

Economic Analysis of Unmanned Aerial Vehicle (UAV) Provided Mobile Services

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Abstract—Due to its agility and mobility, the unmanned aerial vehicle (UAV) is a promising technology to provide high-quality mobile services (e.g., fast Internet access, edge computing, and local caching) to ground users. The Internet Service Providers (ISPs) directly or commission the third-party UAV firms to provide UAV-provided services (UPS) to improve and make up for the shortage of their current mobile services for additional profit. Yet the UAV has limited energy storage and needs to fly to serve users locally, requiring an optimal energy allocation for balancing both hovering time and service capacity. For profit-maximizing purpose, when hovering in a hotspot, how the UAV should dynamically price its capacity-limited UPS according to randomly arriving users with private service valuations is another question. This paper first introduce a threshold-based assignment policy to show how the UAV decides to serve the users or not under complete information that a user's service valuation can be observed when he arrives. Following this benchmark, we analyze the UAV's optimal pricing under incomplete information about the users' random arrival and private service valuations. It is proved that the UAV should ask for a higher price if the leftover hovering time is longer or its service capacity is smaller, and its expected profit approaches to that under complete user information if the hovering time is sufficiently large. Then, based on the optimal pricing, the energy allocation to hovering time and service capacity in a hotspot is optimized. We show that as the hotspot's user occurrence rate increases, a shorter hovering time or a larger service capacity should be allocated. Finally, when a UAV faces multiple hotspot candidates with different user occurrence rates and flying distances, we prove that it is optimal to deploy the UAV to serve a single hotspot, by taking the optimal pricing and energy allocation of each hotspot into consideration. With multiple UAVs, however, this result can be reversed with UAVs' forking deployment to different hotspots, especially when hotspots are more symmetric or the UAV number is large. Perhaps surprisingly, more UAVs may be deployed to the second-best hotspot rather than the first-best one.

Index Terms—UAV-provided mobile services, network economics, dynamic pricing, capacity allocation, incomplete information

1 INTRODUCTION

As the demand for mobile services increases exponentially, it is imperative for the Internet Service Providers (ISPs) to improve existing services' capacity and coverage and provide more customized services for profit maximizing. Due to its agility and mobility, the unmanned aerial vehicle (UAV) emerges as a promising vehicular technology to provide value-added mobile services (e.g., fast Internet access, edge computing, and local caching) to ground users. For example, AT&T has designed a Flying COW to beam high-throughput wireless coverage to the crowds in sports stadiums [2]. By endowing with edge computing capability, the UAV can be also used to offer computation offloading services to mobile users with limited terminal processing capability [3]. The cache-enabled UAV is also implemented recently to improve the quality-of-experience of mobile devices by caching and distributing the popular content to them [4]. Major ISPs want to enable such UAV-provided services (UPS) to improve and enrich their mobile services for additional profit. For example, Verizon hired a specialized drone company Skyward to provide compatible

value-added services to its users [5]. The global revenue of UPS is expected to increase from \$792 million in 2017 to \$12.6 billion by 2025 [6].

The literature focuses on the technological issues of enabling UPS such as exploiting air-to-ground communication to enlarge wireless coverage and addressing UAV energy constraints. For example, [7] analyzes the optimal operating altitude for a UAV's maximum wireless coverage by considering the trade-off between the opportunity of line-of-sight transmission and signal attenuation. In [8], an energy-aware UAV path planning algorithm is proposed to minimize energy consumption of covering users in a specific area. In the UAV-provided edge computing application, [3] studies the optimization of the UAV's trajectory and computing offloading under its energy constraint. As for local caching application, in [4] the optimal UAV's location and the content to cache are jointly investigated according to the users' content request distribution and their mobility patterns. In [9], two fast UAV deployment problems are studied for optimal wireless coverage. [10] further studies the UAV placement games for best serving all the users while ensuring all selfish users' truthfulness in reporting their locations. [11], [12] focus on adapting the UAV deployment to the spatial randomness of mobile users in order to maximize the average throughput of all users in the uplink information transmission.

Still, the economic issues of UPS for serving mobile users are largely overlooked in the literature, hindering the successful development of UPS in the long run. As the UAV's hovering in the air and its service (e.g., computing

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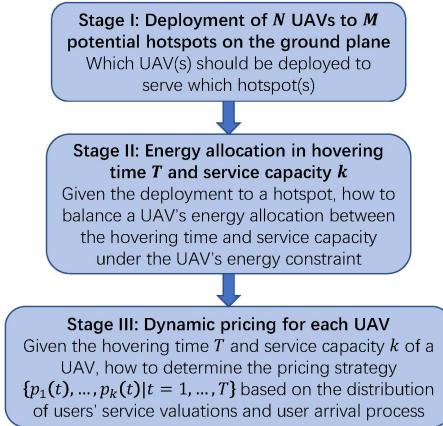


Fig. 1: Three-stage UPS provision of the UAV firm for profit-maximization.

and caching) provision to ground users are both energy-consuming, a longer hovering time helps meet more demands yet leaving less energy for servicing them. How to balance the hovering time and service capacity under the limited energy budget is critical to ensure the economic viability of UPS. Further, when hovering in a hotspot for a given time period, how to dynamically price the capacity-limited UPS to ground users for profit-maximization is another issue. This is challenging under incomplete information about the mobile users' randomness in arriving and their private valuations of buying UPS. What's more, when facing multiple hotspot candidates with different user occurrence rates and flying distances, the optimal deployment of multiple UAVs to cooperatively serve the chosen hotspots needs to be studied. This paper proposes a three-stage UPS provision model to study these economic issues as shown in Fig. 1: first on multiple UAVs' deployment to cooperatively cover heterogeneous hotspots, then on energy allocation of each UAV to balance hovering time and service capacity for its chosen hotspot to deploy, and finally the dynamic UPS pricing for each UAV over its hovering time. These three stages following time sequence are inter-dependent for maximizing the UPS profit and we will apply backward induction for analyzing them.

It should be noted that in the literature there are some related works studying the pricing issues for resource constrained wireless networks (e.g., [13], [14], [15], [16]). For example, [13] discusses both the static and dynamic pricing schemes for a wireless network, depending on whether the network can adapt to the service requirements of their subscribers. An optimal pricing for bandwidth sharing is proposed in [14] for an integrated network using different wireless technologies. [15] studies dynamic WiFi pricing for a fixed user over time under incomplete information of the user's service valuation and utilities. In the broader literature of economics and operations research, there are also some related works about the dynamic admission control and pricing of generic services for users under incomplete demand information ([17], [18], [19]). However, these works assume a fixed service capacity and do not consider users' randomness in arrivals, while in this paper studies a more difficult scenario that each UAV has interchangeable energy capacities for hovering and servicing to further balance and

the mobile users are also randomly moving on the ground.

Our key novelty and main contributions are summarized as follows.

- *Economics of UAV-provided mobile services:* To our best knowledge, this paper is the first work studying the economics of UAV-provided services (UPS) provision, including the interdependent UAV-network deployment, capacity allocation and finally dynamic service pricing for UPS profit maximization in Fig. 1. By applying backward induction, we first analyze each UAV's dynamic pricing under complete and incomplete user information for a given hovering time and service capacity at a given hotspot, then the optimal trade-off between its hovering time and service capacity at a given hotspot, and finally deployment of multiple cooperative UAVs to cover heterogeneous hotspots.
- *Dynamic UPS pricing under complete and incomplete information (Sections 2, 3):* In Section 2, we look at the complete information case for UPS that the UAV can ideally tell a user's service valuation upon his arrival. We develop a threshold-based assignment policy to show how the UAV optimally decides to serve the arriving users or not. As compared to this benchmark case, in Section 3, we further study a more practical case for UPS: incomplete information case regarding users' random arrivals and private service valuations. We propose an optimal dynamic pricing scheme for each UAV, and prove that the UAV should ask for a higher UPS price if its leftover hovering time is longer or its service capacity is smaller. Its expected profit approaches to that under complete user information discussed in Section 2 if the hovering time is sufficiently large. We also show that the UAV may take advantage of a large variance of users' valuation distribution.
- *Optimal energy allocation to balance hovering time and service capacity (Section 4):* Though a longer hovering time ensures a higher service price under incomplete information, it leaves a smaller service capacity under the total energy budget, the UAV should balance its hovering time and service capacity to maximize its profit for providing UPS. To characterize the tradeoff, we develop an optimal threshold-based capacity allocation policy which is easy to implement. We show that as the hotspot's user occurrence rate increases, a smaller hovering time or a larger service capacity should be allocated.
- *Cooperative UAVs' deployment to heterogeneous hotspots (Section 5):* We study the deployment of multiple cooperative UAVs owned by a UAV company to heterogeneous potential hotspots for maximizing the total UPS profit. We aim to reach the best trade-off between different hotspots' user occurrence rates and flying distances for UAV deployment. We prove that it is optimal for a single UAV to only serve one hotspot. However, when we have multiple UAVs for deployment, they should fork to serve different hotspots especially when hotspots are more symmetric or the UAV number is large. We also show

