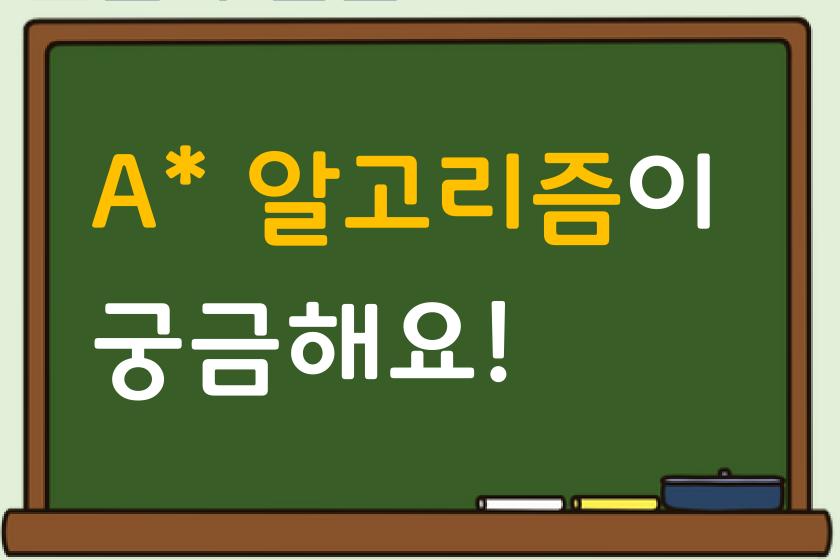


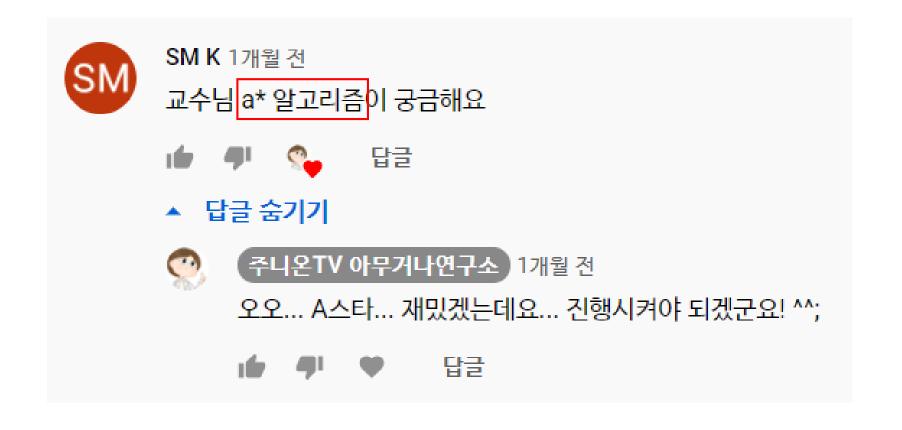


오늘의 질문:





■오늘의 질문:





