Open-Loop Analysis

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Lab 1 Report

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1.1 Step 1

Fig. 1 the Matlab Simulink simulation of the DC motor with a step input as the reference signal.

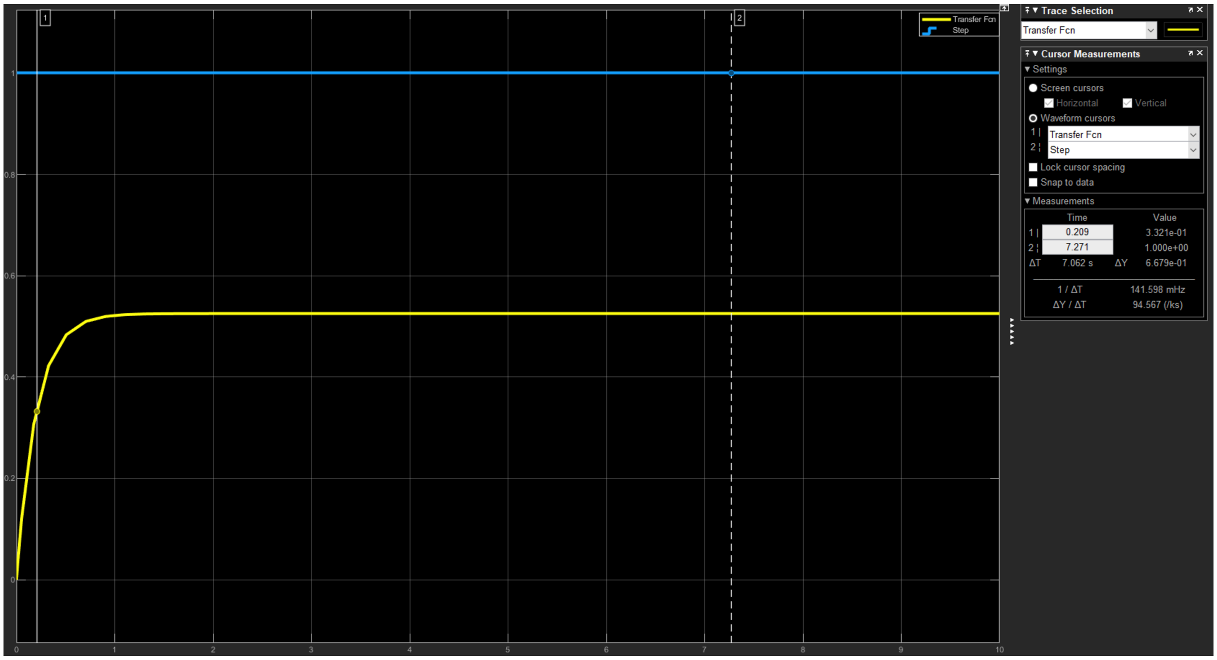

Figure 1: Step input and step response of the DC motor

The transfer function, $H(s)$, of the DC motor can be calculated as follows:

$$\begin{aligned} H_1(s) &= \frac{k_t}{R_a} * \frac{1}{Js+b} \\ &= \frac{k_t}{R_a(Js+b)} * \frac{\frac{1}{R_aJ}}{\frac{1}{R_aJ}} \end{aligned}$$

$$\therefore H_1(s) = \frac{\frac{K_t}{R_aJ}}{s+\frac{b}{J}} \tag{1}$$

$$\begin{aligned} H(s) &= \frac{\frac{\frac{k_t}{R_aJ}}{s+\frac{b}{J}}}{1+\frac{\frac{k_tk_m}{R_aJ}}{s+\frac{b}{J}}} \\ &= \frac{\frac{\frac{k_t}{R_aJ}}{\cancel{s+\frac{b}{J}}}}{\frac{s+\frac{b}{J}+\frac{k_tk_m}{R_aJ}}{\cancel{s+\frac{b}{J}}}} \end{aligned}$$

$$\therefore H(s) = \frac{\frac{k_t}{R_aJ}}{s+[\frac{b}{J}+\frac{k_tk_m}{R_aJ}]} \tag{2}$$

