Control of an Inverted Pendulum

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Lab 3 Report

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1 Preparation

$k_p = \omega_n^2 Ml + (M + m)g$ (1)

$k_d = 2\zeta\omega_n Ml$ (2)

At M = 1000 kg m = 200 kg, l = 10 m, ω n = 0.5 rad/sec, ζ = 0.7, and g = 9.81 m/s^2, we get the following,

k_p = 14 272

k_d = 7000

2 Inverted Pendulum Demo in Simulink

In this case, the underdamped response is preferred over a critically damped or an overdamped response. A critically damped response, where $\theta_o(t)$ is always positive and approaching zero, would prevent the pendulum from falling, but it would take a significant amount of time and the cart would move a significant distance to reach that point. On the other hand, an overdamped system would not be able to prevent the pendulum from falling either since it would undershoot the correction and not be able to stabilize the pendulum before it falls. The underdamped response is quick so when the pendulum is knocked, the underdamped response could help in balancing the car quickly. Since there is an overshooting in the underdamped response, it causes the car to move further than expected, but the θ is still as predicted.

3 Testing the Pendulum

The simulation diagram for the inverted pendulum was built as shown below.

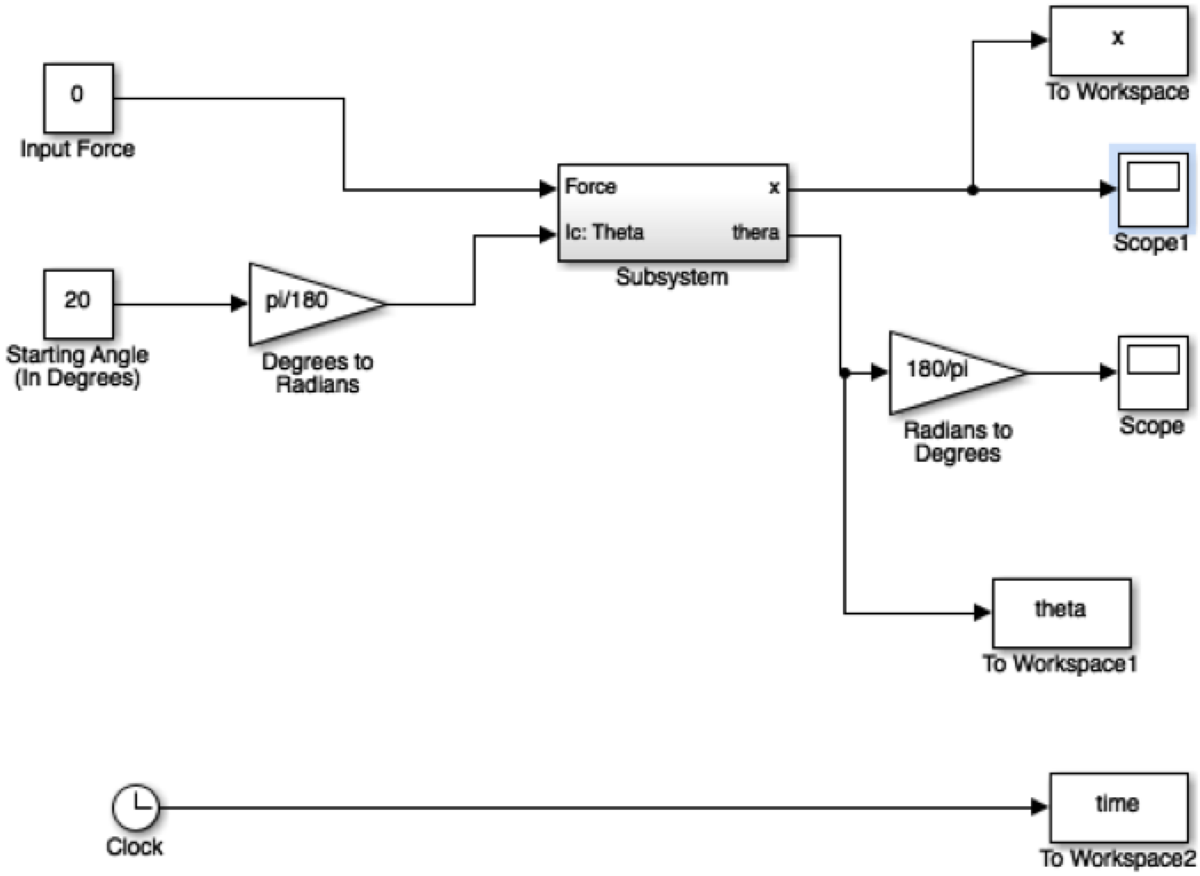

Figure 1 - Simulation Diagram of the Inverted Pendulum [1]

The cart position and the angel of the inverted pendulum were plotted. Figures 2 & 3 below show the plots for the cart position and the inverted pendulum angle form t=0 to t=50.

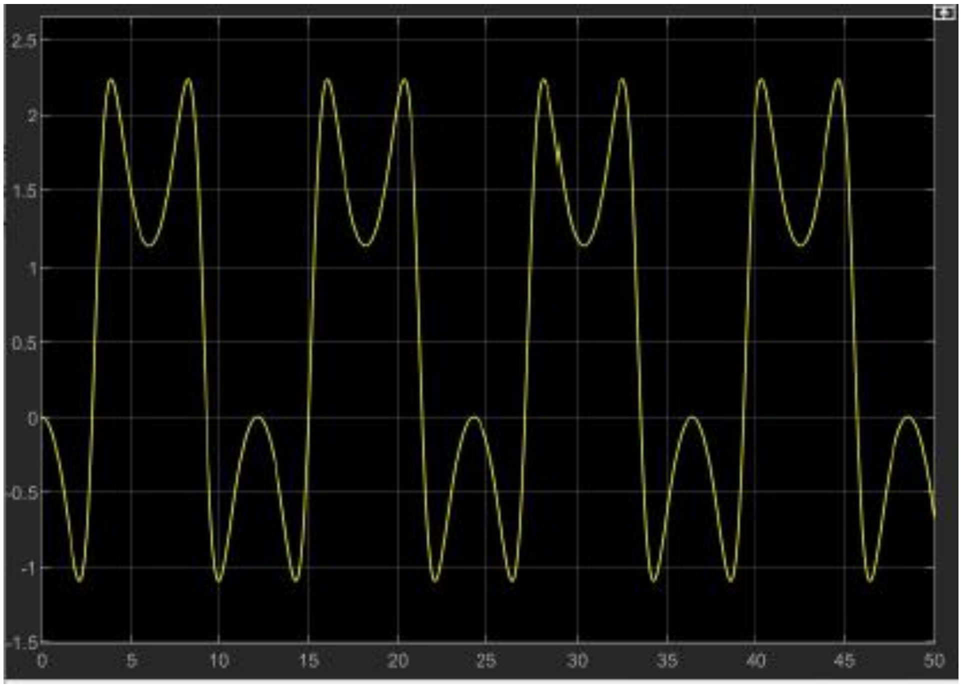

Figure 2 – The Position of the Cart from t=0 to t=50

As shown in Fig. 2 above, the position of the cart is stuck in a pattern of constantly moving from one direction to the other to maintain the angle of the inverted pendulum. Initially, the cart moves towards -1 then suddenly switches directions once the inverted pendulum angle gets over corrected.