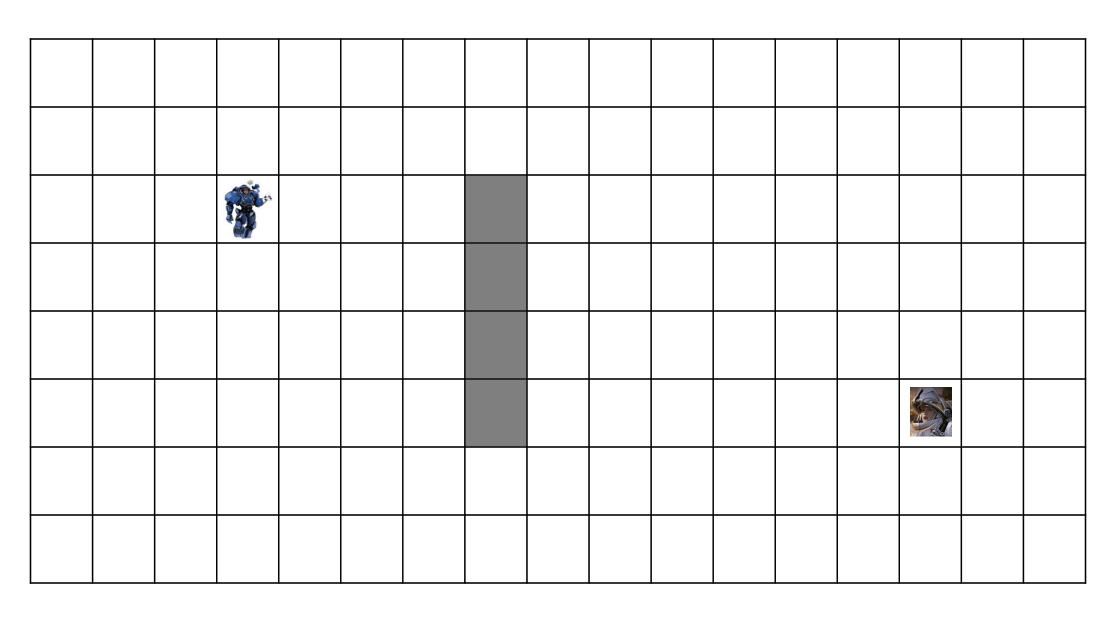
■ 최단 경로 문제와 A* 알고리즘:





