Lab Notebook: Fourier Methods

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Contents

Intro

The Three Experiment Timeline

- 7 Class sessions
- Signal recovery under noise: Ch 6 & 15
- AM Radio Reception: Ch 3 & 11
- The Fluxgate Magnometer: Ch 3 & 13

0.1 Familiarizing with Equipment (Chapter 0-2)

Equipment list:

- SR770 FFT Network Analyzer (main instrument)
- Keysight 33500B Waveform Generator (AC signal source) we will call
- Tektronix TDS 1012 (oscilloscope/scope)
- Teach Spin Fourier Methods Electronic Modules (multi-tool)
- BNC (Bayonet-Neil-Concelman) cable: In short, a coaxial cable with default 50 ohm characteristic impedance for RF applications. All inputs and outputs will be connected via BNC cables.

Figure 0.1: Teach Spin Fourier Methods Electronic Modules

Fourier Series (MAIN CONCEPT)

Any periodic function $f(t)$ with period T can be expressed as a sum of sines and cosines:

$$f(t) = \frac{a_n}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

$$a_0 = \frac{2}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega t)dt$$

where $\omega = 2\pi/T$ is the fundamental frequency and n is the harmonic number. Or for voltage

$$V(t) = V_{dc} + \sum_{n=1}^{\infty} [C_n \cos(2\pi nt/T) + S_n \sin(2\pi nt/T)] \tag{0.1}$$

Observations: From the 33500B, output a 10 kHz, 1 V source to the SIGNAL IN of the SR770 with

- Simple sine wave: Selecting the Sine waveform on the 33500B, we can see the outputs in the time & frequency domains as shown in Fig. 0.2 and 0.3

Figure 0.2: TDS 1012 oscilloscope view

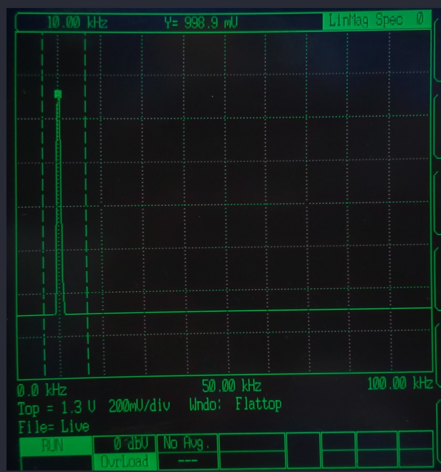

Figure 0.3: SR770 FFT Network Analyzer view

- Square wave: Changing the waveform to SQUARE on the 33500B, we can intuit from the fourier series the coefficients are given by

$$
\begin{aligned}
a_0 &= 0 \\
a_n &= 0 \\
b_n &= \frac{4}{n\pi}\sin(n\pi/2)
\end{aligned}
$$

Thus the square wave is a sum of odd harmonics of the fundamental frequency with amplitudes

$$\left[\frac{4}{\pi}, \frac{4}{3\pi}, \frac{4}{5\pi}, \frac{4}{7\pi}, \dots\right]$$

for odd n as shown in Fig. 0.5. In Fig. 0.4 we can see the shape of the waveform in the time

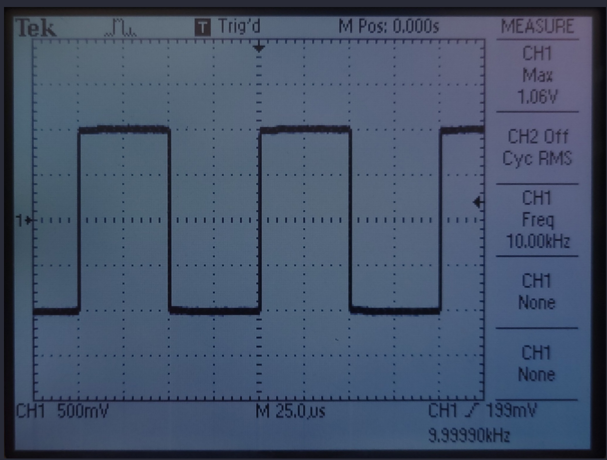

Figure 0.4: TDS 1012 oscilloscope view

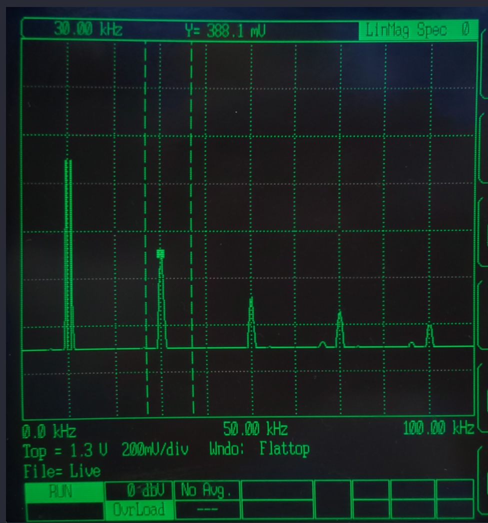

Figure 0.5: SR770 FFT Network Analyzer view

domain, and the odd harmonics are clearly shown in the SR770 frequency domain (Fig. 0.5).

