Servo System Simulation

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SYSC 3600A
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Lab 2 Report

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1 Introduction (GA2.1 Problem Definition)

This laboratory examines the dynamics of a position controller (servo system) for a motor described by a 2nd order differential equation. The problem is to design a controller, analyse each component of the system, the behaviour, the feedback, the parameters, and the damping ratio which will help testing the resulting dynamics of the desired system. Figure 1 below shows a servo system where turning the input dial makes the motor turn the output shaft to the same angular position.

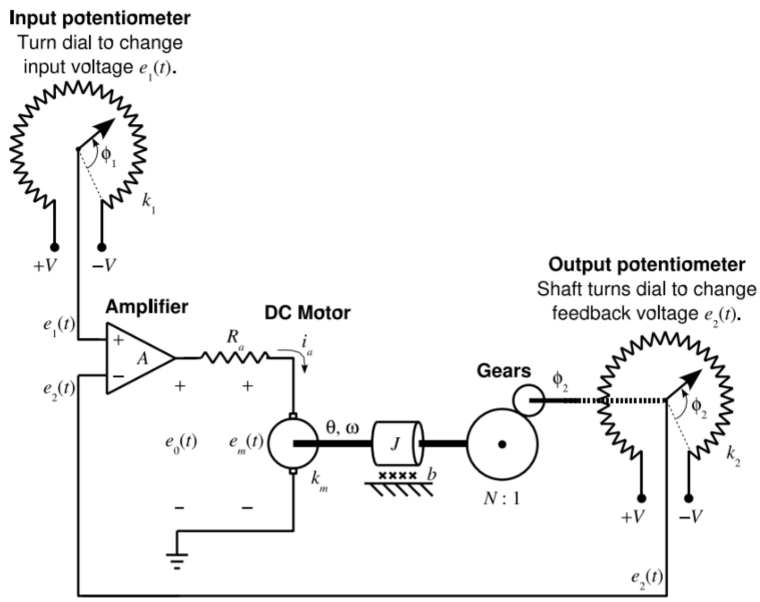

Figure 1 - Servo System with an Input Dial and Output Shaft [1]

This laboratory report consists of two parts. The first part of it will examine the servo system without a rate feedback while the second part of the lab will examine a servo system with a rate feedback.

2 Controller Design (GA4.4 Design Solution)

To achieve an acceptable position controller, several different feedback gains will be implemented such as static gain (k), undamped natural frequency (ω_n), and the damping ratio (ζ). The transfer function of the system, given in the lab manual, is used to demonstrate how the input of a system produces the output with simplistic calculations as compared to the original block diagram. The lab explains the transfer function as follows:

$$H(s) = \frac{k\omega_n^2}{s^2+2\zeta\omega_n+\omega_n^2} \qquad (1)\ [1]$$

Where the following equations were obtained:

$$\omega_n = \sqrt{\frac{A*B*km}{N}} \qquad k = \frac{k1}{k2} = 1 \tag{2}$$

$$\zeta_1 = \frac{C}{2*\omega_n} \qquad \zeta_2 = \frac{C+A*B*Kr}{2*\omega_n} \tag{3}$$

where ζ_1 is the damping ratio without the rate feedback loop and ζ_2 is the damping ratio with the rate feedback loop included.

The undamped natural frequency (ω_n) and damping ratio can also be used to solve for the damped natural frequency, ω_d, using the following equation:

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \text{ rad/s} \qquad (4)\ [1]$$

The frequency of the damped oscillation in step response can be obtained when w_d is a real number. If w_d is a complex number, the system is overdamped and does not exhibit any oscillation. Also, the damping ratio is used to determine whether the system is overdamped, critically damped, underdamped, or undamped, using the following ranges:

$\zeta > 1$: Overdamped; $\zeta = 1$: Critically damped; $\zeta < 1$: Underdamped; $\zeta = 0$: Undamped (5) [1]

2.1 Block Diagram of Controller (GA5.1 Diagrams and Engineering Sketches)

Figure 2 below illustrates the block diagram of the controller. One can see that the rate and position feedback signal is not provided.

