Laboratory Experiment #1

Open-Loop Analysis

A common type of control system is servomechanism whereby a mechanical output variable, such as the position or angular rotation of an element, has to follow a reference input. Therefore, a servomotor is typically characterized by a DC motor and a sensor for measuring the angular rotation. As shown in Figure 1, the servomotor system can be described based on the electrical model of the motor and the mechanical model of the load connected to the motor shaft. In the practical implementation of the armature system, an inductor is in series with the resistance. However, since its magnitude is much less than that of the resistor, it is neglected for simplicity's sake.

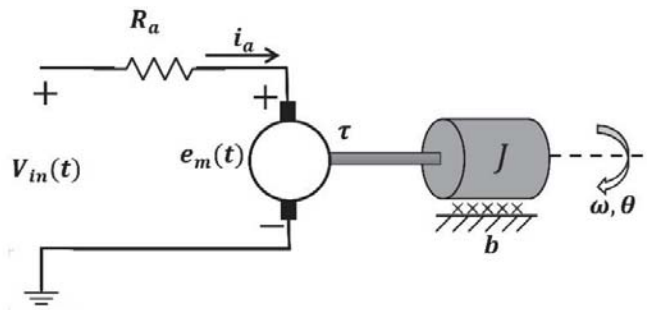


Figure 1

Electromechanical equations of the system shown above have the following form, and the parameters are given in Table 1.

$$i_a(t) = \frac{V_{in}(t) - e_m(t)}{R_a}, \quad e_m(t) = k_m \omega(t), \quad \tau(t) = J\dot{\omega}(t) + b\,\omega(t), \quad \tau(t) = k_t i_a(t)$$

Table 1

symbol	Description	value
k_m	back-EMF constant	1.8350 (v/rad/s)
k_t	Torque constant	1.8350 (Nm/A)
R_a	Armature resistance	3.0484 (Ω)
J	Moment of inertia of the load	0.2296 (kgm^2)
b	Viscous friction factor	0.0412 (Nms)

Model parameter identification

In designing a control system, the engineer is initially faced with the problem of determining a mathematical model that reflects the dynamic behavior of the plant. Two tests are commonly used to identify the parameters that describe the dynamics of a system. The first test is the step response and the second one is the frequency response.

Part 1: The Step Response

In all laboratory experiments, the motor plant is simulated based on the block diagram shown in Figure 2.

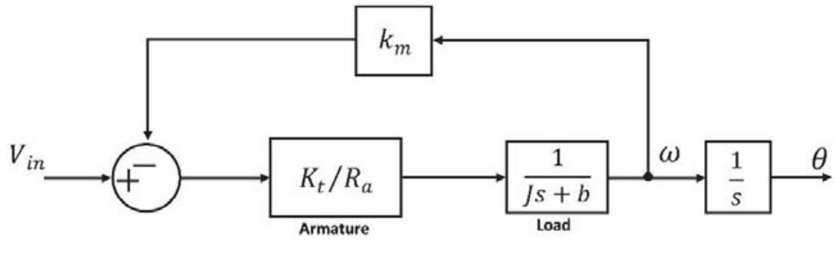


Figure 2

The idea behind the step response is to apply a step input to the DC motor and record the resulting output. The output of the system to be measured is the angular velocity of the DC motor (in practice, the output of the system is the voltage of the tachometer which is proportional to the angular velocity). From the output, one can calculate the time constant of the system and the steady state gain. Denote the gain and time constant of the DC motor by k_G and τ_G, respectively.

Procedure

Step 1 Simulate the system shown in Figure 2 using Matlab Simulink with a step input as the reference signal. Display both the step input and the step response of the system in the same window. Obtain the steady state gain of the system. Once the steady state gain is known, calculate the time constant of the system. Finally, obtain the transfer function of the open-loop DC motor plant in the standard form. Also, explicitly determine the equations for the time constant and steady state gain of the DC motor in terms of the system parameters, explained in Table 1.

Step 2 Examine if there is any discrepancy between the Tacho-generator output (ω) and the reference input (V_{in}) of the plant. Plot the error signal ($e(t) = V_{in}(t) - \omega(t)$) and obtain the time response for the error. Discuss the performance of the open-loop system based on the error signal.

Step 3 Investigate the effect of changes in the time constant of the DC motor on the system performance. If there is a ten percent margin of error in obtaining the value of parameter J, how much error does the system response experience?

Step 4 From the transfer function obtained in Step 1, calculate the bandwidth of the open-loop system.

Part 2: Frequency Response

To verify the results of the step input test, one typically performs a frequency response test. Recall that for a first-order system, the phase lag at the pole is 45 degrees, and the signal amplitude is down by $-3\ dB$. We also know that the high-frequency asymptote goes down at $-20\ dB/decade$. We will use this knowledge to validate the results of part 1.