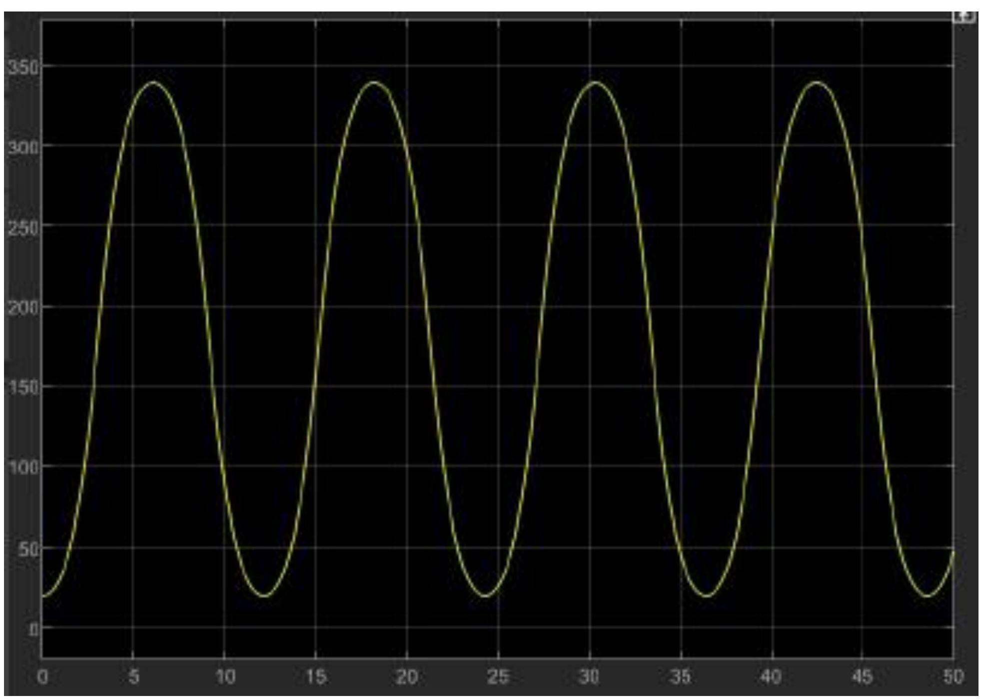

Figure 3 – The Inverted Pendulum Angle from t=0 to t=50

As shown in Fig. 3 above, the pendulum angle dictates the force (and ultimately the position) that is imposed on the cart. This is because the system wants to move the angle of the inverted pendulum to 0 degrees so it can stay up right.

4 Simulation of PD Controlled Non-Linear Inverted Pendulum

For the design of the lowpass filter and its transfer function, the cut-off frequency is set to 100 rad/s, and the following transfer function is as shown below

$$\omega_{cf} = 100\frac{rad}{s} \quad ; \quad H_{LP}(s) = \frac{1}{\frac{s}{100}+1}$$

