The Cumulative Distribution Function (CDF) - Lab

Introduction

In the previous lesson we saw how we can use a discrete random variable used for modeling fair die having a uniform probabilities for all possible values. In this lab, we shall try to calculate a cdf for this variable and visualize it for inspection.

Objectives

You will be able to:

- Calculate the cdf for a given discrete random variable
- Visualize the cdf using matplotlib

Calculating CDF in python

Recall the formula for calculate the cumulative probability from previous lesson:

$$F(x) = P(X \leq x)$$

So given a list of all possible values of x, We can easily calculate the cdf for a given possible value (X) by performing following steps:

- Build a function `calculate_cdf(lst,X)`, where `lst` is a list of all possible values in a discrete variable x (6 values for a die roll), and X is the value for which we want to calculate the cumulative distribution function.
- Initialize a count variable
- for all values in `lst`, if a value is less than X, add one to count - do nothing otherwise. (this will tell us total number of values less than X)
- Calculate the cumulative probability of X dividing the count with total possible values
- Round off by 3 decimals and return the cumulative probability of X.

In [1]:
```
def calculate_cdf(lst, X):
    count = 0
    for value in lst:
        if value <= X:
            count += 1

    cum_prob = count / len(lst) # normalizing cumulative probabilities (as with pmfs)
    return round(cum_prob, 3)

# test data
test_lst = [1,2,3]
test_X = 2

calculate_cdf(test_lst, test_X)
```

Out[1]:
```
0.667
```

Let's now use above function to calculate a cdf for each value in a die roll with an intention of plotting it.

Perform following steps in the cell below:

- Create a list `die_lst` with all possible values of a fair die
- Initialize an empty list `die_cum` for storing cumulative probabilities for these values.
- For each value in the `die_lst` calculate its cumulative probability using the function above and store in `die_cum` list.

In [2]:
```
die_lst = [1,2,3,4,5,6]
die_cum = []
for X in die_lst:
    die_cum.append(calculate_cdf(die_lst, X))
die_cum
```

Out[2]:
```
[0.167, 0.333, 0.5, 0.667, 0.833, 1.0]
```

> cdfs are implemented with two sorted lists: xs, which contains the values, and ps, which contains the cumulative probabilities for xs.

Following this, we now have a list of possible values, and a second list containing cumulative probabilities for each value. Let's go ahead and plot these values in matplotlib using the stem plot.

- Use `die_lst` for x-axis and `die_cum` for y-axis

In [3]:
```
import matplotlib.pyplot as plt
plt.style.use('ggplot')

plt.stem(die_lst, die_cum, '-.', 'ro', 'g-');
plt.title ("Die Roll -  Cumulative Distribution Function");
plt.xlabel('Die values')
plt.ylabel('Cumulative Probabilities')
```

Out[3]:
```
Text(0,0.5,'Cumulative Probabilities')
```

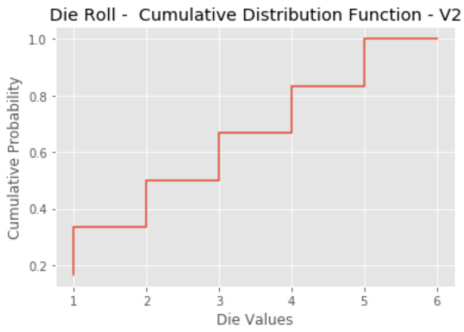
Level Up (optional)

cdfs (and pmfs) can be calculated using built in numpy and matplotlib methods. So we don't have create custom functions to calculate these. We can draw a histogram styled cdf as shown below using following methods.

You would need to perform these steps

- Use `np.histogram()` to automatically calculate the histogram with probabilities. Here is numpy histogram documentation to help you dig deeper.
- Use `plt.step()` method with `np.cumsum()` to calculate and plot cumulative probabilities (just like we did above).

In [4]:
```
import numpy as np
sample = [1,2,3,4,5,6]
hist = np.histogram(sample, bins=6, range=(1, 7), normed=True)
plt.step(hist[1][:-1], np.cumsum(hist[0]))
plt.title ("Die Roll -  Cumulative Distribution Function - V2");
plt.xlabel("Die Values")
plt.ylabel("Cumulative Probability")
plt.show()
```



Summary

In this lesson we looked at developing a cdf a percentile function of a discrete random variable. We looked at how to calculate and visualize a cdf. This technique can also be applied to continuous random variables which we shall see later in this section.