The steady state gain of the DC motor is calculated as follows:

$$\lim_{s\to 0}\left[\frac{1}{s} * H(s)\right] = \lim_{s\to 0}\left[\frac{\cancel{s}\frac{k_t}{R_a J}}{\cancel{s}\left[\cancelto{0}{s} + \left[\frac{b}{J} + \frac{k_t k_m}{R_a J}\right]\right]}\right]$$
$$= \lim_{s\to 0}\left[\frac{\frac{1.8350}{(3.0484)(0.2296)}}{\frac{0.0412}{0.2296} + \frac{1.8350^2}{(3.0484)(0.2296)}}\right]$$

$$\therefore \mathit{Stady\ state\ gain} = 0.5254$$

The time constant, τ, can be calculated as follows:

$$\tau = \frac{1}{a} \tag{3}$$

where $a = \frac{b}{J} + \frac{k_t k_m}{R_a J}$

$$\tau = \frac{1}{\frac{b}{J} + \frac{k_t k_m}{R_a J}}$$
$$= \frac{1}{\frac{0.0412}{0.2296} + \frac{1.8350^2}{(3.0484)(0.2296)}}$$
$$= \frac{1}{4.99}$$

$$\therefore \tau = 0.200\,\mathrm{s}$$

There is another way of finding the time constant, τ. We know that 63.2% of the steady state value is equal to the time constant and thus, we have the following:

$$0.632 * 0.5254 = 0.332$$

Using Fig. 1, the time constant is the x-value when the y-value = 0.332. Therefore, the time constant is approximately 0.200 s.

Explicit equation for the time constant of the DC motor:

$$\therefore \tau = \frac{J}{b + \frac{k_t k_m}{R_a}} \tag{4}$$

Explicit equation for the steady state gain of the DC motor:

$$\therefore k_G = \frac{R_a}{R_a b + k_t k_m} \tag{5}$$

1.2 Step 2

Fig. 2 the Matlab Simulink simulation of the error signal of the DC motor.

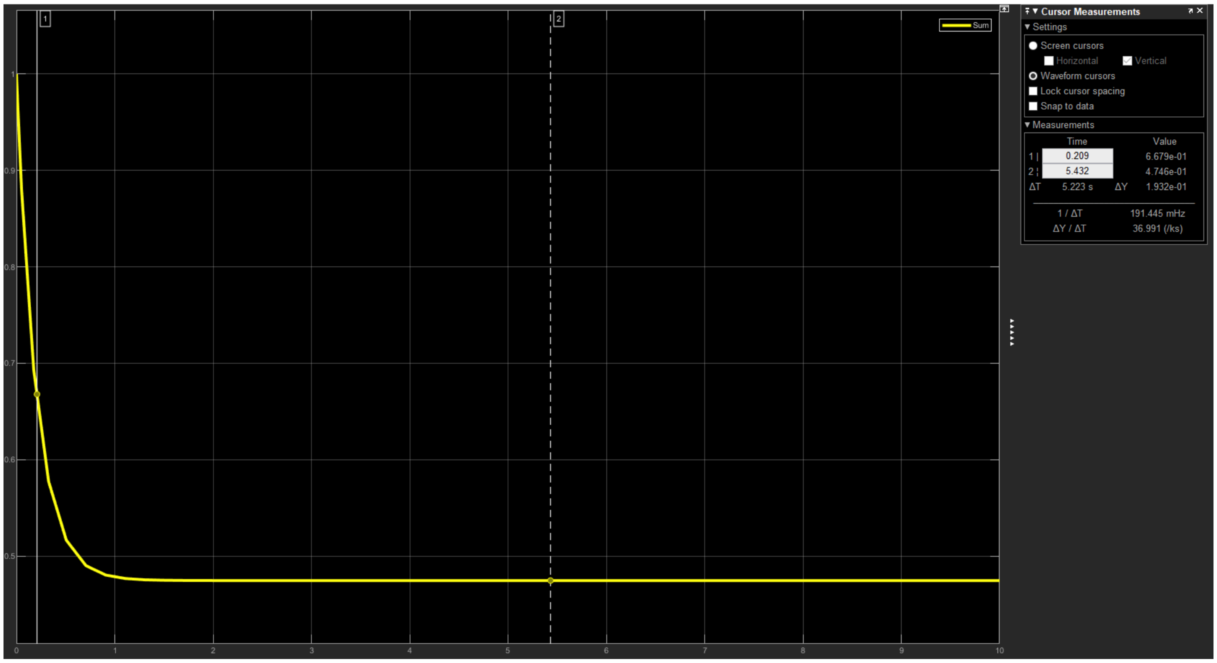

Figure 2: Time response of the error signal

The error signal can be calculated as follows:

$$\begin{aligned} e(t) &= V_{in}(t) - \omega(t) \\ &= 1 - 0.5254 \end{aligned} \tag{6}$$

$$\therefore e(t) = 0.4746$$

It is observed that the error signal is almost half of the input signal which highlights a very important disadvantage of the open loop gain. Ideally, one wants the error signal to be as small as possible, if not negligible.