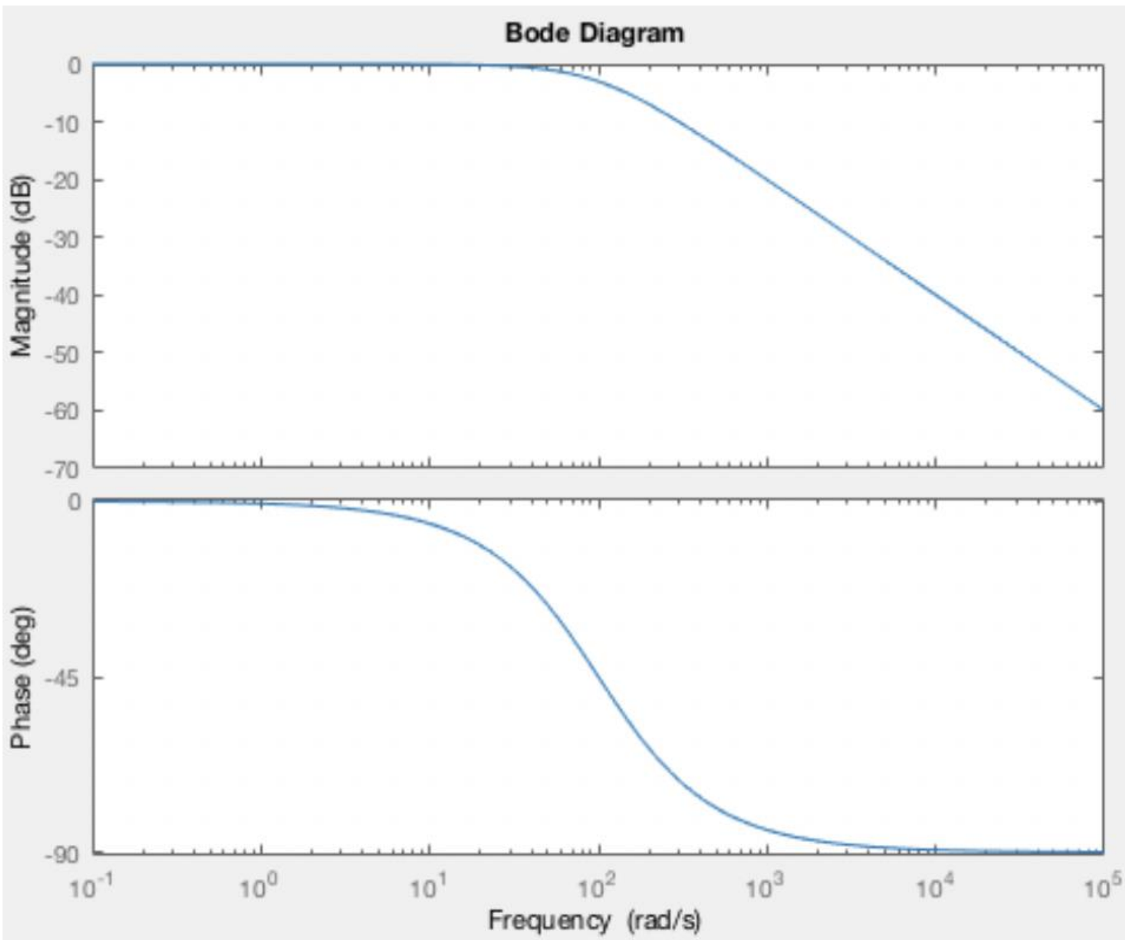

Figure 4 – Bode Plot of the Low Pass Filter

The lowpass filter permits (0dB) signals with frequencies less than 100 rad/s and enlarges signals that are greater than 100 rad/s. After 100 rad/s, the magnitude decreases at -20dB/decade. For the phase diagram, the phase is -45 degrees at the 100 rad/s. After 100 rad/s, the phase decreases at -45 degrees/decade.

5 Simulation of PD Controlled Inverted Pendulum

As shown in Fig. 5 below, the pendulum is starting at an angle of 5 degrees and the cart moves to align to the pendulum. By altering quickly, it overshoots the center line, which makes the angle negative. Then, the cart balances the pendulum at an angle of 0 in the vertical position.