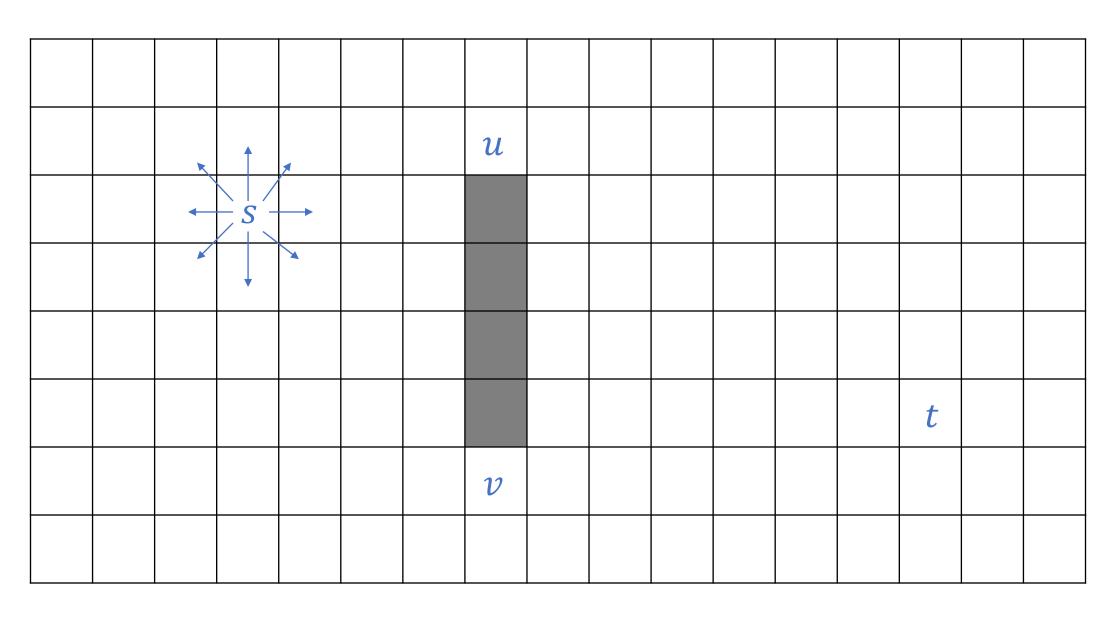




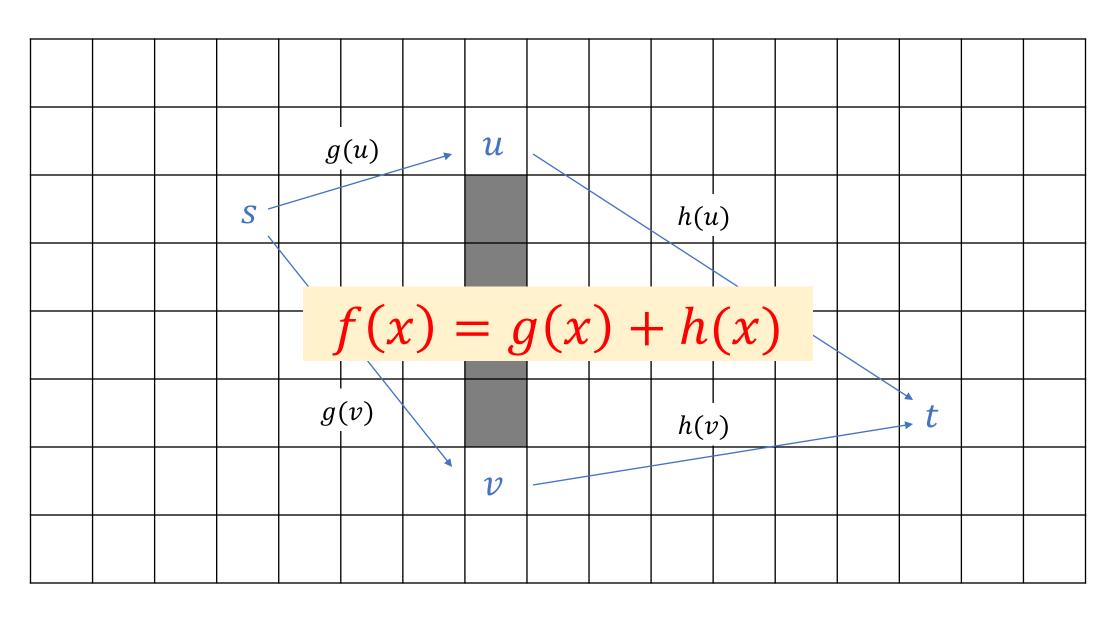










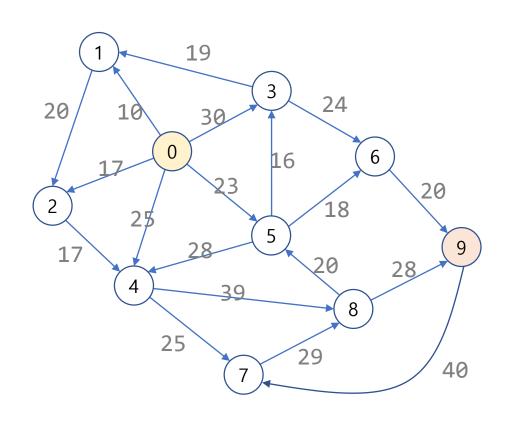




- 최단 경로 알고리즘:
 - 플로이드 알고리즘: Dynamic Programming, $O(n^3)$
 - 모든 쌍(모든 정점에서 다른 모든 정점으로)의 최단 경로 구하기
 - 다익스트라 알고리즘: $Greedy\ Approach,\ O(n^2)$
 - 단일 출발점에서 다른 모든 정점으로의 최단 경로 구하기
 - A* 알고리즘: Heuristic Algorithm
 - 출발점과 도착점이 모두 주어졌을 때 최단 경로 구하기



• 예제: 가중치 있는 방향 그래프 G = (V, E)





■ 그래프 G = (V, E)로부터 인접 행렬 W 만들기:

