attribute: "Outlook"



Decision Tree: ID3

- Example)

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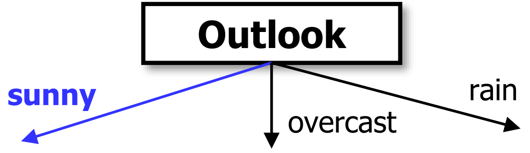
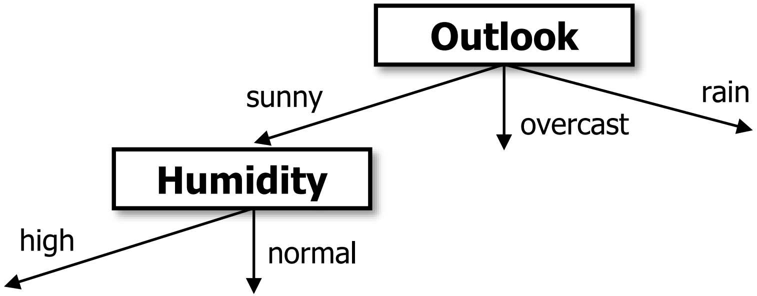


For <u>sunny</u>,

- D_{sunny} = {D1, D2, D8, D9, D11}
- Calculate
  Gain(D_{sunny}, Humidity) = 0.970
  Gain(D_{sunny}, Temperature) = 0.570
  Gain(D_{sunny}, Wind) = 0.019

Pick the Max Gain Attribute, i.e., Humidity



- Example)

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Iterate this procedure.

Final Form