- Saw Wave: (SAW waveform on 33500B) The saw wave is a sum of all harmonics of the fundamental frequency as shown in Fig. 0.7.

Figure 0.6: TDS 1012 oscilloscope view

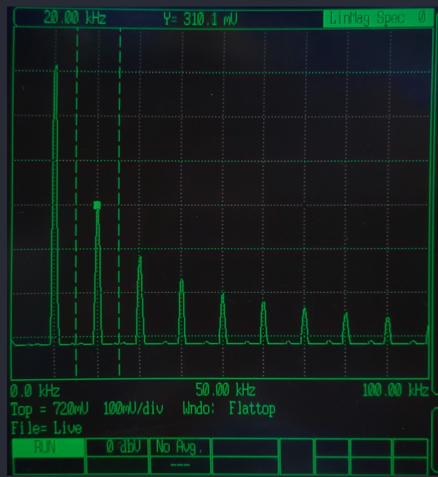

Figure 0.7: SR770 FFT Network Analyzer view

- Triangle Wave: (TRIANGLE waveform on 33500B) The triangle wave is a sum of all odd harmonics of the fundamental frequency as shown in Fig. 0.9. But the amplitude of the harmonics decreases

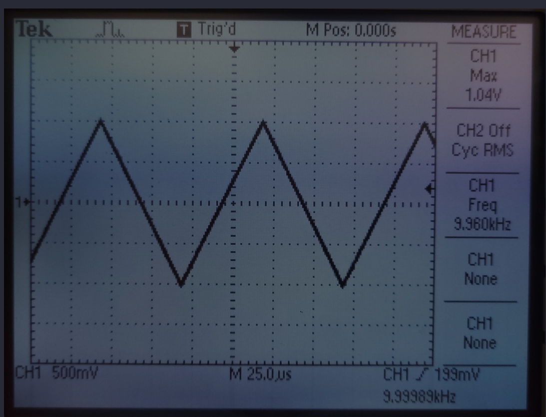

Figure 0.8: TDS 1012 oscilloscope view

Figure 0.9: SR770 FFT Network Analyzer view

by a factor of $1/n^2$. This power law makes the amplitude hard to read in the linear scale, but in the log scale, the amplitudes are clearly visible as shown in Fig. 0.10.

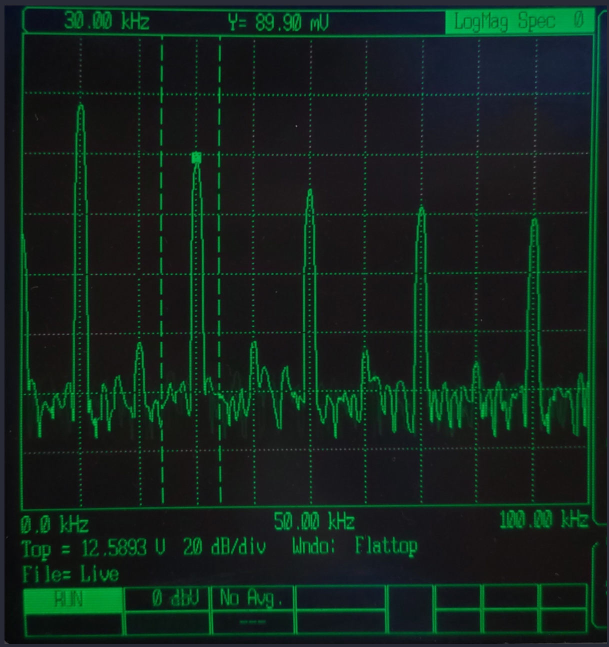

Figure 0.10: SR770 LOG MAGNITUDE

Superposition of sine waves

- 770: 40 kHz, 1 V sine wave → SUMMER input A
- 33500B: 50 kHz, 2 V sine wave → SUMMER input B
- SUMMER output → 770 SIGNAL IN & TDS 1012

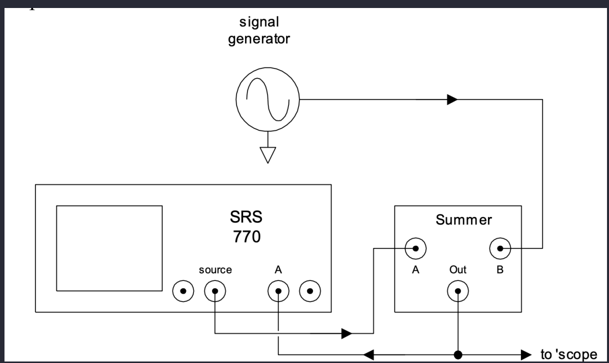

Figure 0.11: Diagram of setup

From the 770, we can easily see the two sine waves in the frequency domain as shown in Fig. [insert fig 0.11], but the time domain (scope) does not clearly describe the summation of the two waves.

