Lab Notebook: Fourier Methods

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Contents

Intro

The Three Experiment Timeline

- 7 Class sessions
- Signal recovery under noise: Ch 6 & 15
- AM Radio Reception: Ch 3 & 11
- The Fluxgate Magnometer: Ch 3 & 13

0.1 Familiarizing with Equipment (Chapter 0-2)

Equipment list:

- SR770 FFT Network Analyzer (main instrument)
- Keysight 33500B Waveform Generator (AC signal source) we will call
- Tektronix TDS 1012 (oscilloscope/scope)
- Teach Spin Fourier Methods Electronic Modules (multi-tool)
- BNC (Bayonet-Neil-Concelman) cable: In short, a coaxial cable with default 50 ohm characteristic impedance for RF applications. All inputs and outputs will be connected via BNC cables.

Fourier Series (MAIN CONCEPT)
Any periodic function $f(t)$ with period T can be expressed as a sum of sines and cosines:

$$f(t) = \frac{a_n}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

$$a_0 = \frac{2}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)dt$$

where $\omega = 2\pi/T$ is the fundamental frequency and n is the harmonic number. Or for voltage

$$V(t) = V_{dc} + \sum_{n=1}^{\infty} [C_n \cos(2\pi nt/T) + S_n \sin(2\pi nt/T)] \tag{0.1}$$

Observations: From the 33500B, output a 10 kHz, 1 V source to the SIGNAL IN of the SR770 with

- Simple sine wave: Selecting the Sine waveform on the 33500B, we can see the outputs in the time & frequency domains as shown in Fig. 0.1 and 0.2

Figure 0.1: TDS 1012 oscilloscope view

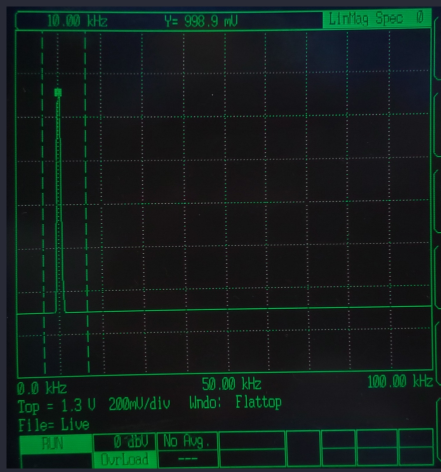

Figure 0.2: SR770 FFT Network Analyzer view

- Square wave: Changing the waveform to SQUARE on the 33500B, we can intuit from the fourier series the coefficients are given by

$$
\begin{aligned}
a_0 &= 0 \\
a_n &= 0 \\
b_n &= \frac{4}{n\pi}\sin(n\pi/2)
\end{aligned}
$$

Thus the square wave is a sum of odd harmonics of the fundamental frequency with amplitudes

$$\left[\frac{4}{\pi}, \frac{4}{3\pi}, \frac{4}{5\pi}, \frac{4}{7\pi}, \dots\right]$$

for odd n as shown in Fig. 0.4. In Fig. 0.3 we can see the shape of the waveform in the time

Figure 0.3: TDS 1012 oscilloscope view

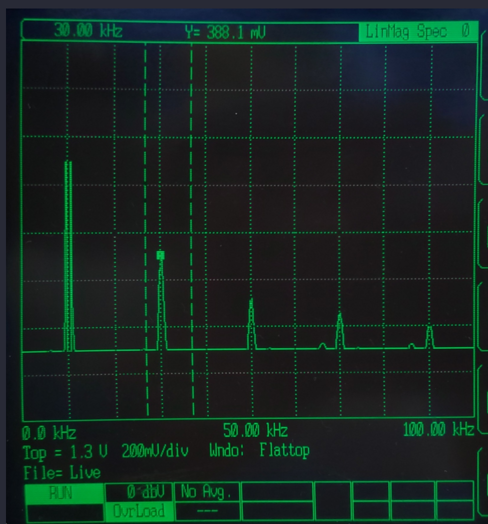

Figure 0.4: SR770 FFT Network Analyzer view

domain, and the odd harmonics are clearly shown in the SR770 frequency domain (Fig. 0.4).

- Saw Wave: (SAW waveform on 33500B) The saw wave is a sum of all harmonics of the fundamental frequency as shown in Fig. 0.6.

Figure 0.5: TDS 1012 oscilloscope view

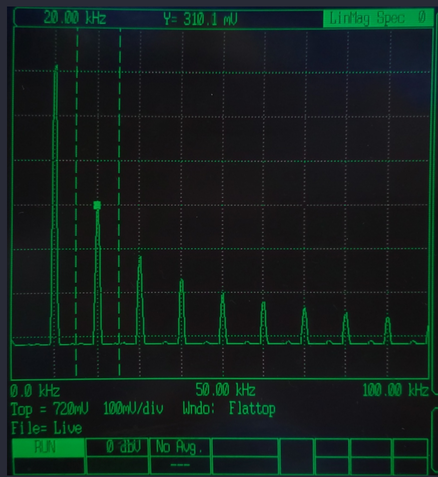

Figure 0.6: SR770 FFT Network Analyzer view

- Triangle Wave: (TRIANGLE waveform on 33500B) The triangle wave is a sum of all odd harmonics of the fundamental frequency as shown in Fig. 0.8. But the amplitude of the harmonics decreases