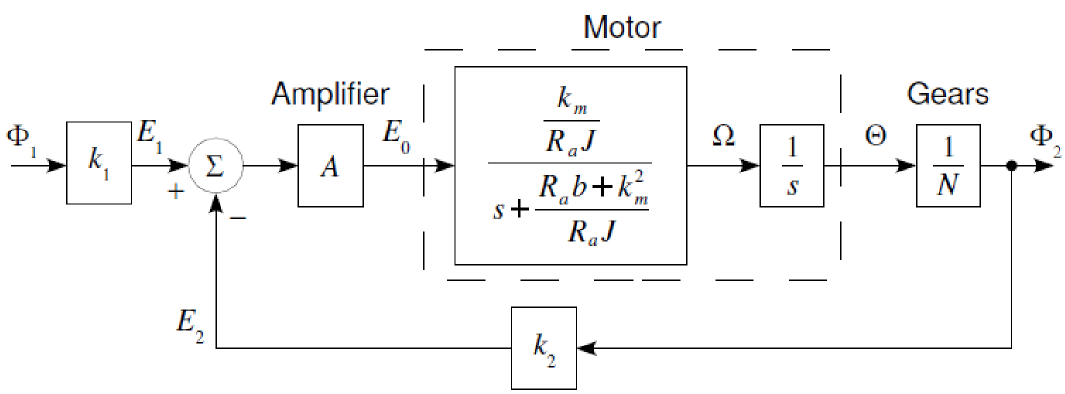

Figure 2 – Servo's Simulation Diagram without a Rate Feedback Loop [1]

Figure 3 below illustrates the block diagram of the controller. One can see that the rate and position feedback signal is provided in the Servo's simulation diagram.

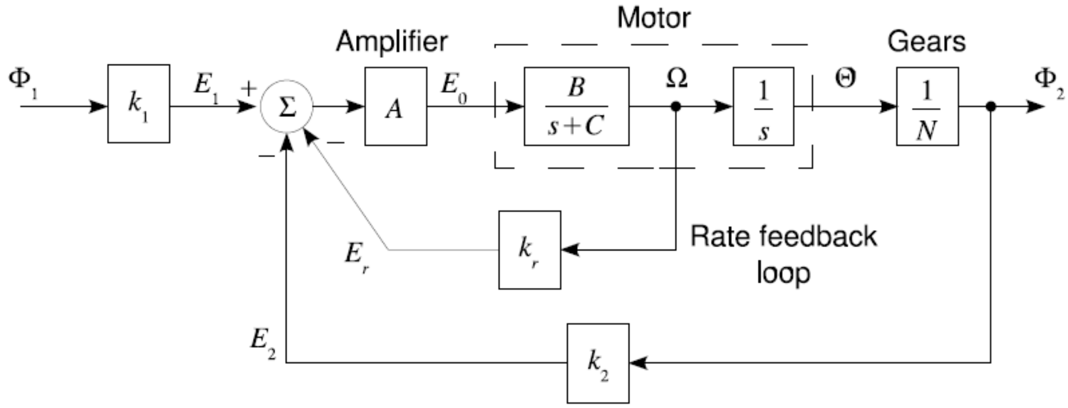

Figure 3 – Servo's Simulation Diagram with a Rate Feedback Loop [1]

3 Results

3.1 Matlab/Simulink Implementation (GA5.3 Tools for Design, Experimentation, Simulation, Visualization, and Analysis)

3.1.1 Values to Use for the Servo Components

Implemented the following component/parameter values in MATLAB:

$$
\begin{aligned}
k_m &= 1.5275\ \text{kg}\cdot\text{m}^2/\text{sec}^2/\text{A} \quad (\text{Nm/A})\\
J &= 100\ \text{kg}\cdot\text{m}^2\\
b &= 100\ \text{kg}\cdot\text{m}^2/\text{sec}\\
R_a &= 1\ \Omega\\
N &= 12\\
k_1 &= 12\ \text{V/rad}\\
k_2 &= k_1
\end{aligned}
$$

3.1.2 Experiments with the Servo System with/without a Rate Position Feedback

Figure 4 illustrates the Simulink diagram used to simulate the position control with no rate feedback loop.