1.3 Step 3

As shown in Eq. 4, the time constant is directly proportional to the parameter J which means an increase in J would cause an increase in τ, and vice verse. On the other hand, the gain is not affected by the changes done to parameter J as Eq. 5 does not have a parameter J in it.

Fig. 3 and Fig. 4 show an increase and a decrease of 10% in the time constant value respectively.

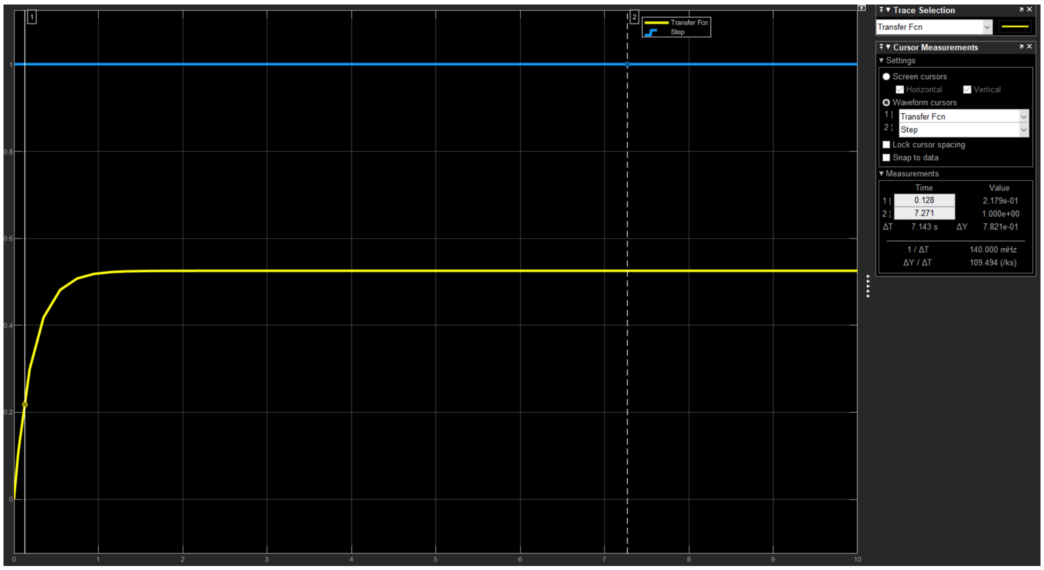

Figure 3: 10% increase in time constant

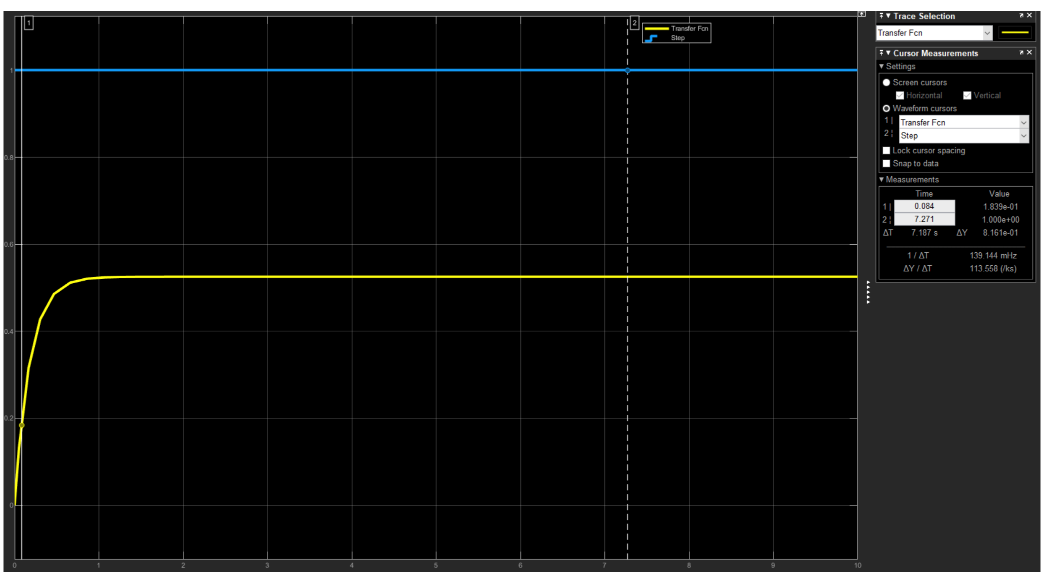

Figure 4: 10% decrease in time constant

1.4 Step 4