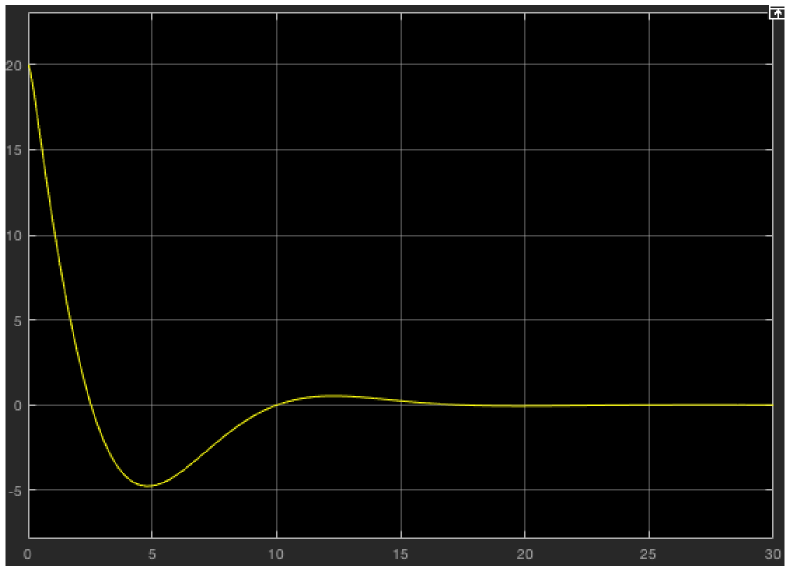

Figure 5 – $\theta_o(t_o^-)$=5°

As shown in Fig. 6 below, the pendulum starts at an angle of 30 degrees and the cart moves to re-center the pendulum. In results, it overshoots the center line, making the angle positive, and then rebalances the pendulum at an angle of 0 degrees in the vertical position.

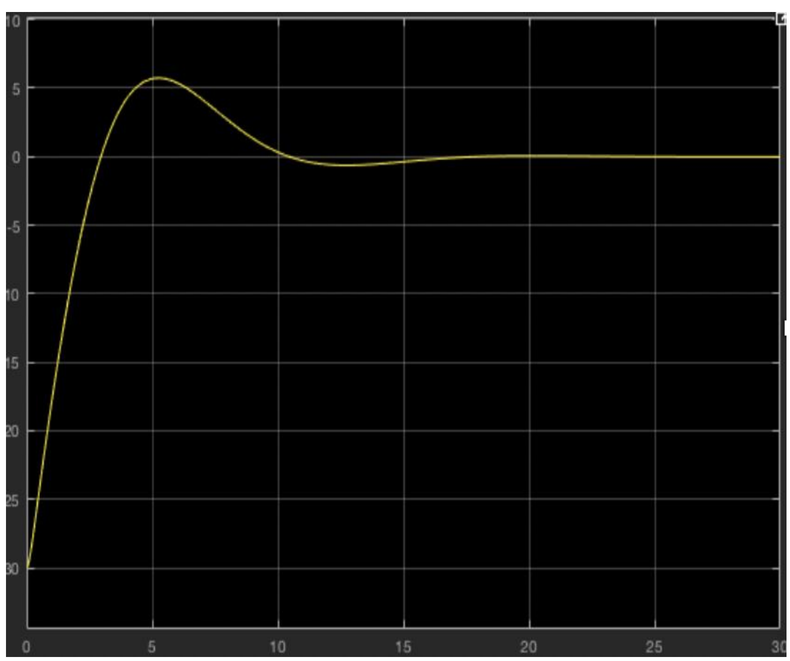

Figure 6 – $\theta_o(t_o^-)$=30°

As shown in Fig. 7 below, the pendulum starts at an angle of 65 degrees and the cart moves to re-center the pendulum. In results, the angle becomes greater which brings the pendulum to a large and negative angle. It bounces back and forth in attempt to correct the massive overshoot, but it can never correctly balance the pendulum which leaves the pendulum bouncing between the two maximum extremes.