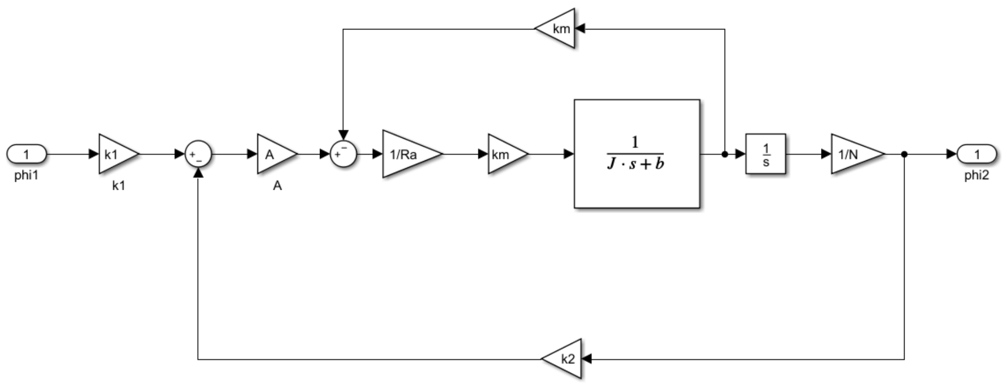

Figure 4 – Simulink Diagram Realizing Servo with No Rate Feedback Loop [1]

Figure 5 illustrates the Simulink diagram used to simulate the position control with a rate feedback loop provided.

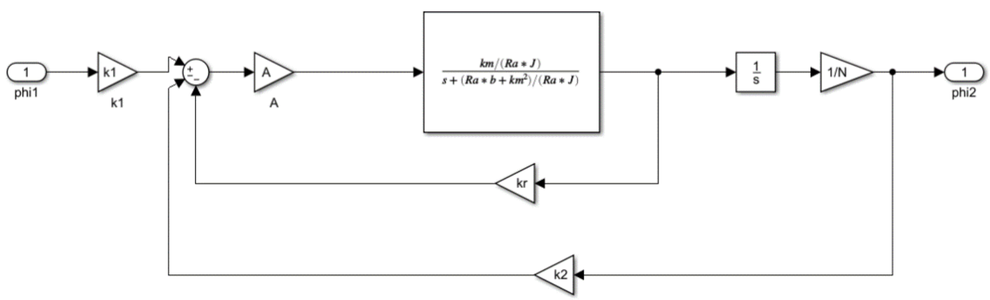

Figure 5 - Simulink Diagram Realizing Servo with a Rate Feedback Loop [1]

3.1.3 Setting Up the Experimental Cases

In this section of the lab, various values of A were implemented and data of a Servo system without a rate feedback loop were obtained in Table 1 below.

Table 1 - Data Obtained from the Characteristics of Servo System

Case	**A**	ζ	ω_n	ω_d	**Behaviour**
1	**4**	2.07	0.2472	0+0.4480j	Over damped
2	**17**	1.0041	0.5096	0+0.0461j	Critically damped
3	**35**	0.6998	0.7312	0.5223	Under damped
4	**300**	0.2390	2.1407	2.0786	Under damped

3.1.4 Simulating the Step Response of the Servo with a Rate Position Feedback

Using the final value theorem, the following data in Table 2 below were obtained. The equation used to obtain is as follows

$$\text{f.v.} = \lim_{t\to\infty} \phi_2(t) = \lim_{s\to 0} s\Phi_2(s) = \lim_{s\to 0} s\,[H(s)\Phi_1(s)] \tag{6}$$ [1]

Table 2 - Servo's System Final Value without Rate Feedback Loop

Case	A	Final Value
1	**4**	1
2	**17**	1
3	**35**	1
4	**300**	1

3.2 Results (GA3.5 Interpretation of Data)

3.2.1 Interpretation of Data of Servo System without Feedback Loop

Using the values given for A, shown in Table 1&2 above, MATLAB plots were generated to show the output signal for each value of A.

