Homework 7 (Homework 2 for Spring Semester): ECON512

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Q1. For this question, I completely follow the guideline in the problem set. For this exercise, I start with the following initial guesses.

$$\mathbf{P} = [c(\omega), c(\omega), c(\omega), \ldots, c(\omega)] \in \mathbb{R}^{L \times L}$$
$$\mathbf{V} = \mathbf{0}_{L \times L}.$$

With this guess, I achieve a equilibrium in about 80 seconds with 170 iterations. Figure 1 is displaying the value and policy functions.

Q2. Let $(\hat{p}_1(\omega), \hat{p}_2(\omega))$ be a equilibrium strategy profile given state ω. Let ω' be a future state. Then in this equilibrium, the transition matrix of state ω can be computed by following:

$$\begin{aligned}\pi(\omega'|\omega) =& D_0(\hat{p}_1(\omega), \hat{p}_2(\omega))P(\omega_1'|\omega_1, q_1 = 0)P(\omega_2'|\omega_2, q_2 = 0)\\ &+ D_1(\hat{p}_1(\omega), \hat{p}_2(\omega))P(\omega_1'|\omega_1, q_1 = 1)P(\omega_2'|\omega_2, q_2 = 0)\\ &+ D_2(\hat{p}_1(\omega), \hat{p}_2(\omega))P(\omega_1'|\omega_1, q_1 = 0)P(\omega_2'|\omega_2, q_2 = 1)\end{aligned}$$

Following the above formula, I construct $L^2 \times L^2$ matrix Π. I start with $1 \times L^2$ vector $p = (1, 0, 0, \ldots, 0)$, and compute the distribution of the state through

$$p^k = p\Pi^k,$$

where $k = 10, 20, 30$. With a proper reshaping process, I could plot the distribution on $\{1, 2, \ldots, L\} \times \{1, 2, \ldots, L\}$. Figure 2 is displaying the distributions of the state after 10, 20, and 30 periods later, respectively.

Q3. Upon getting the transition matrix Π, it is straightforward to obtain the stationary distribution. Start with any initial state vector v^1, I could compute the stationary distribution as Π is ergodic Markov transition matrix. The stationary distribution emerging from the equilibrium computed in Q1 is displayed in Figure 3.