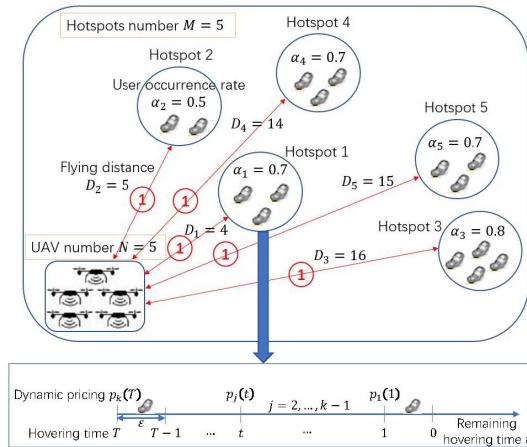


Fig. 2: An example of $N = 5$ UAVs' deployment to $M = 5$ potential hotspots with different user occurrence rates α_m 's and flying distances D_m 's, followed by the dynamic pricing with service capacity k in the hovering period T for each UAV at its hotspot. The number in red circle indicates how many UAVs are assigned to the corresponding hotspot.

a counterintuitive example that more UAVs may be deployed to the second-best hotspot rather than the first-best hotspot to maximize the total profit.

2 SYSTEM MODEL AND PROBLEM FORMULATION

For maximizing the UAV company's profit, we propose a three-stage UPS provision model to study the UAVs' optimal deployment, capacity allocation and dynamic pricing as shown in Fig. 1. In Stage I, we deploy a number N of identical UAVs from the UAV station to M potential heterogeneous hotspots to cooperatively serve users there.¹ Fig. 2 shows an example of deploying $N = 5$ UAVs to $M = 5$ heterogeneous hotspots. Each hotspot m 's user occurrence rate and flying distance from the UAV station are denoted as α_m and D_m , $m = 1, \dots, M$, respectively. Users' random arrivals at a hotspot in a discrete time horizon and each time slot's duration is properly selected such that there is at most one user occurrence at a time.² Here α_m also tells the probability of having a user's occurrence in each time slot at hotspot m . Each user has a one-time-slot service session with the UAV and then leaves the hotspot.³

In Stage II, given an individual UAV's energy storage B upon arrival at its deployed hotspot, this UAV should decide the energy allocation to hovering time T and service capacity k with $T + ck \leq B$, where $c > 0$ is the energy consumption for serving a user (in relative sense to the energy consumption per unit hovering time). Both T and k are integers, telling how many time slots for hovering and how many users to serve, respectively. If the UAV hovers longer, it may encounter more users and charge

1. In practice, most users are clustered in hotspots (e.g., shopping malls and residential areas). Actually, for any unpopular place, we can still model it as a hotspot here, by updating its low user occurrence rate and its flying distance for the UAV to reach.

2. Our analysis and results can be extended to the case that a batch of users with group service valuation v arrive with probability α at a time and the UAV serves them simultaneously by charging this batch of users a price.

3. Our analysis can also be extended to the case that a user stays in service for any fixed duration beyond one time slot.

higher prices under user incomplete information, yet the final number k of users it can serve decreases given the total energy budget B .

In Stage III, given hovering time T and service capacity k for this hotspot, our objective is to maximize the UAV's expected profit $R_k(T)$ at this hotspot by designing the dynamic pricing $\{p_1(t), \dots, p_k(t)|t = 1, \dots, T\}$ in the discrete time horizon as shown in the lower part of Fig. 2, where $p_j(t), j = 1, \dots, k$, is defined as the price for selling the UPS to the j th-to-last user at t time units before the end of hovering/selling period T . Note that $t = 0$ (or $t = T$) is the end (beginning) of the hovering interval and $p_1(t)$ (or $p_k(t)$) is the price for serving the last (first) user. For simplicity, we will use " t time units before the end of hovering period T " and "time t " interchangeably in the rest of the paper. A user (if occurs) will accept the price if his service valuation v is greater than the price asked by the UAV. It is assumed that there is no possibility to recall the users and they may already leave. The users' valuations of the UPS are independent and identically distributed (i.i.d.) according to a probability density function (PDF) $f(v)$, $v \in [a, b]$. Though all potential users' valuations follow the same distribution, their realized valuations are different in general. Under the incomplete information, the UAV does not know the user occurrence for UPS over time t and the user's private service valuation v . It only knows the user occurrence probability in each time slot and the valuation distribution $f(v)$.

For economic purpose, we want to maximize the total UAVs' final profit in the three-stage UPS provision model under their energy budgets. In the following, we will use backward induction to first analyze the optimal dynamic pricing in Stage III given the service capacity and hovering time at a given hotspot, then the energy allocation at the given hotspot in Stage II, and finally the UAVs' optimal deployment to all possible hotspots in Stage I. Before studying the UAV's pricing strategy at Stage III under incomplete information, we first introduce the UAV's assignment policy under complete information, which serves as a benchmark for later analysis. Due to the page limitation, some of the proofs are given in Appendices of the supplemental file and readers can also refer to our online technical report [20].

2.1 Benchmark: Optimal Pricing Strategy under Complete Information in Stage III

In this section, we study the UAV's pricing strategy under complete information, i.e., a user's service valuation v is observed when he arrives. This serves as a performance upper-bound benchmark for comparison with our later analysis under incomplete information. Still, the UAV cannot observe future users' arrival pattern or valuations, otherwise, the analysis is trivial and the result is impractical. For such sequential allocation under complete information, [17] and [18] show that the threshold-based assignment is optimal. We consider the dynamic threshold-based assignment policy $\{y_1(t), \dots, y_k(t)|t \in [0, T]\}$ for serving k users in a finite time interval T : if there are $j, \forall j = 1, \dots, k$, service quotas left and a user with service valuation v arrives at t time units before the end of time horizon T , the UAV serves it if and only if $v \geq y_j(t)$ and charges the user the price equal to his service valuation v . The objective of the UAV is to find the

optimal assignment policy to maximize the expected profit $\hat{R}_k(T)$ at the hotspot.

Before studying the UAV's assignment policy for any service capacity k , we first consider the case of $k = 1$. That is, the UAV can only serve one user finally. By deciding the threshold $y_1(t)$ at time t , a user (if appears with probability α) with the service valuation v will be served and charged the price v if his service valuation v is larger than the threshold, i.e., $v \geq y_1(t)$. Given the cumulative distribution function (CDF) of his service valuation $F(v)$, the probability that a user will appear and be served is $\alpha(1 - F(y_1(t)))$. Then, the UAV's expected total profit at t time units before the end of time interval T is

$$\hat{R}_1(t) = \alpha \int_{y_1(t)}^b v f(v) dv + (1 - \alpha(1 - F(y_1(t)))) \hat{R}_1(t-1), \quad (1)$$

where $1 - \alpha(1 - F(y_1(t)))$ is the probability that no user is served and $\alpha \int_{y_1(t)}^b v f(v) dv$ is the expected profit from successfully serving one user.

We then consider the situation when $k \geq 2$. After successfully serving the last $k-1$ users and using up $k-1$ service capacity, the profit analysis of the case $k=1$ in (1) can be applied for the subsequent analysis of serving the k th-to-last user. Note that the expected profit received from the last $k-1$ users at t time units before the end of selling interval T is $\hat{R}_{k-1}(t)$. By successfully serving the k th-to-last user with the service valuation v based on the threshold $y_k(t)$ at time t , i.e., $v \geq y_k(t)$, the UAV's total profit at time t is $v + \hat{R}_{k-1}(t-1)$. Otherwise, the UAV will keep $\hat{R}_k(t-1)$ with k service quota for the remaining $t-1$ time slots. Then, the expected profit with service capacity k at time slot t is

$$\begin{aligned} \hat{R}_k(t) = & \alpha \left(\int_{y_k(t)}^b v f(v) dv + (1 - F(y_k(t))) \hat{R}_{k-1}(t-1) \right) \\ & + (1 - \alpha(1 - F(y_k(t)))) \hat{R}_k(t-1), \end{aligned} \quad (2)$$

where $\hat{R}_k(0) = 0$ due to zero remaining hovering time and $\hat{R}_0(t) = 0$ due to zero leftover service capacity. Note that if the remaining hovering time t is less than the maximum number of users k that the UAV can serve, i.e., $t < k$, the UAV can at most serve t users (one per time slot) and the extra service capacity is wasted, thus $\hat{R}_k(t) = \hat{R}_t(t)$ for $t = 1, \dots, k-1$, and the corresponding $y_k(t), t < k$ does not exist.