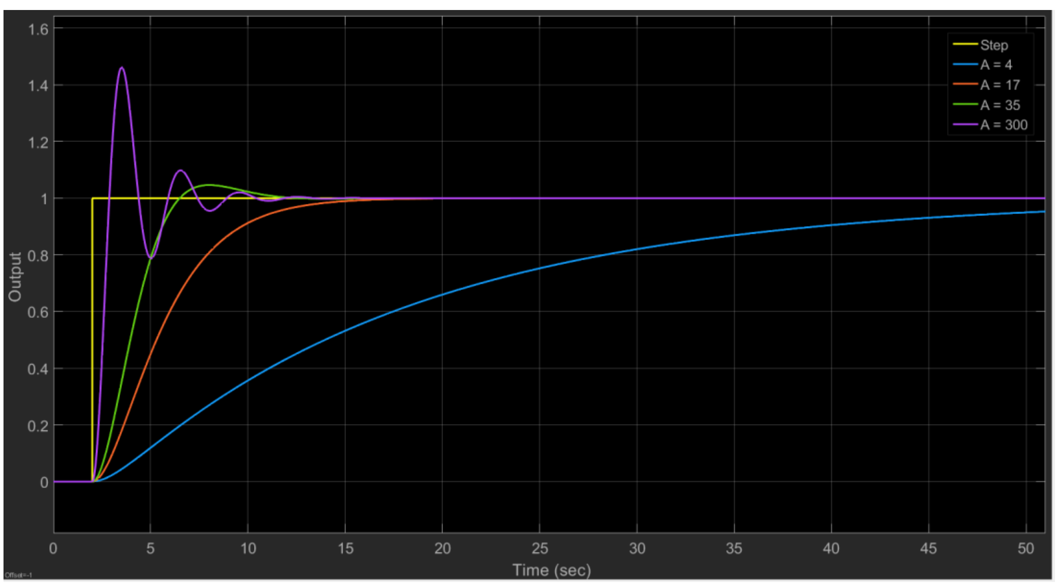

Figure 6 - Output Step Response with Different Values of A

3.2.2 Interpretation of Data of Servo System with Feedback Loop

Using a value for A=300 and values of k_r = 0.2, 0.5, 1, 3, 5, & 10. MATLAB plots were generated to show the output signal for each value of k_r.

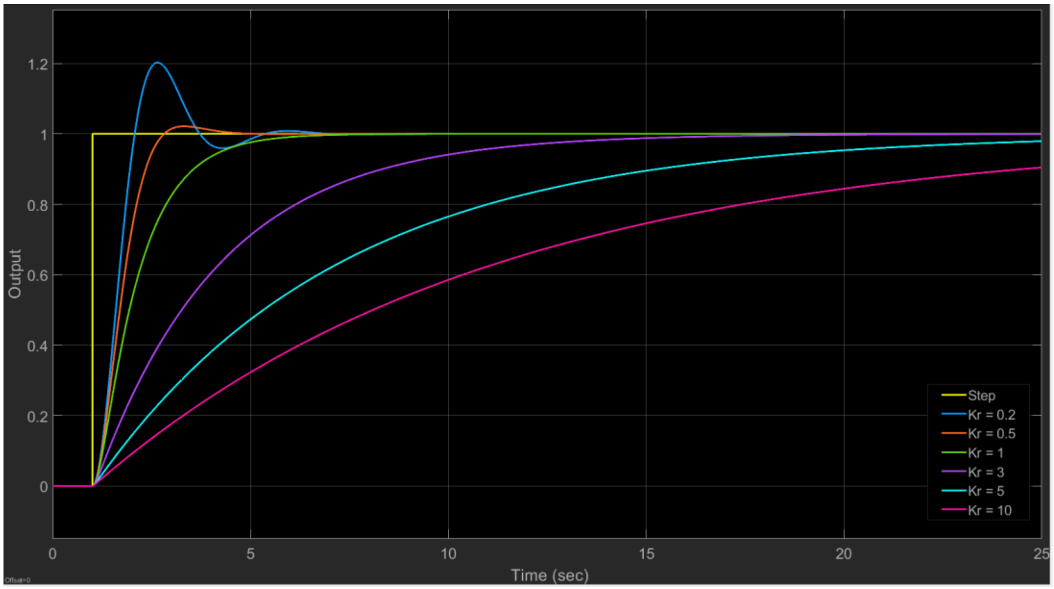

Figure 7 - Output Step Response with Different Values of k_r

4 Discussion (GA7.2 Professional Documents: Writing, Design Notes, Drawings, Attributions, and References)

4.1 Discussion of Results of Servo System without Feedback Loop

Figure 6 above shows no oscillation when A=4 which satisfies the data calculated in Table 2 above. This output is an overdamped system trying to reach a final value of 1. When A=17, the plot shows no oscillation which satisfies the data calculated in Table 2 above. This output is a critically damped system with ω_d being a complex number. In both cases, the output system has a damping ratio that is greater than 1, $\zeta > 1$.