The bandwidth, BW, of the transfer function can be calculated as follows:

$$BW = \frac{1}{\tau} = \frac{1}{0.200} \tag{7}$$

$$\therefore BW = 5\,\text{rad/s}$$

2 The Frequency Response

2.1 Step 1

Fig. 5 shows the bode diagram (magnitude and phase) based on the transfer function obtained using Matlab built in function.

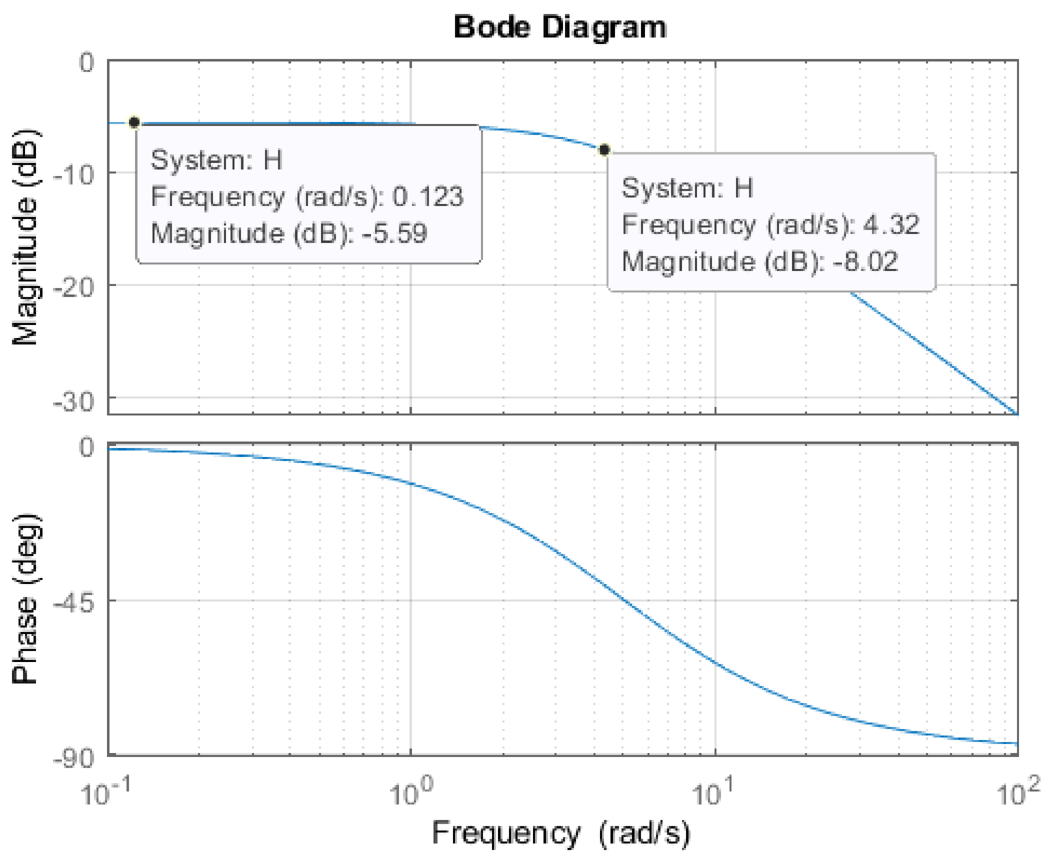

Figure 5: Matlab plot of the DC motor transfer function

2.2 Step 2

Parameters shown in Table 1 are used to construct the frequency response of the system including the phase and the gain using Eq. 8 and Eq. 9.