By taking the derivative of $\hat{R}_k(t)$ with respect to $y_k(t)$ and let $\frac{d\hat{R}_k(t)}{dy_k(t)} = 0$, we obtain the following proposition.

Proposition 2.1. The optimal assignment policy for any $k \geq 1$ at time $t \geq k$ is

$$y_k(t) = \hat{R}_k(t-1) - \hat{R}_{k-1}(t-1), \quad (3)$$

with $y_1(1) = a$.

Here, the boundary condition $y_1(1) = a$ tells that at the end of time horizon, the UAV would serve any arriving user to earn profit by setting the threshold as the lowest service valuation of the users.

Algorithm 1 with computation complexity $O(kT)$ shows how to obtain the optimal threshold-based assignment policy: based on the initial condition $y_1(1) = a$, we can obtain $\hat{R}_1(1)$ according to (2), which can be used to calculate $y_1(2)$ and $y_2(2)$ according to (3). Iteratively, we can obtain $\hat{R}_j(t)$

Algorithm 1 UAV's threshold-based assignment policy for serving k users in hovering time T

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1: for  $j = 1$  to  $k$  do
2:   for  $t = 1$  to  $T$  do
3:     if  $t \leq j-1$  then
4:        $R_j(t) = R_t(t);$ 
5:     else
6:       Compute  $\hat{R}_j(t)$  according to (2) with initial condition  $y_1(1) = a;$ 
7:       compute  $y_j(t+1)$  according to (3);
8:     end if
9:     return  $y_j(t), \hat{R}_j(t)$ 
10:   end for
11: end for
12: return  $\{y_1(t), \dots, y_k(t) | t = 1, \dots, T\}$  and  $\hat{R}_k(T)$ 

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according to $y_j(t)$ and (2), and thus $y_j(t+1)$ and $y_{j+1}(t+1)$ according to (3). Note that $y_j(t), t < j$ does not exist as the leftover time slots t is not enough to serve the j th users. As it is difficult to obtain the closed-form solution for $\{y_1(t), \dots, y_k(t) | t = 1, \dots, T\}$, in the following subsection, we extend the discrete-time analysis to continuous-time case to reveal the insights of dynamic profit-maximizing policy.

2.1.1 Continuous Time Relaxation under Complete Information

In the discrete time model as in Fig. 2, we can only use a recursive and numerical way to derive $\hat{R}_k(t)$ and $y_k(t)$ according to (2) and (3). To obtain more analytical results for the threshold-based assignment policy, we next apply continuous-time relaxation on the discrete-time model. Assume users arrive according to a Poisson process with arrival rate α' . Denote the time duration of each time slot for the discrete time model as ε in Fig. 2. To keep the same user occurrence rate α per time slot as in the discrete time case, we have $\varepsilon \alpha' + o(\varepsilon) = \alpha$ as $\varepsilon \rightarrow 0$. Then, for the continuous time case, the dynamic profit-maximizing policy $\{y_1(t), \dots, y_k(t) | t \in [0, T]\}$ can be derived according to the following proposition.

Proposition 2.2. The optimal assignment policy $y_k(t), t \in [0, T]$ is the solution to

$$\frac{dy_k(t)}{dt} = \alpha' \int_{y_k(t)}^{y_{k-1}(t)} (1 - F(v)) dv = 0, \quad (4)$$

where $y_0(t) = b$. And the corresponding expected profit is

$$\hat{R}_k(t) = \sum_{i=1}^k y_i(t). \quad (5)$$

The proof of Proposition 2.2 is given in Appendix A of the supplemental material.

Example 2.1. Assume the users' service valuation follows the exponential distribution $F(v) = 1 - e^{-\lambda v}$. Then, according to (4), the optimal assignment policy over time for $k = 3$ are

$$y_1(t) = \frac{1}{\lambda} \log(1 + \alpha' t), \quad (6)$$

$$y_2(t) = \frac{1}{\lambda} \log\left(\frac{\lambda(1 + \alpha' t)^{1+\frac{1}{\lambda}} + 1}{(\lambda + 1)(1 + \alpha' t)^{\frac{1}{\lambda}}}\right), \quad (7)$$

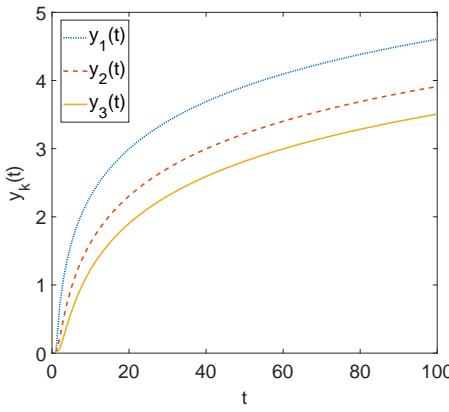


Fig. 3: Optimal assignment policy $y_k(t)$ versus time t when $\lambda = 1$ and $\alpha' = 1$ for $k = 3$.

$$y_3(t) = \frac{1}{\lambda} \log\left(\frac{\alpha't + \frac{\lambda^2(1+\alpha')t^{2+\frac{1}{\lambda}}}{2\lambda+1} + \frac{\lambda^2+3\lambda+1}{2\lambda+1}}{\lambda(1+\alpha')t^{1+\frac{1}{\lambda}} + 1}\right). \quad (8)$$

In Fig. 3, it is shown that $y_k(t)$ decreases with k and increases with t , which means that the UAV would like to charge a higher price if there are less service quotas left or it has more leftover time to search for users with high service valuation.

3 DYNAMIC UPS PRICING UNDER INCOMPLETE INFORMATION IN STAGE III

In this section, we study the UAV's pricing strategy under incomplete information, i.e., the UAV does not know the user's service valuation but its distribution $f(v)$. Note that in Stage III, each UAV's hovering time T and service capacity k are given for its deployed hotspot m . The UAV at this hotspot has probability α_m or simply α of meeting a user request in each time slot. Here we skip the subscript as the analysis holds for a UAV at any hotspot. The UAV should decide the dynamic pricing $p_j(t)$ at t time slots before the end of hovering interval T given any leftover service capacity j , $j = 1, \dots, k$ as shown in Fig. 2. Note that fixed pricing rate is not optimal and is just a special case of our dynamic pricing here. It is possible to deploy more than one UAV at the same hotspot and their cooperative pricing is studied later in Section 5.

Before studying the UAV's optimal pricing strategy for any service capacity k , we first consider the case of $k = 1$. That is, the UAV can only serve one user finally. By announcing price $p_1(t)$ at time t , a user (if appears with probability α) will accept and pay the price if his service valuation v is greater, i.e., $v \geq p_1(t)$. Given the CDF of the service valuation $F(v)$, the probability that a user will appear and accept the price is $\alpha(1 - F(p_1(t)))$. Then, the UAV's expected total profit in the remaining t time slots is

$$R_1(t) = \alpha p_1(t)(1 - F(p_1(t))) + R_1(t-1)(1 - \alpha(1 - F(p_1(t)))) \quad (9)$$

Similar to the analysis in Section 2.1, for the general $k \geq 2$ case, the expected total profit at time t can be derived recursively as

$$R_k(t) = \alpha(p_k(t) + R_{k-1}(t-1))(1 - F(p_k(t))) + R_k(t-1)(1 - \alpha(1 - F(p_k(t)))) \quad (10)$$

Algorithm 2 UAV's dynamic pricing for serving k users in hovering time T

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1: for  $j = 1$  to  $k$  do
2:   for  $t = 1$  to  $T$  do
3:     if  $t \leq j - 1$  then
4:        $R_j(t) = R_t(t)$ ;
5:     else
6:       compute  $p_j(t)$  as the unique solution to (11);
7:       update  $R_j(t)$  according to  $p_j(t)$  and (10);
8:     end if
9:   return  $p_j(t)$ ,  $R_j(t)$ 
10: end for
11: end for
12: return  $\{p_1(t), \dots, p_k(t)|t = 1, \dots, T\}$  and  $R_k(T)$ 

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where $R_k(0) = 0$ due to zero remaining hovering time and $R_0(t) = 0$ due to zero leftover service capacity.

By taking the derivative of $R_k(t)$ with respect to $p_k(t)$, the optimal price $p_k(t)$ satisfies

$$\frac{dR_k(t)}{dp_k(t)} = \alpha \left(1 - F(p_k(t)) - f(p_k(t)) \times (p_k(t) - (R_k(t-1) - R_{k-1}(t-1))) \right) = 0. \quad (11)$$

According to (11), it is easy to check that $p_k(t) \geq R_k(t-1) - R_{k-1}(t-1)$.