Procedure

Step 1 Using the Matlab built-in function, plot the Bode diagram (magnitude and phase) based on the transfer function obtained in Part 1.

Step 2 In the Simulink model, apply the sinusoidal input at frequencies of $0.2\ Hz$, $0.5\ Hz$, $1\ Hz$, $2\ Hz$, $4\ Hz$, and $6.5\ Hz$. For one frequency point, display both the sinusoidal input and the Tacho-generator output in the same window. The result should be similar to the one in Figure 3.

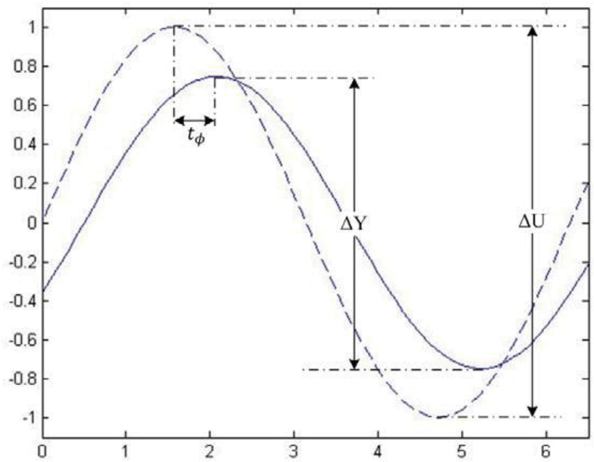


Figure 3

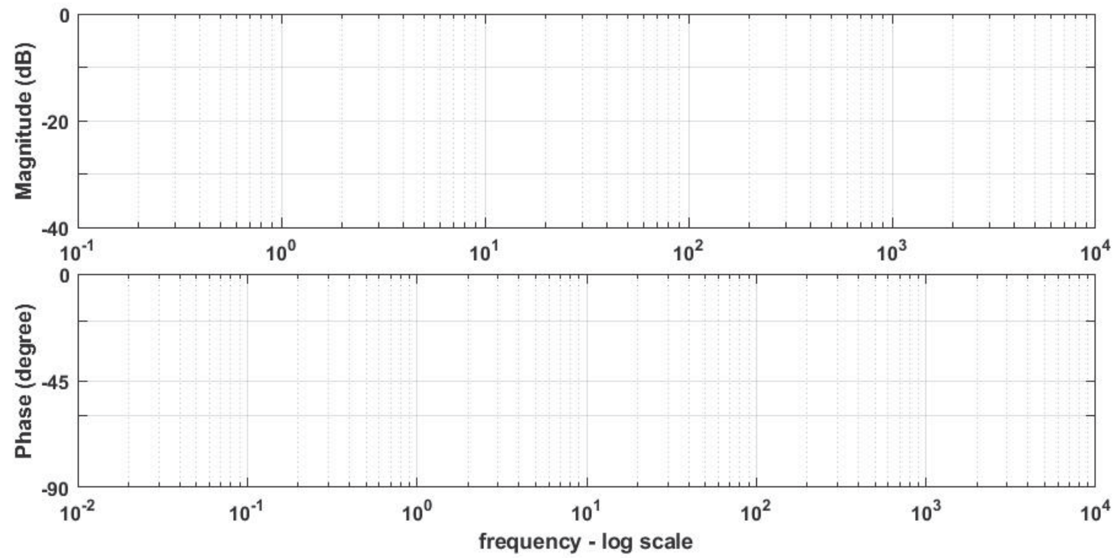
Fill out Table 2 based on the parameters defined in Figure 3 at different frequencies. Note that the frequency response of the system including the phase lag $\measuredangle H(j\omega)$ and the gain $|H(j\omega)|$ can be calculated using the following equations, where T is the period of the sinusoid ($T = 1/f$).

$$\measuredangle H(j\omega) = -360^\circ\, {}^{t_\phi}/_{T} \quad , \quad |H(j\omega)| = {}^{\Delta Y}/_{\Delta U}$$

Table 2

$f\ [hz]$	T	t_ϕ	ΔY	ΔU	$\measuredangle H(j\omega)$	$\lvert H(j\omega)\rvert$
0.2						
0.5						
1.0						
2.0						
4.0						
6.5						

Step 3 Using the information in Table 2, plot the results of magnitude and phase at the specified frequencies in the Bode paper. Remember to convert f to ω using $\omega = 2\pi f$ and $|H(j\omega)|$ into dB using $dB = 20log|H(j\omega)|$. Besides, add the Bode plots for the transfer function obtained in Part 1.



Step 4 Identify the bandwidth of the open-loop system from the Bode plot and compare it with the result obtained in Part 1. What is the general equation of the bandwidth for a first-order system?

Step 5 What do you think about the open-loop system performance characteristics? Do they need any improvement/control? If yes, explain your reasons and, suggest ways to improve them.