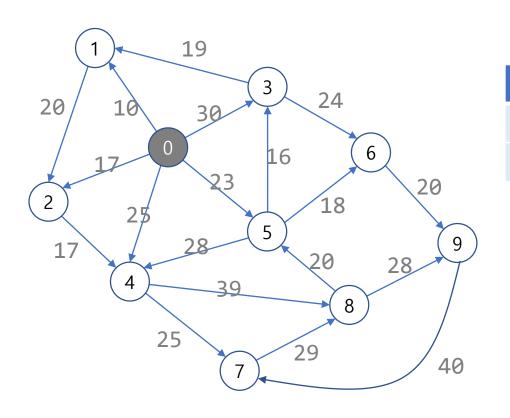
```
INF = 0 \times FFFF
n, m = map(int, input().split())
W = [[0 if i == j else INF for i in range(n)] for j in range(n)]
for _ in range(m):
   u, v, w = map(int, input().split())
    W[u][v] = w
s, t = 0, 9
bypass = dijkstra(s, n, W)
path = shortest(s, t, bypass)
print(bypass)
print(path)
```



- 다익스트라 알고리즘: 출발 정점 S가 주어질 때.
 - length[v]: s로부터 v에 도달하는 최단 경로 길이
 - bypass[v]: S로부터 v에 도달하기 위해 거쳐가는 정점

```
def dijkstra(s, n, W):
    bypass = [s for _ in range(n)]
    length = [W[s][i] for i in range(n)]
    for _ in range(n - 1):
        vnear = nearest(s, length)
        for v in range(n):
            if length[v] > length[vnear] + W[vnear][v]:
                length[v] = length[vnear] + W[vnear][v]
                bypass[v] = vnear
        length[vnear] = -1
    return bypass
```

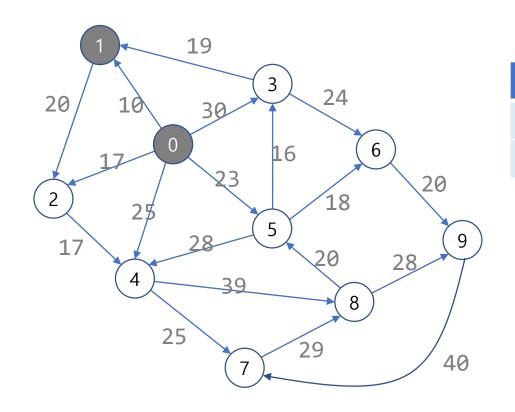




$$V = \{0\}$$

V	0	1	2	3	4	5	6	7	8	9
length	0	10	17	30	25	23	∞	∞	∞	∞
bypass	0	0	0	0	0	0	0	0	0	0

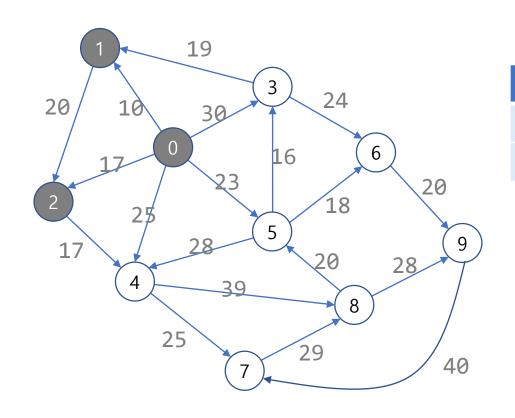
(60)