Proposition 3.1. For $\forall t \in \{1, \dots, T\}$ and $k \geq 1$, we have $R_k(t) \geq R_k(t-1)$. Moreover, if $t < k$, $R_k(t) = R_t(t)$, and otherwise $kR_1(\lfloor \frac{t}{k} \rfloor) \leq R_k(t)$.

The proof of Proposition 3.1 is given in Appendix B of the supplemental material.

Intuitively, if the remaining hovering time t is less than the service quota k , i.e., $t < k$, the UAV can at most serve t users, and thus $R_k(t) = R_t(t)$ and the price $p_k(t)$ at time $t < k$ does not exist. If the UAV has reasonable leftover time t to sell k (i.e., $t \geq k$), it is better to optimize its expected profit $R_k(t)$ via jointly pricing k service capacities over t time slots, rather than independently pricing for each service capacity with separate selling time $\lfloor \frac{t}{k} \rfloor$. This implies that $R_k(t)$ concavely increases with k and t .

In the following, we assume the distributions of the users' service valuations are regular, which is widely adopted in the realm of mechanism design [21]: $\phi(v) = v - \frac{1-F(v)}{f(v)}$ is an increasing function of v , where $F(v)$ and $f(v)$ are the CDF and PDF of each user's valuation v , respectively. Regularity holds for many distributions such as uniform, normal, exponential and Rayleigh distributions.

Proposition 3.2. For $\forall t \in \{1, \dots, T\}$ and $k \geq 1$, Algorithm 2 optimally returns the dynamic pricing scheme with computation complexity $O(kT)$. Especially, when $k = 1$, the optimal price $p_1(t)$ is a non-decreasing function of leftover time t and mean user occurrence rate α in the hotspot.

The proof of Proposition 3.2 is given in Appendix C of the supplemental material.

In Algorithm 2, we first compute the optimal price $p_j(t)$ according to (11), which is a function of $R_{j-1}(t-1)$ and $R_j(t-1)$ with initial conditions $R_1(0) = 0$ and $R_0(t) = 0$. Note that the price $p_j(t)$ at time $t \leq j - 1$ does not exist

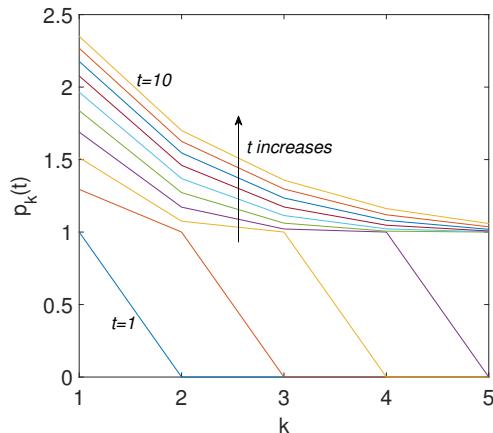


Fig. 4: Optimal price $p_k(t)$ at each time t versus the service capacity k for exponential distributions of users' service valuations when $\alpha = 0.8$, $\lambda = 1$ and $T = 10$.

as the leftover time slots t is not enough to serve the j th users. Finally, we can obtain the expected profit $R_j(t)$ based on $p_j(t)$. Proposition 3.2 also shows that the UAV should ask for a higher price if it has more leftover time t for encountering more users or a larger user demand (characterized by a higher user occurrence rate α). Actually, for general k , we can numerically show that the optimal price $p_k(t)$ has the following properties (see Fig. 4):

- For any given t , the optimal price $p_k(t)$ decreases with k , as the UAV has more service capacity supply k to meet the users' demand.
- For any given k , the optimal price $p_k(t)$ increases with t , as the UAV has more leftover time for encountering more users.

3.1 Continuous-time Relaxation For More Tractable Analysis

To obtain more analytical results for dynamic pricing design, we apply continuous-time relaxation on the discrete-time model in this section, by assuming that users arrive according to a Poisson process with arrival rate α' . Similar to the analysis of the discrete time case in (10), as $\varepsilon \rightarrow 0$, the expected total profit that the UAV can obtain at continuous time $t + \varepsilon$ is

$$R_k(t + \varepsilon) = \alpha' \int_t^{t+\varepsilon} (p_k(x) + R_{k-1}(x))(1 - F(p_k(x)))dx + R_k(t)(1 - \alpha' \int_t^{t+\varepsilon} (1 - F(p_k(x)))dx) + o(\varepsilon). \quad (12)$$

Note that $R_k(0) = 0$. According to (12), the UAV's expected total profit with service capacity k at time t can be derived as

$$R_k(t) = \alpha' \int_0^t (p_k(x) + R_{k-1}(x) - R_k(x))(1 - F(p_k(x)))dx. \quad (13)$$

To ensure positive profit, $p_k(x) \geq R_k(x) - R_{k-1}(x)$. The optimal price $p_k(t)$ that maximizes the expected profit $R_k(t)$ is simplified to

$$p_k(t) = \arg \max_{p \geq R_k(t) - R_{k-1}(t)} (p + R_{k-1}(t) - R_k(t))(1 - F(p)). \quad (14)$$

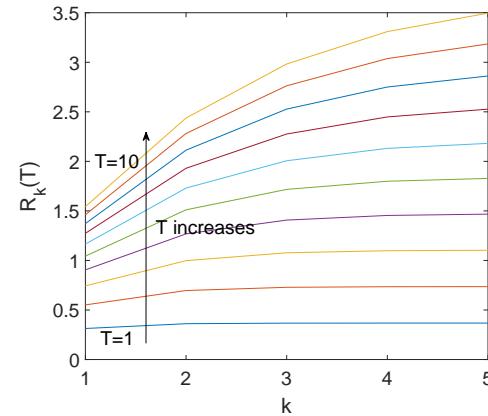


Fig. 5: Optimal expected profit $R_k(T)$ at given hovering time T vs the number of serving users k for exponential distributions of users' service valuations when $\alpha' = 1$, $\lambda = 1$ and $T = 10$.

To analytically obtain the expected profit, we further consider the case that the users' i.i.d. service valuations follow exponential distributions, i.e., $F(v) = 1 - e^{-\lambda v}$. Then, by solving (14), the optimal price $p_k(t)$ is

$$p_k(t) = \frac{1}{\lambda} + R_k(t) - R_{k-1}(t), \quad (15)$$

which is greater than the mean valuation $\frac{1}{\lambda}$ by considering the future pricing opportunity characterized by $R_k(t) - R_{k-1}(t) \geq 0$. Insert (15) into (13), the expected profit for serving k users in total hovering time T can be derived in closed-form, given by

$$R_k(T) = \frac{1}{\lambda} \log \left(\sum_{i=0}^k \frac{1}{i!} \left(\frac{\alpha' T}{e} \right)^i \right). \quad (16)$$

From (15) and (16), we can obtain the closed-form dynamic price at time $t \in [0, T]$ explicitly as

$$p_k(t) = \frac{1}{\lambda} + \frac{1}{\lambda} \log \left(\frac{\sum_{i=0}^k \frac{1}{i!} \left(\frac{\alpha' t}{e} \right)^i}{\sum_{i=0}^{k-1} \frac{1}{i!} \left(\frac{\alpha' t}{e} \right)^i} \right). \quad (17)$$

Proposition 3.3. The optimal expected profit $R_k(T)$ in (16) concavely increases with both k and T , respectively. Further, the optimal price $p_k(t)$ in (17) increases with t and convexly decreases with k .

The proof of Proposition 3.3 is given in Appendix D of the supplemental material.

Proposition 3.3 shows that the growth rate of the expected profit decreases with service capacity k given the fixed hovering time T as shown in Fig. 5. This is because the partitioned hovering time $\frac{T}{k}$ for pricing an individual service capacity decreases with k in average sense. Similarly, the growth rate of the expected profit decreases with hovering time T given the fixed service capacity k . We can also see that Proposition 3.3 is consistent with the numerical results under discrete-time case in Fig. 4. Moreover, the optimal price increases faster as k decreases due to the scarce service

4. The exponential distribution is assumed here for analysis tractability of closed-form results. It also characterizes some practices that many users have low service valuations and are not willing to pay high price for the UPS. The analysis method also holds for other continuous distributions though the analysis is more involved without closed-form.