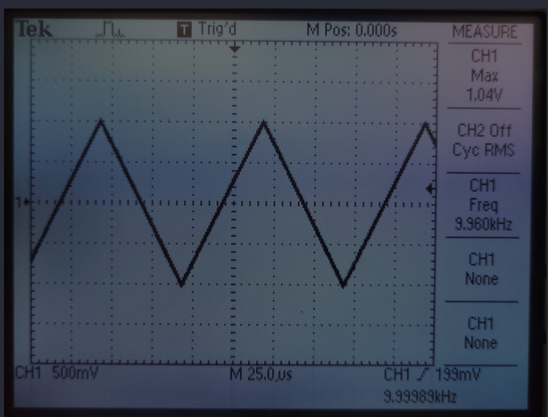

Figure 0.7: TDS 1012 oscilloscope view

Figure 0.8: SR770 FFT Network Analyzer view

by a factor of $1/n^2$. This power law makes the amplitude hard to read in the linear scale, but in the log scale, the amplitudes are clearly visible as shown in Fig. 0.9.

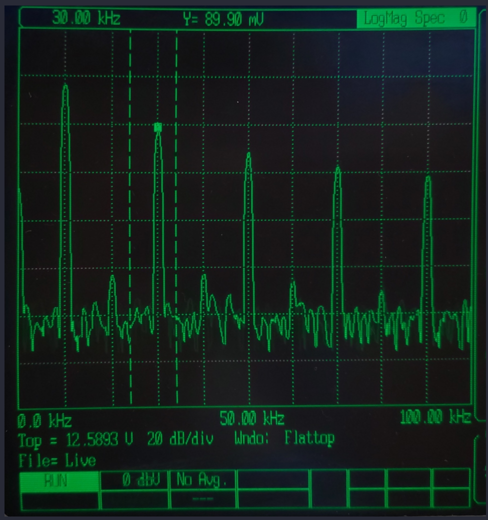

Figure 0.9: SR770 LOG MAGNITUDE

Superposition of sine waves

- 770: 40 kHz, 1 V sine wave → SUMMER input A
- 33500B: 50 kHz, 2 V sine wave → SUMMER input B
- SUMMER output → 770 SIGNAL IN & TDS 1012

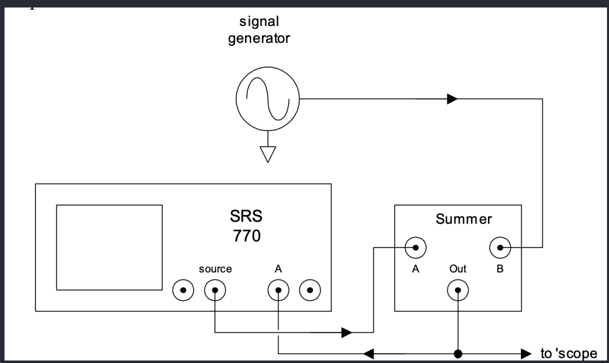

Figure 0.10: Diagram of setup

From the 770, we can easily see the two sine waves in the frequency domain as shown in Fig. [insert fig 0.11], but the time domain (scope) does not clearly describe the summation of the two waves.

Similar amplitude

- 770: 50 kHz, 1 V sine wave → SUMMER input A
- 33500B: 51 kHz, 1 V sine wave → SUMMER input B

In the full (100 kHz) span view, we can't see the two peaks. To increase the frequency resolution, we can reduce the span in the 770 FREQ menu, but this will increase the acquisition time.

e.g. a full span of 100 kHz has an acquisition time of 4 ms; the 'voltage sampling' rate is 256 kSa/s, or 256 samples per ms i.e. 1024 samples in 4 ms.

For our experiment, we set the span to 1.56 kHz to clearly see the two peaks, but this costs us an acquisition time of 256 ms or $256 * 256\,\text{samples/ms} = 65536$ samples. This trade-off can be described by the 'frequency duration uncertainty principle':

$$\text{(frequency resolution achievable)} \cdot \text{(acquisition time required)} \geq \text{a number}$$

The 770 magic number is

$$100\,\text{kHz} \cdot 4\,\text{ms} \geq 400\,\text{kHz}\,\text{ms}$$

which we can use to find the minimum acquisition time for a given frequency resolution e.g. the 1.5625 kHz span required

$$\begin{aligned}\text{(acquisition time req)} &\geq 400/\text{(freq resolution)}\\ &= 400/1.5625 = 256\,\text{ms}\end{aligned}$$

Windowing & Different amplitude Recommended windowing:

- Uniform: close spanced peaks with similar amplitudes
- Flattop: accurate peak height measurement
- Hanning: good for spectral resolution
- BMH: good for weak peak near strong peak, but not the best resolution for top peak & amplitude accuracy

1 Signal Recovery Under Noise

Chapter 6: Noise Waveforms

2 AM Radio Reception

3 The Fluxgate Magnometer