Furthermore, when A=35, the plot shows no oscillation and the value of ω_d is around 0.52. This output is an underdamped system as it reaches the its final value of 1 too fast which satisfies the data in Table 2. Also, when A=300, the plot shows no oscillation with a ω_d value of around 2. This output is an underdamped system as it reaches the its final value of 1 too quickly which satisfies the data in Table 2. In these cases, the output system has a damping ratio that is less than 1, $\zeta < 1$.

In comparison between Figure 6 and Table 2, it is seen that all the final values in Figure 6 are the same as the calculated ones in Table 2 regardless of the system being underdamped or overdamped. Moreover, the step response output behaviour corresponded with the expected damping ratio.

The frequency of the oscillations, in the case where A=300, is compared with ω_d using the following equation as it converts ω_d to a time value

$$\text{Time Between Peaks} = 2\pi / \omega_d \tag{7}$$

This given a calculated value of 3.02 seconds which lines up with inspection of Figure 6, A=300, where the value between he two peaks is roughly 3 seconds.

4.2 Discussion of Results of Servo System with Feedback Loop

As shown in Figure 7, as the feedback rate increases, the function flattens out as it approaches the final value. When the feedback rate is less than 1, the functions are still able to oscillate before they flatten out in attempt of reaching their final value. However, as the feedback rate is increased, the oscillations are further dampened. When the feedback rate is greater than 1, the oscillations are no longer present, and the functions are fast approaching their final value.

To calculate the rate feedback constant, k_r, the following equation is used:

$$k_r = \frac{2\omega_n\zeta - C}{AB} \tag{8}$$

Using the dampening ratio of 1, an appropriate value of k_r, can be obtained for a critically dampened system. The value of k_r is calculated to be $k_r = 0.711$. If the value of k_r of this critically damped system, is slightly decreased, it will change its behaviour to an underdamped system. It becomes an underdamped system when it passes through the Servo system with no rate feedback.

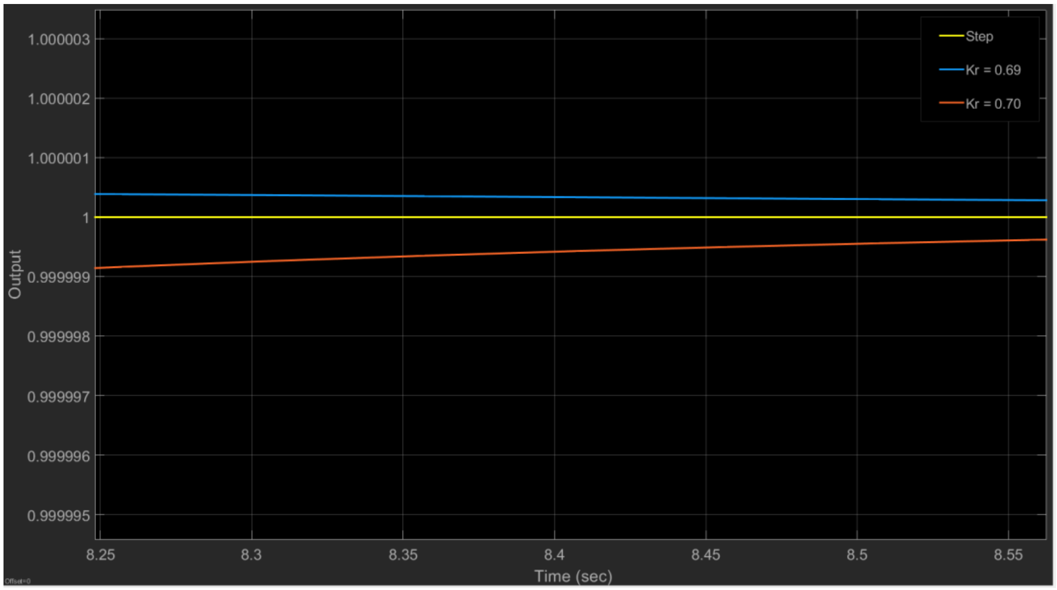

Figure 8 - Output Step Response with Two Different Values of k_r

References

[1] "Lab #2: Servo System Simulation," Carleton Univeristy, Ottawa, 2019.