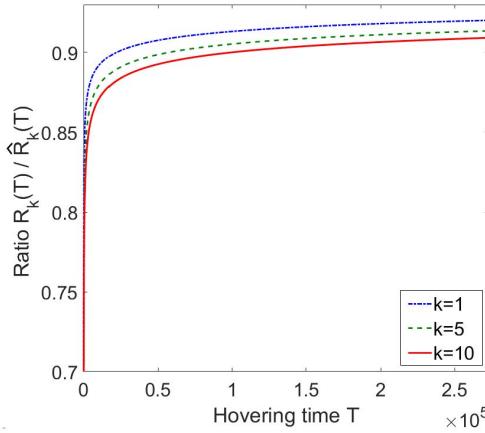


Fig. 6: Ratio of the expected profits under incomplete information and complete information $R_k(T)/\hat{R}_k(T)$ versus hovering time T and service capacity k .

capacity to sell within t time period. Thus, $p_k(t)$ convexly decreases with k .

3.2 Comparison with complete information benchmark

We have finished analyzing the dynamic pricing under incomplete information above. We wonder the performance gap with ideally complete information as discussed in Section 2.1. According to the closed-form solution (6)–(8) and L'Hospital's rule, we have the following proposition.

Proposition 3.4. For $k \in \{1, 2, 3\}$, $\lim_{T \rightarrow \infty} \frac{R_k(T)}{\hat{R}_k(T)} = 1$,

where $R_k(T)$ and $\hat{R}_k(T)$ are the expected profits under incomplete and complete information for exponential distribution of users' service valuations, respectively.

The proof of Proposition 3.4 is given in Appendix E of the supplemental material.

Actually, for any finite $k < \infty$, we can iteratively obtain $\hat{R}_k(T)$ in non-closed-form according to Proposition 2.2. Then we can also show the convergence of $\frac{R_k(T)}{\hat{R}_k(T)}$ to 1 in a numerical way. As shown in Fig. 6, $R_k(t)$ approaches $\hat{R}_k(t)$ if the hovering time T is sufficiently large. Moreover, $R_k(t)$ converges faster to $\hat{R}_k(t)$ as the service capacity k decreases. This is because, as k decreases, the partitioned hovering time $\lfloor \frac{T}{k} \rfloor$ for pricing an individual service capacity increases in average sense and is easier to become sufficient.

We also wonder how the expected profit changes with the user variance given a fixed mean for the distribution of each user's service valuation. For the uniform distribution's CDF $F(v) = \frac{v-a}{b-a}, v \in [a, b]$, Fig. 7 shows the maximum profits under both complete and incomplete information, and the former is greater. When the hovering time T is big (e.g., $T = 12$), the expected profits under incomplete and complete information always increase with the variance. This is because the users' maximum service valuation b increases due to the increased variance and the UAV has enough time to wait for the user with higher service valuation to pay. However, when the hovering time T is small (e.g., $T = 3$), the expected profits first decrease with the variance due to the more information loss and then increase due to the larger upper bound b to exploit. Note that even for the complete information, the UAV still has

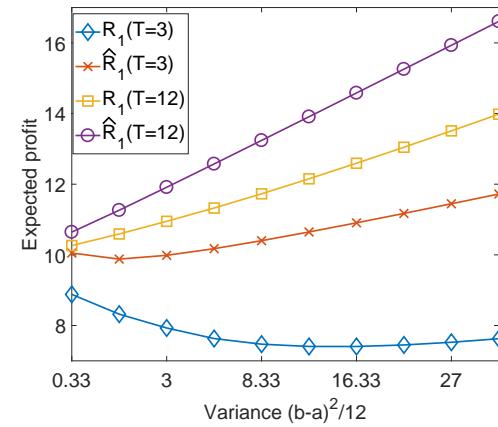


Fig. 7: Expected profits under complete information $\hat{R}_1(T)$ and incomplete information $R_1(T)$ versus the variance with fixed mean 10 of uniform distribution.

some information loss as it can only observe an immediate arrival's valuation rather than any future information.

4 UAV'S ENERGY ALLOCATION IN HOVERING TIME AND SERVICE CAPACITY IN STAGE II

Based on the analysis of optimal pricing in Section 3, a longer hovering time T results in a higher service price at the cost of smaller service capacity k . Therefore, in Stage II the UAV under the total energy budget B should balance T and k optimally for profit maximization. Its optimal energy allocation problem at the given hotspot is

$$\max_{k, T \in \mathbb{Z}^+} R_k(T), \quad (18)$$

s.t.

$$T + ck \leq B, \quad (19)$$

where $R_k(T)$ is returned by Algorithm 2 for discrete-time case, and B can be viewed as the maximum hovering time if the UAV does not use any energy to serve any user.

At the optimality, (19) is tight to use up all the budget and the problem can be rewritten as

$$\max_{k \in \mathbb{Z}^+} R_k(B - ck). \quad (20)$$

Recall that $\forall t < k, R_k(t) = R_t(t)$ in Proposition 3.1 and the UAV would not set k to be larger than the maximum time, i.e., $k \leq B - ck$. Thus, integer decision k is upper bounded by $\lfloor \frac{B}{1+c} \rfloor$. By using Algorithm 2 for any $k \in \{1, \dots, \lfloor \frac{B}{1+c} \rfloor\}$, we can recursively calculate the corresponding expected profit $R_k(B - ck)$. Then, the UAV compares and chooses the best k^* with maximal expected profit, i.e., $k^* = \arg \max_k R_k(B - ck)$.

Fig. 8 shows a numerical example for uniform distribution of users' service valuations under the discrete-time case.

- For low user occurrence rate α , it is better to only serve one user, i.e., $k^* = 1$. It is worthwhile for the UAV to hover the longest possible time to encounter a user.
- For medium user occurrence rate α , it is easier for the UAV to encounter more users and it should choose $k^* \in \{2, \dots, \lfloor \frac{B}{c+1} \rfloor - 1\}$, telling an optimal balance between encountered demand and service capacity supply.

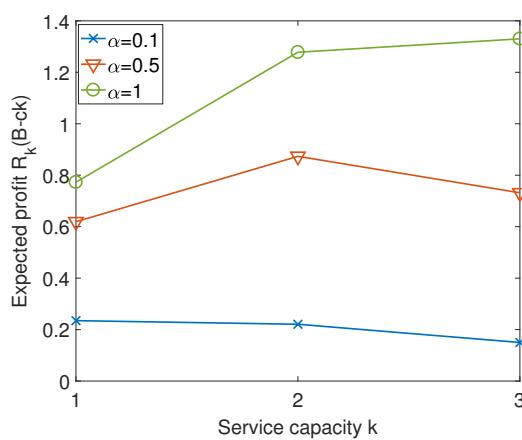


Fig. 8: Expected profit versus service capacity k for uniform service valuation distributions when $B = 15$ and $c = 3$. Here, integer decision k is upper bounded by $\lfloor \frac{B}{1+c} \rfloor = 3$.

- For high user occurrence rate α , the UAV will meet many users and it will choose to serve as many users as possible, i.e., $k^* = \lfloor \frac{B}{c+1} \rfloor$.

To analytically obtain the energy allocation policy, similar to Section 3.1, we apply continuous-time relaxation, where the maximum service capacity is $\lfloor \frac{B}{c} \rfloor$. Recall Proposition 3.3 shows that $R_k(T)$ concavely increases with both k and T and α' is the arrival rate of Poisson process under the continuous-time model. Based on this result, we successfully develop an optimal energy allocation policy by assuming that the users' service valuations follow exponential distributions.

Theorem 4.1. The optimal service capacity k^* depends on the user occurrence rate and is given as follows.

- In the low user occurrence regime ($\alpha' \leq \frac{2ce}{(B-2c)^2}$), the UAV will decide $k^* = 1$ for serving one user only.
- For medium user occurrence regime ($\frac{2ce}{(B-2c)^2} < \alpha' < \bar{\alpha}'$), the UAV will decide

$$k^* = \arg \max_{k \in \mathbb{Z}^+} \sum_{i=0}^k \frac{1}{i!} \left(\frac{\alpha'(B-ck)}{e} \right)^i \quad (21)$$

$$\in \{2, \dots, \lfloor \frac{B}{c} \rfloor - 1\}.$$

- For high user occurrence regime ($\alpha' \geq \bar{\alpha}'$), the UAV will decide $k^* = \lfloor \frac{B}{c} \rfloor$ for serving as many users as possible, where $\bar{\alpha}'$ is the unique solution to

$$\frac{\alpha'}{e \lfloor \frac{B}{c} \rfloor!} (B - c \lfloor \frac{B}{c} \rfloor)^{\lfloor \frac{B}{c} \rfloor} - \sum_{i=1}^{\lfloor \frac{B}{c} \rfloor - 1} \left(\frac{e}{\alpha'} \right)^{\lfloor \frac{B}{c} \rfloor - i - 1} \frac{1}{i!} \quad (22)$$

$$\times \left((B - c(\lfloor \frac{B}{c} \rfloor - 1))^i - (B - c \lfloor \frac{B}{c} \rfloor)^i \right) = 0.$$

The optimal hovering time is $T^* = B - ck^*$.