$$V = \{0, 1\}$$

V	0	1	2	3	4	5	6	7	8	9
length	0	-1	17	30	25	23	∞	∞	∞	∞
bypass	0	0	0	0	0	0	0	0	0	0



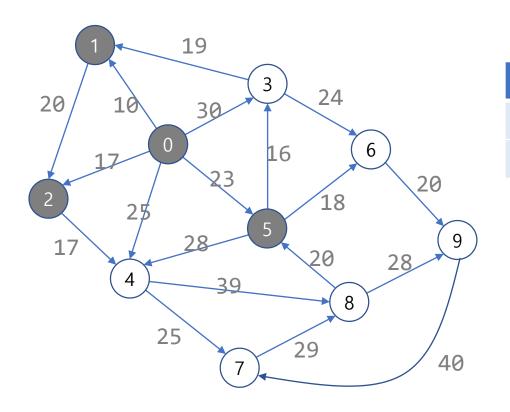


$$V = \{0, 1, 2\}$$

V	0	1	2	3	4	5	6	7	8	9
length	0	-1	-1	30	25	23	∞	∞	∞	∞
bypass	0	0	0	0	0	0	0	0	0	0

14



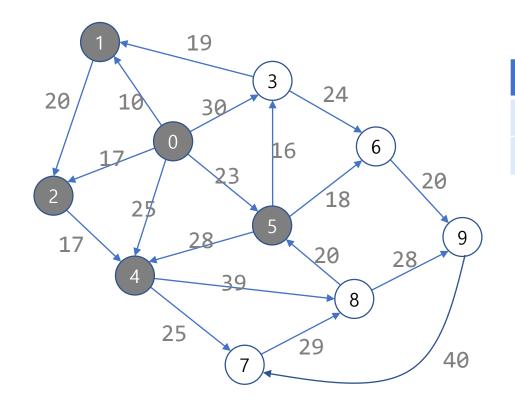


$$V = \{0, 1, 2, 5\}$$

V	0	1	2	3	4	5	6	7	8	9
length	0	-1	-1	30	25	-1	41	∞	∞	∞
bypass	0	0	0	0	0	0	5	0	0	0

15



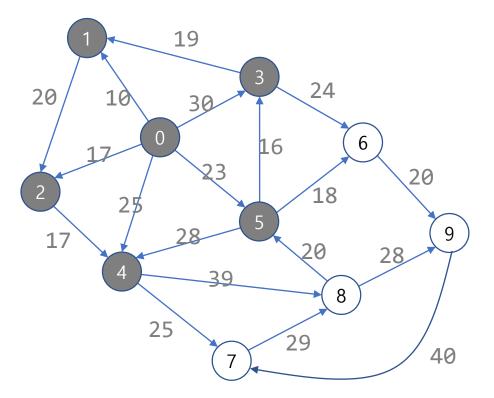


$$V = \{0, 1, 2, 5, 4\}$$

V	0	1	2	3	4	5	6	7	8	9
length	0	-1	-1	30	-1	-1	41	50	64	∞
bypass	0	0	0	0	0	0	5	4	4	0

16

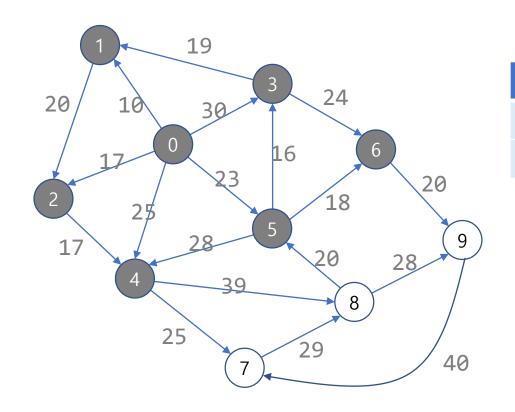




$$V = \{0, 1, 2, 5, 4, 3\}$$

V	0	1	2	3	4	5	6	7	8	9
length	0	-1	-1	-1	-1	-1	41	50	64	∞
bypass	0	0	0	0	0	0	5	4	4	0

