Covariance and Correlation - Lab

Introduction

In this lab, we shall working towards calculating covariance and correlation for a given dataset in python. We shall use the formulas shown in previous lesson and verify our results with python libraries.

Objectives

You will be able to:

- Calculate and and interpret correlation and covariance for given variables
- Build density and scatter plots to visually identify the level of dependence between variables
- Perform covariance and correlation using python and numpy

Dataset

Included dataset (heightWeight.csv) includes 20 heights (inches) and weights(pounds). Yes, it is a particularly small dataset and will help us focus more on seeing covariance and correlation in action. At this point, you should be able to calculate the average height and average weight. You can also explain the medians, variances and standard deviations for this dataset.

But all of those measurements are only concerned with a **single variable**. What if we want to see:

How height interacts with weight ?

Does weight increase as height increases ?

Are Weight and Height not related at all ?

Note while there are plenty of fat short people and overly skinny tall people, but when you look at the population at large, taller people will tend to weigh more than shorter people. This generalization of information is very common as it shows you a bigger picture that you can build your intuitions upon.

Let's first load this dataset into pandas. Read the file "heightWeight.csv" and for header, length of the records and basic stats.

In [16]:

```
# Load the dataset into pandas and perform basic inspection

import pandas as pd
data =pd.read_csv('heightWeight.csv')

print (len(data))

print(data.head())

print (data.describe())
```

```
20
   height  Weight
0      68     165
1      71     201
2      61     140
3      69     170
4      71     192
          height      Weight
count  20.000000   20.000000
mean   66.850000  165.800000
std     5.112163   28.971129
min    58.000000  115.000000
25%    63.250000  143.750000
50%    68.500000  170.000000
75%    71.000000  192.750000
max    74.000000  210.000000
```

Calculate covariance

Here's the covariance formula once again.

$$COV(x,y) = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{n-1}$$

We would use (n-1) due to the fact that we are working with samples of a bigger population here.

Mean normalization

But before we do this, we have to ensure the that both variables are **Mean Normalized** (as shown in the numerator above). i.e. both variables have mean values = 0 . This allows us to calculate how much they vary while disregarding their distance from each other. A bit like standardization that we saw before, but here we are not standardizing the spread (standard deviation), as that is what needs to be studied. So the formula to mean normalize a data set is :

> **xi - X(mean)**

Pretty simple, take each element of the variable and subtract the mean value from it. This will create a new "mean-normalized" dataset. Let's write a function that takes in a vector, calculates the mean of vector and subtracts the calculated mean value from each element to calculate xi - X(mean).

Hint: use `np.mean()` to calculate the mean for above formula

In [5]:

```
import numpy as np

# Write a function to take in an iterable, calculate the mean and subtract the mean value
# from each element , creating and returning a new list.

def mean_normalize(var):

    norm = [] # Vector for storing output values
    n = 0     # a counter to identify the position of next element in vector
    mean = np.mean(var)

    # for each element in the vector, subtract from mean and add the result to norm
    for i in var:
        diff = var[n] - mean
        norm.append(diff)
        n = n + 1

    return norm

mean_normalize([1,2,3,4,5]), mean_normalize([11,22,33,44,55])

# ([-2.0, -1.0, 0.0, 1.0, 2.0], [-22.0, -11.0, 0.0, 11.0, 22.0])
```

Out[5]:

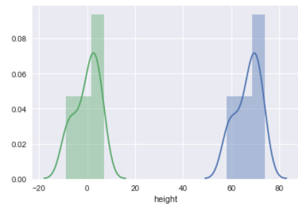
```
([-2.0, -1.0, 0.0, 1.0, 2.0], [-22.0, -11.0, 0.0, 11.0, 22.0])
```

Great so you see, our function maintains the variance of list elements and moves their mean to zero. As a quick test, we can visualize what exactly happens to the data with mean normalization. Plot the height variable distribution before and after the normalization process.

In [43]:
```
# Visualize the height data distribution before and after mean normalization

height = mean_normalize(data.height)
import seaborn as sns
sns.distplot(data.height)
sns.distplot(height)
```

Out[43]: <matplotlib.axes._subplots.AxesSubplot at 0x1a1b9ee668>



So there you go, not much changes in the shape of the data. Try repeating above with weight.

The dot product

So now that we have our new normalized datasets. According to the numerator in the formula,we have to take the **DOT PRODUCT** of these two vector values. A dot product let's us apply the directional growth of one vector to another. Dot products are very important in vector calculus for a number of applications. [Here is a great article explaining this in detail.](#)

For two vectors a and b, a dot product is calculated by multiplying each element of one vector to its counterpart in the second , and then adding them up together.

```
a[0] * b[0] + a[1] * b[1] + a[2] * b[2] ...
```

So lets write a function that will take two iterables and return their dot product.