The proof of Theorem 4.1 is given in Appendix F of the supplemental material.

Note that the high user occurrence regime may not always exist since $\bar{\alpha}'$ as the solution to (22) can be infinity. It happens when $\frac{B}{c}$ is an integer and the hovering time $B - ck^* = B - c \frac{B}{c}$ is zero. In this case, we only have low and medium user occurrence regimes with $k^* < \frac{B}{c}$.

The policy shown in Theorem 4.1 is threshold-based and easy to implement. As the total energy budget B increases, the low user occurrence regime is less likely to happen and the optimal service capacity k^* in (21) increases with B and user occurrence rate α' . This is because the UAV has more energy budget and thus it is better for it to serve more users. Moreover, we can see that Theorem 4.1's result under the continuous-time model is consistent with Fig. 8 under the discrete-time model.

5 OPTIMAL UAV DEPLOYMENT IN STAGE I

Given M hotspots with heterogeneous user occurrence rates α_m 's and distances D_m 's with $m \in \{1, \dots, M\}$ from the UAV station in Fig. 2, we now study how to deploy N UAVs to such hotspots. Here each UAV has an identical initial energy budget B_0 fully charged at the UAV station. In the following, we will analyze the optimal UAV deployment strategy for a single UAV first and then extend to multiple cooperative UAVs.

5.1 Deployment of a Single UAV to Heterogeneous Hotspots

Given a single UAV's route going through $M' \leq M$ hotspots in sequence $\mathcal{H} = \{H_1, H_2, \dots, H_{M'}\}$, the route distance is $D_{H_1} + \sum_{m=1}^{M'-1} D_{H_m, H_{m+1}}$, where $D_{H_m, H_{m+1}}$ is the flying distance from hotspot H_m to H_{m+1} . Given the initial energy budget B_0 , the UAV spends energy traveling in the route and its remaining energy profile is partitioned among M' hotspots in the meantime for serving users there, denoted as $\mathcal{B}_{M'} = \{B_{H_1}, \dots, B_{H_{M'}}\}$ with $\sum_{m=1}^{M'} B_{H_m} = B_0 - D_{H_1} - \sum_{m=1}^{M'-1} D_{H_m, H_{m+1}}$. Here we normalize the energy consumption per unit flying distance as one. Based on the discussion in Sections 3 and 4, given the energy budget B_{H_m} for hotspot H_m , we can decide the optimal energy allocation to hovering time T_{H_m} and service capacity k_{H_m} in Stage II as well as the optimal dynamic pricing for serving k_{H_m} users during hovering time T_{H_m} in Stage III according to (20) and Algorithm 2. Then, given routing strategy \mathcal{H} and energy partition $\mathcal{B}_{M'}$, the UAV's overall expected profit by covering M' hotspots in the whole route is

$$\Psi_{\mathcal{H}}(\mathcal{H}, \mathcal{B}_{M'}) = \sum_{H_m \in \mathcal{H}} \max_{k_{H_m} \in \mathbb{Z}^+} R_{k_{H_m}}^{\alpha_{H_m}} (B_{H_m} - ck_{H_m}), \quad (23)$$

where $R_{k_{H_m}}^{\alpha_{H_m}}(t)$ is the expected profit from hotspot H_m with user occurrence rate α_{H_m} for serving k_{H_m} users at any time t .

Still, the UAV needs to decide the route by considering $\sum_{M'=1}^M C_M^{M'} M'!$ possibilities, where the UAV needs to first choose M' out of M hotspots and each M' introduces $M'!$ possible ordering sequences. Among these routing possibilities, the UAV finds the optimal route with hotspot sequence \mathcal{H} and energy allocation $\mathcal{B}_{M'}$ to maximize the overall expected profit in (23).

This routing problem followed by energy allocation and pricing is complicated and we can only solve it in a numerical way. For analysis tractability, we still apply continuous-time relaxation on the discrete-time horizon and analyze the optimal UAV routing by assuming exponential distributions for users' service valuations. We also consider the situation

when the hovering time increases in the energy allocated to the hotspot and the optimal service capacity increases in the user occurrence rate for any given energy budget.⁵

Theorem 5.1. For any number M of heterogeneous hotspots distributed on the ground plane, it is optimal to deploy the single UAV to only one hotspot denoted by $m^* = \arg \max_{m \in \{1, \dots, M\}} \max_{k_m} R_{k_m}^{\alpha_m}(B_0 - D_m - ck_m)$ without serving any other hotspot in the route.

The proof of Theorem 5.1 is given in Appendix G of the supplemental material, and we use the results of Theorem 4.1 in Stage II and (16) in Stage III for the proof.

Theorem 5.1 tells that the single UAV will only serve the first-best hotspot with maximum individual expected profit. If a part of energy budget is removed from serving the first-best hotspot to also serve the second-best, the overall profit decreases as the UAV's marginal profit from serving the second-best hotspot is lower. Even if the UAV has to bypass a hotspot before reaching the first-best hotspot, it will not spend any hovering time or service capacity there in the mean time.

Without loss of generality, we sort the M heterogeneous hotspots according to their individual expected profits served by the single UAV, i.e.,

$$R_{k_1^*}^{\alpha_1}(B_0 - D_1 - ck_1^*) \geq \dots \geq R_{k_M^*}^{\alpha_M}(B_0 - D_M - ck_M^*), \quad (24)$$

where

$$k_m^* = \arg \max_{k_m \in \mathbb{Z}^+} R_{k_m}^{\alpha_m}(B_0 - D_m - ck_m), \quad (25)$$

$m \in \{1, \dots, M\}$, and now hotspot 1 is the first-best followed by hotspot 2 and others. Other than profit maximization objective, if the UAV further has the fairness commitment constraint to serve a minimum number of hotspots with certain hovering time in each, then we can still apply the sorting in (24) for choosing profitable hotspots and deciding the energy allocation differently among them.

5.2 Deployment of Multiple UAVs to Hotspots: Forking or Not

We are now ready to study how to assign multiple identical UAVs simultaneously from the common UAV station to heterogeneous hotspots, and we want to answer the key question: whether should all the UAVs still be deployed to the first-best hotspot only as in Theorem 5.1 or should they fork to serve different hotspots.⁶ It is possible for more than one UAV to cooperatively serve the same hotspot by "pooling" their service capacities, e.g., when only one hotspot is close and reachable within the energy budget B_0 . Given a number n_m of UAVs cooperatively serving hotspot m , $m \in \{1, \dots, M\}$, they will stay for the same amount of hovering time T_m due to their symmetry. To keep the same energy consumption rates during hovering, n_m UAVs take turns to serve users. For example, in the caching application, each UAV can sequentially distribute $1/n_m$ segment of the requested popular file to the same user.

5. Based on our exhaustive simulation, this assumption will not change the result in Theorem 5.1.

6. We assume UAVs are deployed at the same time to rapidly provide users with UPS. Otherwise, for profit maximization purpose, all the UAVs will be deployed one by one without any overlap to independently serve the first-best hotspot only, as in Theorem 5.1.

According to Theorem 5.1, the same group of n_m UAVs will only travel to one hotspot. It is possible that $n_m = 0$ or 1. We should note that there is an upper bound \bar{n}_m for the number n_m of UAVs serving a particular hotspot m to avoid collision and save energy.

Given the UAV deployment profile $\mathcal{N} = \{n_1, \dots, n_M\}$ to M hotspots, for a particular hotspot m with n_m cooperative UAVs deployed there, we still need to decide the energy allocation to hovering time T_m and total service capacity k_m in Stage II. By pooling the residual energy $n_m(B_0 - D_m)$ of n_m UAVs upon reaching hotspot m , the optimal energy allocation problem for n_m UAVs is

$$\max_{k_m, T_m \in \mathbb{Z}^+} R_{k_m}^{\alpha_m}(T_m), \quad (26)$$

s.t.

$$n_m T_m + ck_m \leq n_m(B_0 - D_m), \quad (27)$$

At the optimality, (27) should be tight. Then, the problem can be rewritten as

$$\max_{k_m \in \mathbb{Z}^+} R_{k_m}^{\alpha_m}(B_0 - D_m - \frac{ck_m}{n_m}). \quad (28)$$

In order to decide the optimal energy allocation above, we first need to find out the optimal dynamic pricing during hovering time $T_m = B_0 - D_m - \frac{ck_m}{n_m}$ given the total service capacity k_m in Stage III. According to the analysis in Section 3, by pooling n_m UAVs' service capacity k_m to jointly serve the users, the optimal dynamic pricing $\{p_1^{\alpha_m}(t), \dots, p_{k_m}^{\alpha_m}(t) | t = 1, \dots, \lfloor B_0 - D_m - \frac{ck_m}{n_m} \rfloor\}$ can be obtained according to Algorithm 2. The cooperation among UAVs helps pool their service capacities to jointly decide pricing, yet they waste more energy in hovering at the same time.