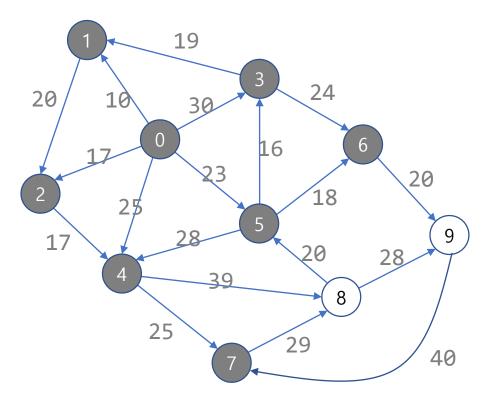




$$V = \{0, 1, 2, 5, 4, 3, 6\}$$

V	0	1	2	3	4	5	6	7	8	9
length	0	-1	-1	-1	-1	-1	-1	50	64	61
bypass	0	0	0	0	0	0	5	4	4	6

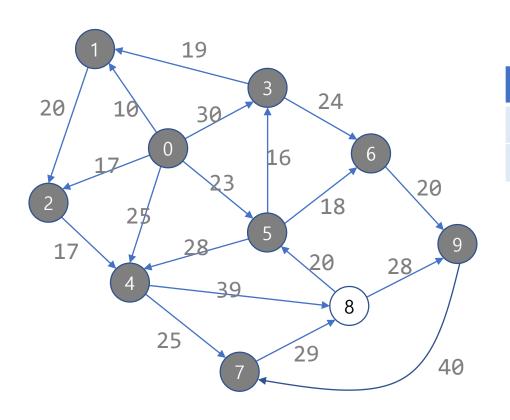




$$V = \{0, 1, 2, 5, 4, 3, 6, 7\}$$

V	0	1	2	3	4	5	6	7	8	9
length	0	-1	-1	-1	-1	-1	-1	-1	64	61
bypass	0	0	0	0	0	0	5	4	4	6

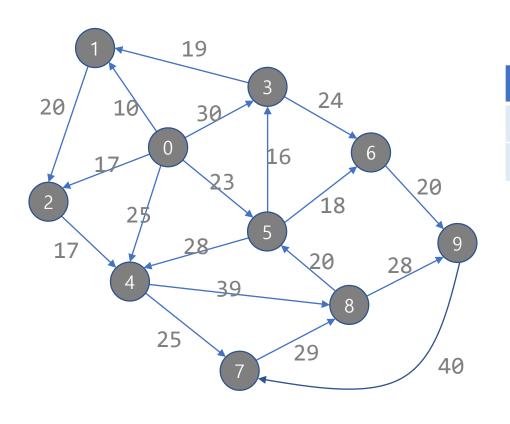




$$V = \{0, 1, 2, 5, 4, 3, 6, 7, 9\}$$

V	0	1	2	3	4	5	6	7	8	9
length	0	-1	-1	-1	-1	-1	-1	-1	64	-1
bypass	0	0	0	0	0	0	5	4	4	6





$$V = \{0, 1, 2, 5, 4, 3, 6, 7, 9, 8\}$$

V	0	1	2	3	4	5	6	7	8	9
length	0	-1	-1	-1	-1	-1	-1	-1	-1	-1
bypass	0	0	0	0	0	0	5	4	4	6

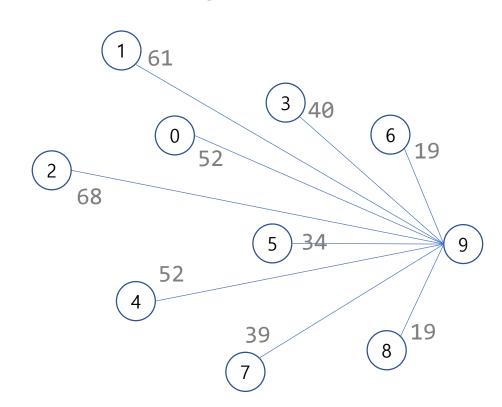


```
def nearest(s, length):
    minimum, vnear = INF, s
    for v in range(len(length)):
        if v == s: continue
        if 0 <= length[v] < minimum:</pre>
            minimum, vnear = length[v], v
    return vnear
                                                                        6
def shortest(s, t, bypass):
                                    length
    if bypass[t] == s:
                                    bypass
                                                                   0
        return [s, t]
    else:
        return shortest(s, bypass[t], bypass) + [t]
```