In [44]:
```
# Write a function to calculate the dot product of two iterables

def dot_product(x,y):
    n = 0  # a counter pointing to the current element of vector(s)
    prod_vec = [] # Initliaze an empty list to store the results

    # For all elements in the vectors, multiply and save results in prod_vec
    for i in range(len(x)):
        prod = x[i]* y[i]
        prod_vec.append(prod)
        n += 1

    dot_prod = np.sum(prod_vec)
    return dot_prod

a = [1,2,3]
b = [4,5,6]

dot_product(a,b)

#  32  calculated as (1*4 + 2*5 + 3*6)
```

Out[44]: 32

So we have the numerator of the formula sorted out. Let's finally write a function `covariance()` that will take height and weight lists we created earlier and return the covariance value using the functions we created earlier.

In [30]:
```
# Calculate covariance using functions above

def covariance(var1, var2):

    # Formula for covariance is:
    # [Sum (x_i - X)(y_i - Y)] / N-1

    # Sanity Check : Check to see if both vectors are of same length
    # Exit the function if variables have different lengths

    if len(var1) != len(var2):
        return None
    else:

        # Mean normalize both variables
        x = mean_normalize(var1)
        y = mean_normalize(var2)

        # Take the dot product of mean normalized variables
        result = dot_product(x,y)

        # divide the dot product by n-1
        return result /((len(var1)) -1)

covariance(data['height'], data['Weight'])

# 144.75789473684208
```

Out[30]: 144.75789473684208

Let's verify our results with pandas built in `dataFrame.cov()` method.

In [31]:
```
data.cov()
```

Out[31]:

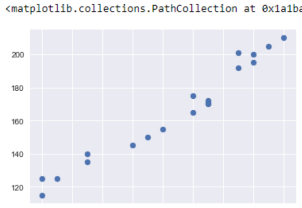
	height	Weight
height	26.134211	144.757895
Weight	144.757895	839.326316

Okay so covariance (as well as correlation) are usually shown in matrix form. the covariance between height and weight is exactly what we calculated. the matrix also shows the covariance of a variable with itself. So this gives us magnitude which is a bit hard to interpret. How about we visualize height and weight on a scatter plot !

In [45]:
```
# Plot a scatter graph between height and weight to visually inspect the relationship

import matplotlib.pyplot as plt
plt.scatter(data.height, data.Weight)
```

Out[45]: <matplotlib.collections.PathCollection at 0x1a1ba379e8>



So we can see there is quite a bit of positive relationship between the two, but a covariance value is a bit hard to interpret. So let's try calculating correlation.

Calculate Correlation

Once again, heres the formula to calculate the correlation.

$$\sum (x-\bar{x})(y-\bar{y})$$

$$r = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}}$$

lots of mean normalizations going on here. It shouldn't be too hard now to implement this using our functions above.

In [46]:

```
# Calculate Correlation between two variables using formula above
import math
def correlation(var1,var2):

    if len(var1) != len(var2):
        return None
    else:

        mean_norm_var1 = mean_normalize(var1)
        mean_norm_var2 = mean_normalize(var2)

        # Try the numpy way for calculating doc product
        var1_dot_var2 = [a * b for a, b in list(zip(mean_norm_var1, mean_norm_var2))]

        var1_squared = [i * i for i in mean_norm_var1]
        var2_squared = [i * i for i in mean_norm_var2]

        return np.round(sum(var1_dot_var2) / math.sqrt(sum(var1_squared) * sum(var2_squared)), 2)

correlation(data['height'], data['Weight'])
```

Out[46]: 0.98

Wow, 0.98, thats very close to one. So that means height and weight are like TOTALLY dependent on each other. Well, only for this particular sample. And there is a takeaway in this. sample size plays a major rule in determining the nature of a variable and its relationship with other variables. the set of 20 records we have seem to correlate highly, but this might be different for a different set of samples. We shall talk about how to further test such a finding to either reject it , or confirm it as a FACT.

As a last check , let's use pandas `dataframe.corr()` method to see how that works.

In [42]:

```
data.corr()
```

Out[42]:

	height	Weight
height	1.0000	0.9774
Weight	0.9774	1.0000

Another matrix similar to above. And we see that a correlation of a variable to itself will always be = 1. The correlation between height and weight can be rounded off to our results. That is great. Now we know how this works.

Summary

In this lab we saw how to calculate the covariance and correlation between variables. We also looked at mean normalization and dot products which will be revisited later in the course. Finally we saw how to calculate these measures using pandas built in methods.