$$\measuredangle H(j\omega) = -360\frac{t_\phi}{T} \tag{8}$$

$$\mid H(j\omega) \mid = \frac{\Delta Y}{\Delta U} \tag{9}$$

Table 1: Data Points

f (Hz)	T (sec)	t_ϕ	ΔY	ΔU	∡H(jω)	| H(jω)|	20log|H(jω)| (dB)
0.2	5	0.198	1.017	2	-14.256	0.5085	-5.8741
0.5	2	0.169	0.8819	2	-30.42	0.44095	-7.1122
1	1	0.129	0.6878	2	-46.44	0.3439	-9.2713
2	0.5	0.085	0.4323	2	-61.20	0.21615	-13.3049
4	0.25	0.054	0.1909	2	-77.76	0.09545	-20.4045
6.5	0.153846	0.035	0.134	2	-81.90	0.0670	-23.4785

2.3 Step 3

Fig. 6 shows the bode diagram (magnitude and phase) based on the transfer function plotted by hand using data point in Table 1.

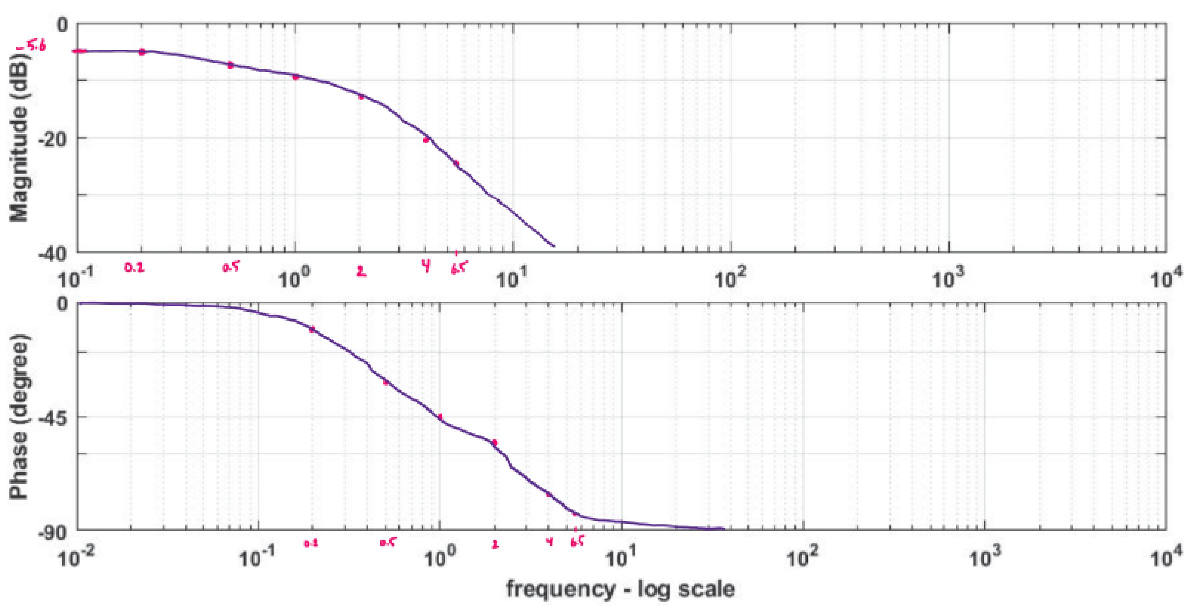

Figure 6: Frequency response data points

2.4 Step 4

Using Fig. 6, the bandwidth, BW, is determined using the 3 dB drop from the DC gain of the DC motor.

$$BW \approx -5.8741\,\text{dB} - 3\,\text{dB}$$

$$BW \approx -8.8741\,\text{dB}$$

As shown in Eq. 7, the general equation of the bandwidth is:

$$BW = \frac{1}{\tau}$$

2.5 Step 5

Based on this experiment and as shown in Fig. 2, the open loop gain of the DC motor has a very large error signal value which is approximately 50% of the input signal. This highlights a very important disadvantage of the open loop gain. The open loop gain is a type of analysis that can be used to analyze a system of dynamics. However, it should be avoided, if possible, as it has a very large error signal relative to the input signal.

3 References

[1] "Laboratory Experiment #1: Open-Loop Analysis", Carleton Univeristy, Ottawa, 2020.