- lacktriangleA* 알고리즘: 출발 정점 \mathcal{S} 와 도착 정점 \mathcal{L} 가 주어질 때.
 - 휴리스틱: 각 정점 간의 추정 거리를 알 수 있다고 가정함
 - g(v): 출발 정점 s에서 임의의 정점 v에 이르는 최단 거리
 - h(v): 임의의 정점 v에서 도착 정점 t에 이르는 추정 거리
 - f(v) = g(v) + h(v)

```
graph.2.in: // graph.in.in
// .....
9
0 9 52
1 9 61
2 9 68
3 9 40
4 9 52
5 9 34
6 9 19
7 9 39
8 9 19
```





```
INF = 0 \times FFFF
n, m = map(int, input().split())
W = [[0 if i == j else INF for i in range(n)] for j in range(n)]
for _ in range(m):
    u, v, w = map(int, input().split())
    W[u][v] = w
H = [[0 if i == j else INF for i in range(n)] for j in range(n)]
k = int(input())
for _ in range(k):
    u, v, h = map(int, input().split())
    H[u][v] = h
```

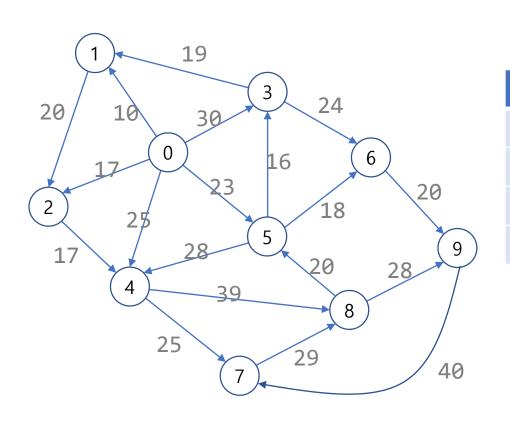




■ A* 알고리즘의 구현:

```
def a_star(s, t, n, W, H):
    bypass = [s for _ in range(n)]
   g = [0 if i == s else INF for i in range(n)]
   h = [H[i][t] for i in range(n)]
    f = [h[s] if i == s else INF for i in range(n)]
   for _ in range(n - 1):
        vnear = nearest(s, f)
        if vnear == t:
            return bypass
        for v in range(n):
            if g[v] > g[vnear] + W[vnear][v]:
                g[v] = g[vnear] + W[vnear][v]
                f[v] = g[v] + h[v]
                bypass[v] = vnear
        f[vnear] = -1
    return bypass
```

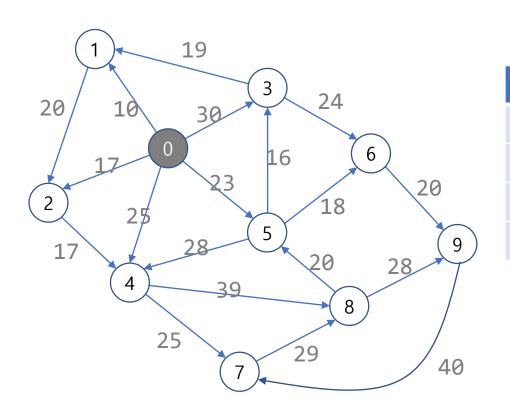




$$V = \{ \}$$

V	0	1	2	3	4	5	6	7	8	9
h	52	61	68	40	52	34	19	39	19	0
g	0	∞								
f	52	∞								
bypass	0	0	0	0	0	0	0	0	0	0