Then, the overall expected profit given the UAV deployment profile $\mathcal{N} = \{n_1, \dots, n_M\}$ to M hotspots with $\sum_{m=1}^M n_m = N$ and $n_m \leq \bar{n}_m, m \in \{1, \dots, M\}$ can be written as

$$\Phi(\mathcal{N}) = \sum_{m=1}^M \max_{k_m \in \mathbb{Z}^+} R_{k_m}^{\alpha_m}(B_0 - D_m - \frac{ck_m}{n_m}). \quad (29)$$

Note that $R_{k_m}^{\alpha_m} = 0$ for hotspot m if $n_m = 0$. For the discrete time model, the service capacity k_m at hotspot m should be no larger than the maximum hovering time, i.e., $k_m \leq B_0 - D_m - \frac{ck_m}{n_m}$. Thus, $k_m \leq \lfloor \frac{B_0 - D_m}{1 + \frac{c}{n_m}} \rfloor$, where $n_m \leq \bar{n}_m$.

We propose Algorithm 3 to find out the optimal deployment of N UAVs to M hotspots. Given any UAV deployment profile \mathcal{N} , we first compute the optimal energy allocation and the dynamic pricing for each hotspot m in Stages II and III. Then the expected profit $\max_{k_m} R_{k_m}^{\alpha_m}(\lfloor B_0 - D_m - \frac{ck_m}{n_m} \rfloor)$ under optimal energy allocation and dynamic pricing for each hotspot m can be found. Finally, by comparing the overall expected profits $\Phi(\mathcal{N})$ in (29) under all possible UAV deployment profiles $\mathcal{A}(\mathcal{N})$, the optimal UAV deployment $\mathcal{N}^* = \arg \max_{\mathcal{N} \in \mathcal{A}(\mathcal{N})} \Phi(\mathcal{N})$ can be obtained. The computation complexity of Algorithm 3 is $O(N^M M \max_{m \in \{1, \dots, M\}} (B_0 - D_m)^2)$.

In the following, we further analyze whether N UAVs should all center at the first-best hotspot 1 or fork to hotspot 2 (or more hotspots), by assuming exponential distributions of users' service valuations for the relaxed continuous-time model with α' as the user arrival rate of Poisson process.

$$\varphi = \frac{\max_{k_1} \sum_{i=0}^{k_1} \frac{1}{i!} \left(\frac{\alpha'_1 (B_0 - D_1 - \frac{c k_1}{N})}{e} \right)^i - \max_{k_1} \sum_{i=0}^{k_1} \frac{1}{i!} \left(\frac{\alpha'_1 (B_0 - D_1 - \frac{c k_1}{N-1})}{e} \right)^i}{\left(\max_{k_1} \sum_{i=0}^{k_1} \frac{1}{i!} \left(\frac{\alpha'_1 (B_0 - D_1 - \frac{c k_1}{N-1})}{e} \right)^i \right) \left(\sum_{i=1}^{k_2^*} \frac{1}{i!} \left(\frac{\alpha'_1 (B_0 - D_2 - c k_2^*)}{e} \right)^i \right)}. \quad (31)$$

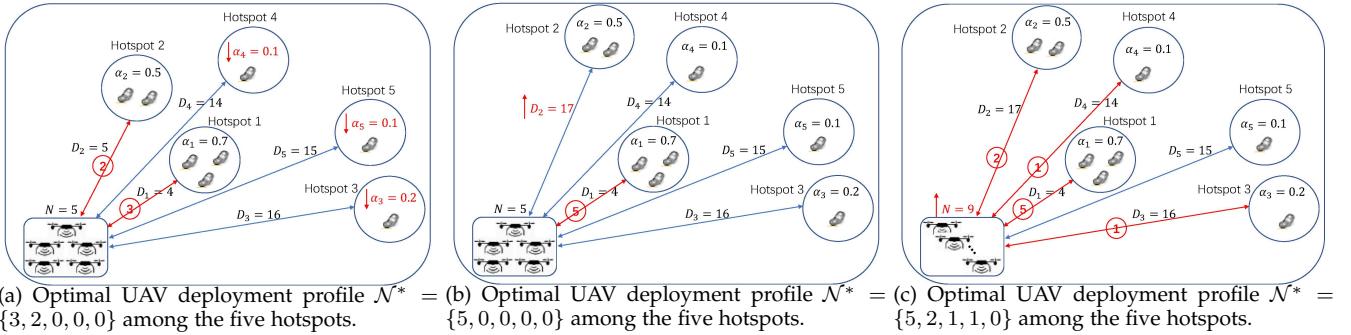


Fig. 9: Illustration of the optimal deployment of multiple UAVs to different hotspots when $B_0 = 20$, $c = 2$. The number in red circle indicates how many UAVs are assigned to the corresponding hotspot.

Algorithm 3 Optimal deployment of N UAVs to M hotspots

```

1: for Any UAV deployment profile  $\mathcal{N}$  do
2:   for  $m = 1$  to  $M$  do
3:     if  $n_m = 0$  then
4:        $R_{k_m}^{\alpha_m} = 0$ 
5:     else
6:       for  $k_m = 1$  to  $\lfloor \frac{B_0 - D_m}{1 + \frac{c}{n_m}} \rfloor$  do
7:         for  $t = 1$  to  $\lfloor B_0 - D_m - \frac{c k_m}{n_m} \rfloor$  do
8:           if  $t \leq k_m - 1$  then
9:              $R_{k_m}^{\alpha_m}(t) = R_t^{\alpha_m}(t)$ 
10:            else
11:              compute  $p_{k_m}^{\alpha_m}(t)$  as the unique solution to
12:                (11);
13:              update  $R_{k_m}^{\alpha_m}(t)$  according to  $p_{k_m}^{\alpha_m}(t)$  and
14:                (10);
15:            end if
16:          end for
17:          return  $R_{k_m}^{\alpha_m}(\lfloor B_0 - D_m - \frac{c k_m}{n_m} \rfloor)$ 
18:        end for
19:      end if
20:    end for
21:  end for
22: return  $\mathcal{N}^* = \arg \max_{\mathcal{N}} \Phi(\mathcal{N})$ 
```

Here we assume $\bar{n}_1 \geq N$. Otherwise, the UAVs will definitely fork to serve different hotspots.

Proposition 5.1. Given $N \geq 2$ UAVs for $M \geq 2$ hotspots, the UAVs will fork in their deployment to serve different hotspots rather than all centering at the first-best hotspot 1 if

$$\frac{\alpha'_2}{\alpha'_1} > \max(\varphi^{\frac{1}{k_2^*}}, \varphi), \quad (30)$$

where φ is given in (31) and k_2^* is given in (25). The forking condition in (30) is more likely to hold for a smaller flying distance D_2 to hotspot 2 or a larger user occurrence rate α'_2 .

The proof of Proposition 5.1 is given in Appendix H of the supplemental material.

According to Proposition 5.1, we can see that the UAVs are more likely to fork to serve hotspot 2 (and others) if the latter has a similar user density α'_2 and flying distance D_2 as hotspot 1.

As a numerical example, we apply Algorithm 3 to consider $N = 5$ UAVs to be deployed to $M = 5$ hotspots in Fig. 2, Fig. 9(a), and Fig. 9(b) under the same setup and show how the user occurrence rate and traveling distance of the hotspots affect the optimal UAV deployment. Later in Fig. 9(c), we will increase the number of UAVs from 5 to 9 to show the impact of the number of UAVs on the forking deployment. By considering the exponential distribution of users' valuations with $\lambda = 1$ and $\bar{n}_m \geq N, m \in \{1, \dots, M\}$, we observe that

- In Fig. 2, it is optimal to deploy 5 UAVs to all 5 hotspots. As the user occurrence rates of hotspots 3, 4, 5 decrease from Fig. 2 to Fig. 9(a), we will not serve these hotspots but deploy 3 UAVs to hotspot 1 and 2 UAVs to hotspot 2. Now hotspot 1 (2) is the first (second) best.
- As the flying distance to hotspot 2 increases from Fig. 9(a)'s $D_2 = 5$ to Fig. 9(b)'s $D_2 = 17$, all the UAVs will stop forking and only serve hotspot 1 without considering hotspot 2. This is consistent with Proposition 5.1.
- Finally, Fig. 9(c) shows that as the number of UAVs increases from Fig. 9(b), the UAVs fork to serve different hotspots again. This is because when many cooperative UAVs center at the same hotspot, they waste a lot of energy in hovering for the same group of users as shown in (27). Therefore, it is better for some UAVs to fork to serve different hotspots (though distant) and meet more demands.