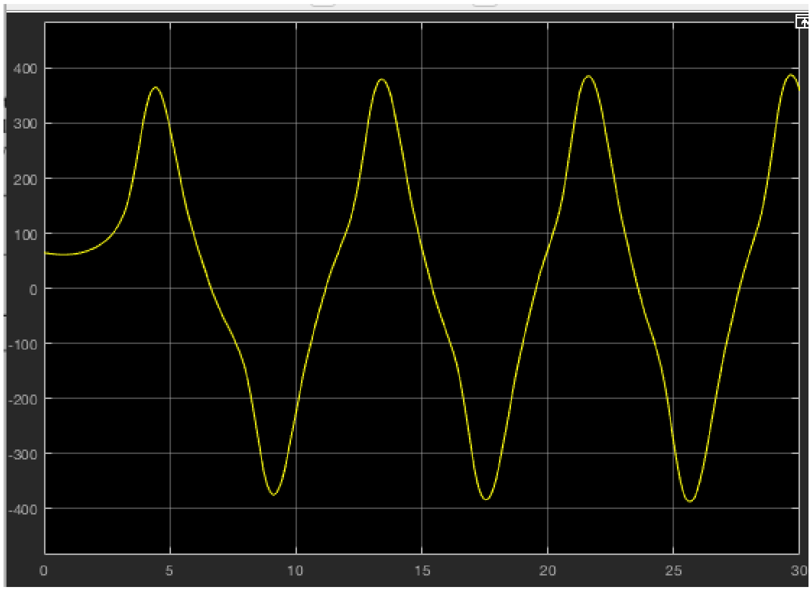

Figure 7 – $\theta_o(t_o^-)$=65°

As show in Fig. 8 below, the pendulum starts at an angle of 75 degrees and the cart moves to re-center the pendulum. In results, the angle becomes greater, then attempts to readjust. This makes the angle large and negative, then it readjusts, bouncing back and forth between the two maximum extremes on the positive and negative ends, never balancing the pendulum.

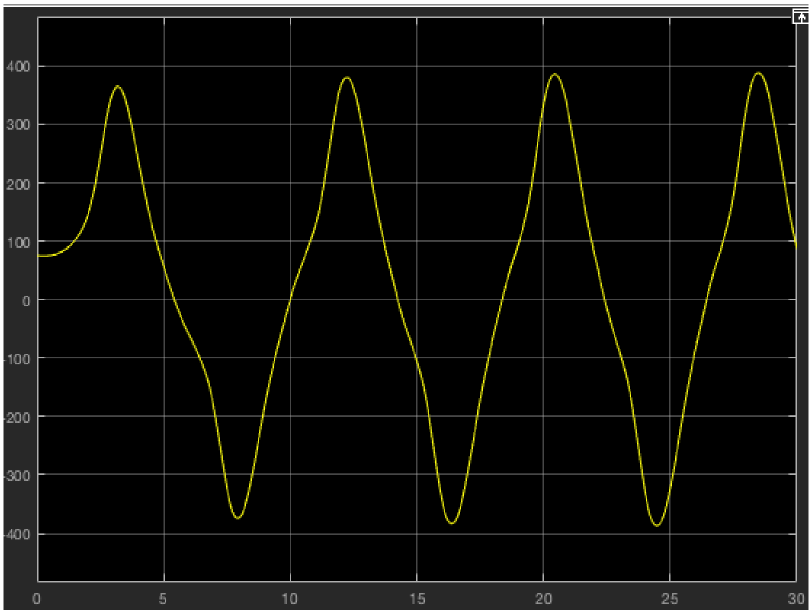

Figure 8 – $\theta_o(t_o^-)$=75°

In the previous two cases, the PD controller fails to keep the pendulum inverted. This is due to the pendulum starting from an angle greater than 45 degrees. With angles greater than 45 degrees, the carts motion can never get properly underneath the pendulum to balance. This will simply push the pendulum sideways as it falls.

6 Additional Questions

If the reference angle is set to something other than zero, then the closed loop system would not keep the pendulum at that angle, however if the angle is zero or very close to zero, for example 0.001 degrees, the pendulum, system would be able to balance. This is because as the system reached the reference angle, it would stop attempting to balance the pendulum, and since the pendulum reference angle is not vertical (for reference angles other than zero), that would lead the pendulum mass to push it down again. This would continue forever in an attempt by the system to stabilise the pendulum position. Therefore, for the system to balance, the reference angle should either be zero or very close to zero.

There should be two initial conditions for Eq. 13 of the lab manual. One of them is the initial angular position of theta of the pendulum and the other one, which is missing the initial angular velocity of the pendulum. The initial angular velocity is assumed to be zero in the first implementation of the open looped system. The initial angular velocity of the pendulum is the velocity of the pendulum before the PD controlled system started to balance itself.

References

[1] “Lab #3: Control of an Inverted Pendulum,” Carleton Univeristy, Ottawa, 2019.