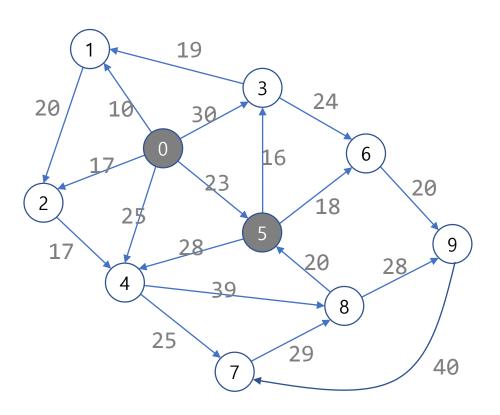




$$V = \{ 0 \}$$

V	0	1	2	3	4	5	6	7	8	9
h	52	61	68	40	52	34	19	39	19	0
g	0	10	17	30	25	23	∞	∞	∞	∞
f	-1	71	85	70	77	57	∞	∞	∞	∞
bypass	0	0	0	0	0	0	0	0	0	0

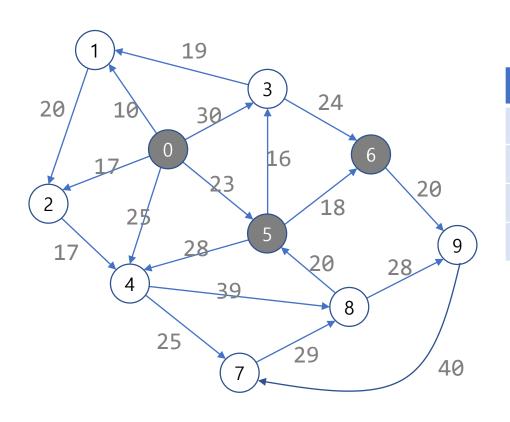




$$V = \{ 0, 5 \}$$

V	0	1	2	3	4	5	6	7	8	9
h	52	61	68	40	52	34	19	39	19	0
g	0	10	17	30	25	23	41	∞	∞	∞
f	-1	71	85	70	77	-1	60	∞	∞	∞
bypass	0	0	0	0	0	0	5	0	0	0

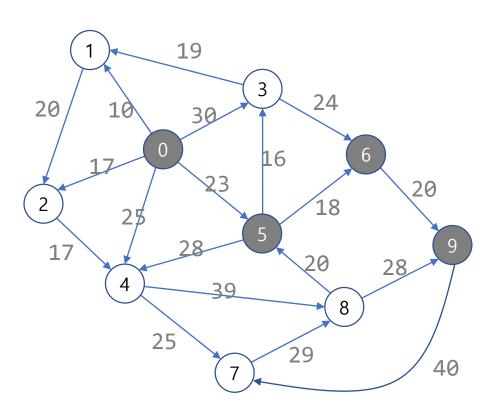




$$V = \{ 0, 5, 6 \}$$

V	0	1	2	3	4	5	6	7	8	9
h	52	61	68	40	52	34	19	39	19	0
g	0	10	17	30	25	23	41	∞	∞	61
f	-1	71	85	70	77	-1	-1	∞	∞	61
bypass	0	0	0	0	0	0	5	0	0	6





$$V = \{0, 5, 6, 9\}$$

V	0	1	2	3	4	5	6	7	8	9
h	52	61	68	40	52	34	19	39	19	0
g	0	10	17	30	25	23	41	∞	∞	61
f	-1	71	85	70	77	-1	-1	∞	∞	-1
bypass	0	0	0	0	0	0	5	0	0	6



```
s, t = 0, 9
bypass = a_star(s, t, n, W, H)
path = shortest(s, t, bypass)
print(bypass)
print(path)
```



■ A* 알고리즘:

- 출발지와 도착지가 주어질 때 최단 경로를 찾는 휴리스틱 알고리즘
- 발견한 최단 경로와 남은 추정 거리의 합이 최소가 되는 경로를 선택
- 남은 추정 거리 계산: 유클리드 거리, 맨하탄 거리, 코사인 유사도 등
- 응용 분야: 컴퓨터 게임, 자동차 네비게이션, 15-퍼즐 문제 등