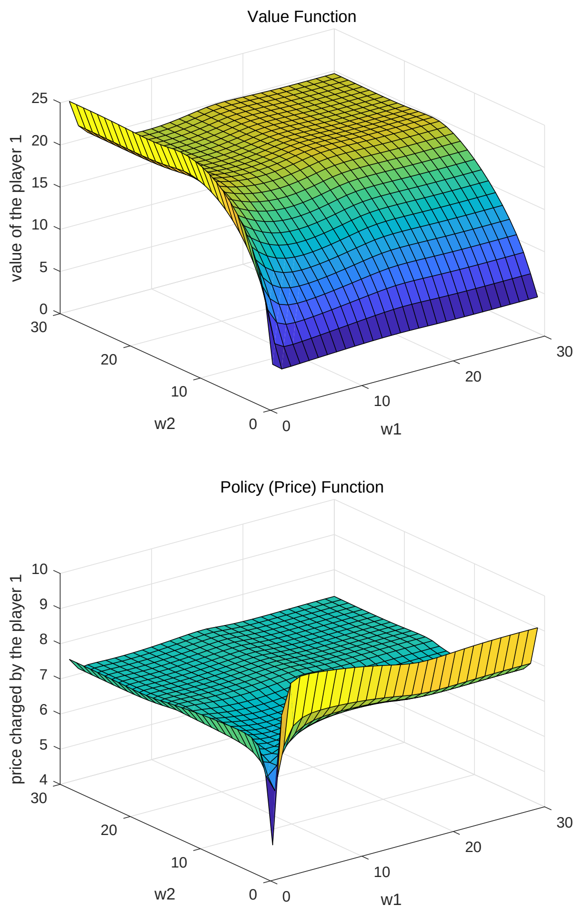

Figure 1: Value and Policy Functions

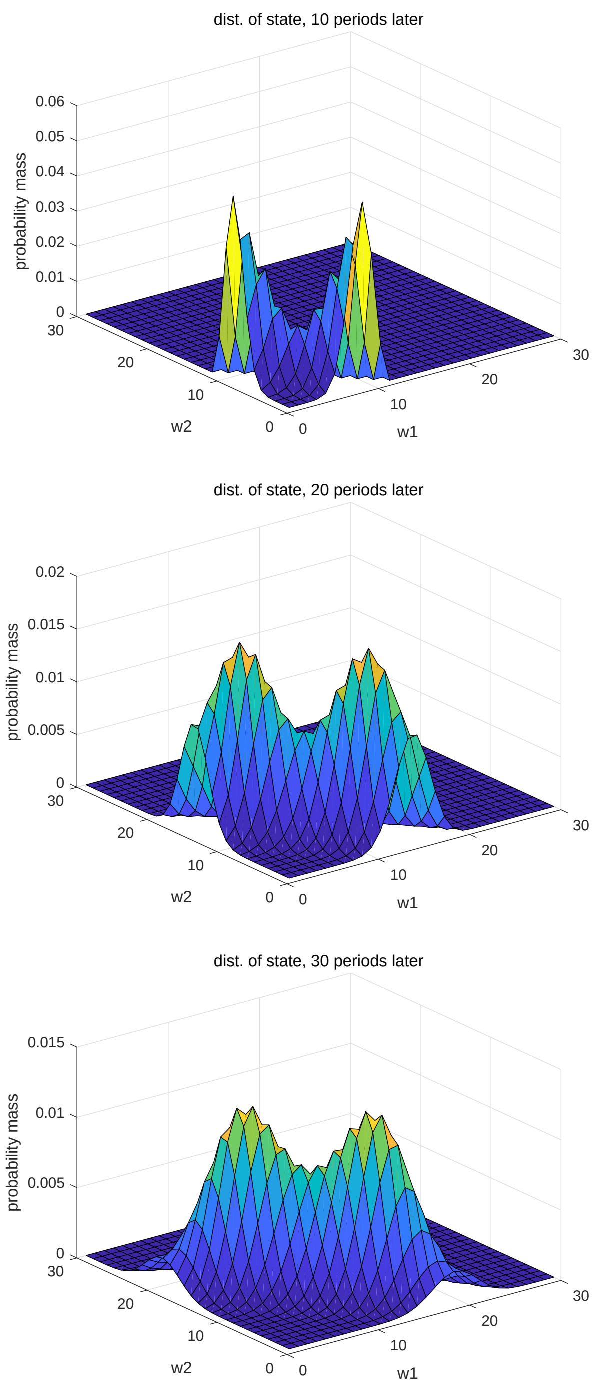

Figure 2: Evolution of Distribution of State

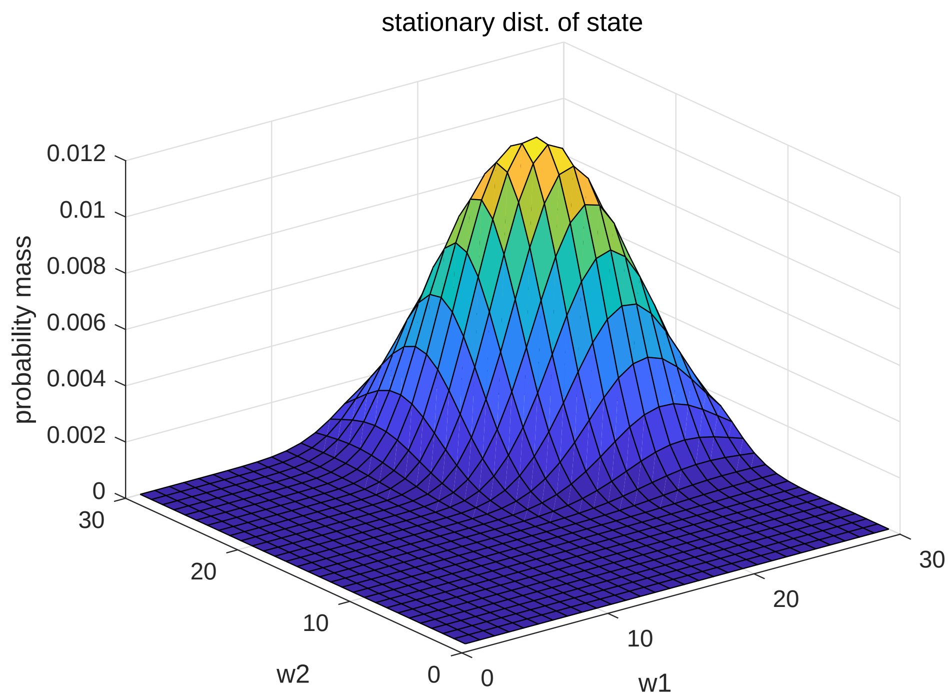

Figure 3: Stationary Distribution of State

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% Code for HW2 in Spring (HW7) ECON512
% Written by Joonkyo Hong
% Build upon codes contained in code_QLD in ECON512 lecture
% Feb 16, 2019

clear;