We wonder if the first-best hotspot should always be allocated most UAVs, and the following simulation result shows not.

Example 5.1. Consider $N = 5$ UAVs to be deployed to $M = 2$ hotspots with $\alpha'_1 = \alpha'_2 = 0.5, D_1 = 4, D_2 = 4.5$,

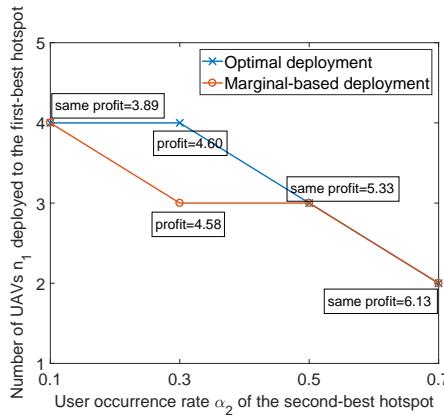


Fig. 10: Comparison between marginal-based UAV deployment and optimal UAV deployment when $N = 5$ UAVs are deployed to $M = 2$ hotspots with $\alpha_1 = 0.6$, $D_1 = 1$, $D_2 = 4.5$. After deploying n_1 UAVs to the first-best hotspot, the rest UAVs $N - n_1$ are deployed to the second-best hotspot.

$B_0 = 20$, $c = 2$, $\lambda = 1$. According to Algorithm 3, the optimal UAV deployment profile is $\mathcal{N}^* = \{2, 3\}$ by allocating 2 UAVs to hotspot 1 (the first-best) and 3 to hotspot 2. We note that this UAV deployment profile is counterintuitive as more UAVs are deployed to the second-best hotspot. Actually, due to the concavity of the expected profit $R_k(T)$ in both service capacity k and hovering time T (see Proposition 3.3), the increased profit that a hotspot receives from hovering one additional unit of time decreases in the hovering time T given the service capacity k . Here, given 2 UAVs deployed to each hotspot, when deciding the last UAV's location, we should note that the hovering time of the second-best hotspot may be less than that of the first-best hotspot due to longer flying distance. Therefore, the increased profit of the second-best hotspot when deploying one more UAV to it could be larger than that of the first-best hotspot and thus more UAVs should be deployed to the second-best hotspot to maximize the overall profit.

5.2.1 Marginal-based Deployment Scheme for Multiple UAVs

Note that the computation complexity of Algorithm 3 is $O(N^M M \max_{m \in \{1, \dots, M\}} (B_0 - D_m)^2)$, which is formidably high by increasing exponentially in M . Thus, we propose a marginal-based deployment algorithm as shown in Algorithm 4 with low computation complexity $O(NM \max_{m \in \{1, \dots, M\}} (B_0 - D_m)^2)$, which is linearly increasing in both N and M and significantly reduces the computation complexity. In Algorithm 4, start from the initial deployment $n_1 = 1, n_2 = \dots = n_M = 0$ with one UAV deployed to the first-best hotspot 1, each time we deploy one UAV to the hotspot with the largest marginal profit, which is the increased profit $DR_{n_m}^{\alpha_m}$ that hotspot m receives when the number of UAVs deployed to it increases from $n_m - 1$ to n_m , until $\sum_{i=1}^M n_i = N$. Then, the marginal-based UAV deployment $\mathcal{N}_m = \{n_1, \dots, n_M\}$ can be obtained.

In Fig. 10, we compare the optimal UAV deployment (Algorithm 3) and marginal-based UAV deployment (Algorithm 4) in terms of deployment profiles to two hotspots as well as their achieved profit gap. By increasing the user occurrence rate α_2 of the second-best hotspot 2, the number

Algorithm 4 Marginal-based deployment of N UAVs to M hotspots

```

1: for  $m = 1$  to  $M$  do
2:   for  $n_m = 1$  to  $\bar{n}_m$  do
3:     for  $k_m = 1$  to  $\lfloor \frac{B_0 - D_m}{1 + \frac{c}{n_m}} \rfloor$  do
4:       for  $t = 1$  to  $\lfloor B_0 - D_m - \frac{ck_m}{n_m} \rfloor$  do
5:         if  $t \leq k_m - 1$  then
6:            $R_{k_m}^{\alpha_m}(t) = R_t^{\alpha_m}(t)$ 
7:         else
8:           compute  $p_{k_m}^{\alpha_m}(t)$  as the unique solution to
9:             (11);
10:          update  $R_{k_m}^{\alpha_m}(t)$  according to  $p_{k_m}^{\alpha_m}(t)$  and (10);
11:        end if
12:      end for
13:      return  $R_{k_m}^{\alpha_m}(\lfloor B_0 - D_m - \frac{ck_m}{n_m} \rfloor)$ 
14:    end for
15:    return  $\max_{k_m} R_{k_m}^{\alpha_m}(\lfloor B_0 - D_m - \frac{ck_m}{n_m} \rfloor)$ 
16:  end for
17: for  $m = 1$  to  $M$  do
18:    $DR_{n_m=1}^{\alpha_m} = \max_{k_m} R_{k_m}^{\alpha_m}(\lfloor B_0 - D_m - ck_m \rfloor)$ 
19:   for  $n_m = 2$  to  $\bar{n}_m$  do
20:      $DR_{n_m}^{\alpha_m} = \max_{k_m} R_{k_m}^{\alpha_m}(\lfloor B_0 - D_m - \frac{ck_m}{n_m} \rfloor) -$ 
21:        $\max_{k_m} R_{k_m}^{\alpha_m}(\lfloor B_0 - D_m - \frac{ck_m}{n_m-1} \rfloor)$ 
22:   end for
23:   for  $n_m = \bar{n}_m + 1$  to  $N$  do
24:      $DR_{n_m}^{\alpha_m} = 0$ 
25:   end for
26: end for
27: while  $\sum_{i=1}^M n_i < N$  do
28:   for  $m = 1$  to  $M$  do
29:      $DR(m) = DR_{n_m+1}^{\alpha_m}$ 
30:   end for
31:    $I = \arg \max_m DR(m)$ 
32:    $n_I = n_I + 1$ 
33: end while
34: return  $\mathcal{N}_m = \{n_1, \dots, n_M\}$ 

```

of UAVs deployed to the first-best hotspot 1 decreases and the total profit of serving these two hotspots increases. When $\alpha_2 = 0.1, 0.5, 0.7$, the UAV deployments obtained from the marginal-based algorithm are exactly the optimal UAV deployments. When $\alpha_2 = 0.3$, the optimal UAV deployment is $\mathcal{N}^* = \{4, 1\}$ with total profit 4.60 according to Algorithm 3. Differently, the UAV deployment obtained according to the marginal-based UAV deployment scheme in Algorithm 4 is $\mathcal{N}_m = \{3, 2\}$ with total profit 4.58, which is just a little smaller than the optimal profit. Actually, in our most simulation results, the performance gap between Algorithms 3 and 4 is mild.

6 CONCLUSION

In this paper, we first analyze the UAV's optimal pricing strategy under complete user information that a user's service valuation can be observed as he arrives. Following this benchmark, we propose a dynamic pricing scheme under incomplete information including random user arrivals and

unknown service valuations. It is shown that the UAV should ask for a higher price if the leftover hovering time is longer or its service capacity is smaller, and its expected profit approaches to that under complete user information if the hovering time is sufficiently large. Then, given a hotspot, the energy allocation to hovering time and service capacity is optimized. We show that, given the energy budget at the hotspot, a shorter hovering time or a larger service capacity should be allocated as the hotspot's user occurrence rate increases. For the UAV deployment, we prove that it is optimal for a single UAV to only serve the best hotspot. While for multiple UAVs, they prefer to fork to serve different hotspots when hotspots are more symmetric or the UAV number is large.

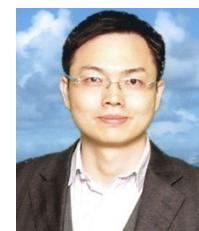
There are some possible directions to study in the future. For example, now we plan the UAVs' dynamic pricing and energy allocation strategies beforehand at the UAV station, which saves the implementation complexity for UAVs to decide and operate in real time. Yet given more artificial intelligence, the UAVs in the future may learn and adapt prices to realized user occurrence over time and decide to hover longer or not. Another future direction is to decide the UPS provision under minimum service requirements (e.g., minimum number of hotspots or demands to serve) for fairness concern besides profitability objective.

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