%% Question 1. Solve for the equilbrium through policy and value function iteration

run prmtr.m
global L eta kappa l nu delta betta lambda eps cost trans0 trans1
iter=0;
diff=10;
maxiter = 5000;

P0=repmat(cost,1,L);                      % competitive price
V0=zeros(L,L);

 tic
 while diff > eps && iter < maxiter

     % Policy function updating

           g = @(x) P_FOC(V0,x);
           P1 = fsolve(g,P0);

      % With updated P1, update the value function

           V1 = updateV(V0, P1, P0);

      % difference is

           diff = max(max(max(abs((V1-V0)./(1+V1)))),  max(max(abs((P1-P0)./(1+P1)))));

      % Dampening

           V0 = (1-lambda)*V0 + lambda*V1;
           P0 = (1-lambda)*P0 + lambda*P1;

      % Go Next Iteration

       iter = iter+1;
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        clc;
        disp('current iteration');
        disp(num2str(iter));
        disp('current difference');
        disp(num2str(diff));


end
toc


figure(1)
surf((1:1:L),(1:1:L),V1);
title("Value Function");

figure(2)
surf((1:1:L),(1:1:L),P1);
title("Policy (Price) Function");


%% Question 2. Compute the distribution of the state as time evolves

   % D0, D1, D2 in the equilibrium computed in Section 1.
   [D0eqm, D1eqm, D2eqm] = computeD(P1,P1');

   D0state = repmat(reshape(D0eqm',L*L,1),1,L*L);
   D1state = repmat(reshape(D1eqm',L*L,1),1,L*L);
   D2state = repmat(reshape(D2eqm',L*L,1),1,L*L);

   % Conditional on each learning status, compute the transition matrix
   State0 = kron(trans0, trans0);
   State1 = kron(trans1, trans0);
   State2 = kron(trans0, trans1);

   % Finally the transition martrix is
   Pi = D0state.*State0 + D1state.*State1 + D2state.*State2;
   Pi = Pi./repmat(sum(Pi,2),1,L*L);     % To rule out the numerical noises making the sum exceed one

   % Compute the distribution of states after 10, 20, and 30 periods

   Pi10 = Pi^10;
   Pi20 = Pi^20;
   Pi30 = Pi^30;
```

```
    start = [1 zeros(1,899)];

    state10 = start*Pi10;
    state10 = reshape(state10,L,L);    % distribution of states, 10 periods later, L by L matrix
    state20 = start*Pi20;
    state20 = reshape(state20,L,L);    % distribution of states, 20 periods later, L by L matrix
    state30 = start*Pi30;
    state30 = reshape(state30,L,L);    % distribution of states, 30 periods later, L by L matrix

    figure(3)
    surf(1:1:L,1:1:L,state10);
    xlabel('w1');
    ylabel('w2');
    zlabel('probability mass');
    title('dist. of state, 10 periods later');

    figure(4)
    surf(1:1:L,1:1:L,state20);
    xlabel('w1');
    ylabel('w2');
    zlabel('probability mass');
    title('dist. of state, 20 periods later');

    figure(5)
    surf(1:1:L,1:1:L,state30);
    xlabel('w1');
    ylabel('w2');
    zlabel('probability mass');
    title('dist. of state, 30 periods later');


%% Question 3. Compute the stationary distribution of the state

   start = [1 zeros(1,899)];

         diff=1;

         while diff>10e-16
               update=start*Pi;
               diff=norm(update-start)/norm(start);
               start=update;
         end
```

```
stationary = reshape(update,L,L);

 figure(6)
 surf(1:1:L,1:1:L,stationary);
 xlabel('w1');
 ylabel('w2');
 zlabel('probability mass');
 title('stationary dist. of state');
```