Similar amplitude

- 770: 50 kHz, 1 V sine wave → SUMMER input A
- 33500B: 51 kHz, 1 V sine wave → SUMMER input B

In the full (100 kHz) span view, we can't see the two peaks. To increase the frequency resolution, we can reduce the span in the 770 FREQ menu, but this will increase the acquisition time.

e.g. a full span of 100 kHz has an acquisition time of 4 ms; the 'voltage sampling' rate is 256 kSa/s, or 256 samples per ms i.e. 1024 samples in 4 ms.

For our experiment, we set the span to 1.56 kHz to clearly see the two peaks, but this costs us an acquisition time of 256 ms or $256 * 256\,\text{samples/ms} = 65536$ samples. This trade-off can be described by the 'frequency duration uncertainty principle':

$$\text{(frequency resolution achievable)} \cdot \text{(acquisition time required)} \geq \text{a number}$$

The 770 magic number is

$$100\,\text{kHz} \cdot 4\,\text{ms} \geq 400\,\text{kHz}\,\text{ms}$$

which we can use to find the minimum acquisition time for a given frequency resolution e.g. the 1.5625 kHz span required

$$\begin{aligned}\text{(acquisition time req)} &\geq 400/\text{(freq resolution)}\\ &= 400/1.5625 = 256\,\text{ms}\end{aligned}$$

Windowing & Different amplitude Recommended windowing:

- Uniform: close spaced peaks with similar amplitudes
- Flattop: accurate peak height measurement
- Hanning: good for spectral resolution
- BMH: good for weak peak near strong peak, but not the best resolution for top peak & amplitude accuracy

Summary

We have learned the basic concepts of Fourier series and the Fourier transform, and analyzed basic waveforms from the 33500 in both the time and freqency domains using the scope and 770 respectively. Furthermore, there are trade offs between frequency resolution and acquisition time, and the choice of windowing function which can affect the accuracy of the measurement.

1 Signal Recovery Under Noise

Chapter 6: Noise Waveforms

After reading chapters 6 & 15 we will be proceeding with the Signal Recovery from under noise experiment.

Initial Setup T-Spin's "buried treasure" (BT) module through a F splitter to both the 'scope and the SR770 as shown in Fig. 1.1 and 1.2.

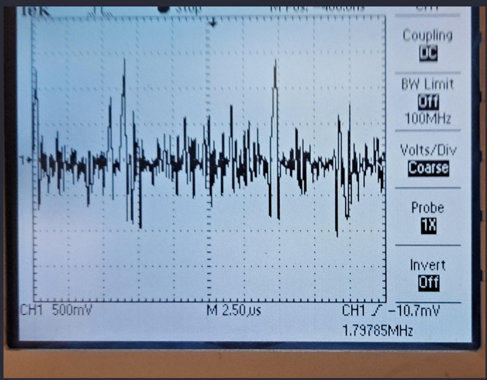

Figure 1.1: BT Noise on scope (Time Domain)

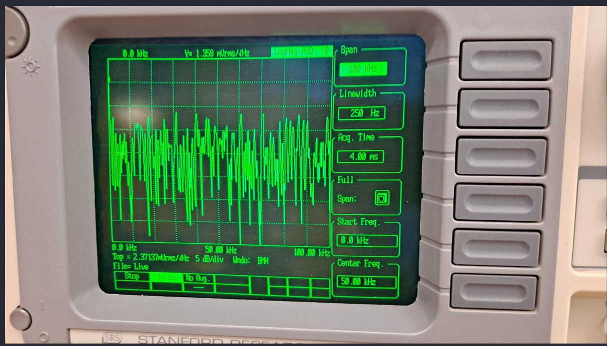

Figure 1.2: BT Noise on SR770 (Frequency domain)

Changing the rotary switch on the BT module to "filtered noise" (0.2) will filter out all frequencies about the full span of the 770, i.e., all frequencies above 100 kHz will be filtered out which is shown in Fig. 1.3 and 1.4.

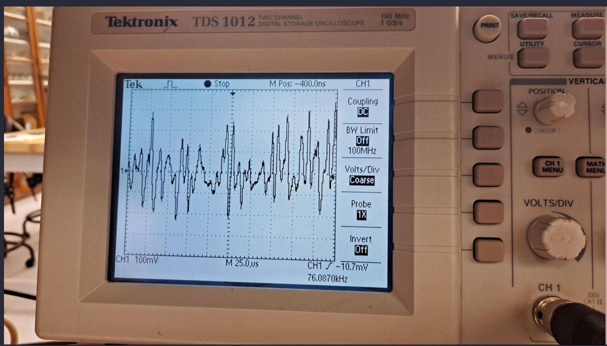

Figure 1.3: Filtered Noise on scope

Figure 1.4: Filtered Noise on SR770

By averaging out the noise using 64 averages we achieved a flat waveform, demonstrating that the power spectrum for white noise, because it is statistically random, is flat across frequencies. The 64 in this case, refers to the idea that during acquisition we are taking an average for ever 4 ms

Figure 1.5: Filtered Noise on SR770 with 64 averages

We then switched the noise to “pink” noise and averaged out the waveform to its power spectrum.

Figure 1.6: Pink Noise on SR770 with 64 averages

In attempting to record the V_{rms} for a monochromatic 1 V, 50 kHz sinusoidal wave, we erroneously plug our BNC cable into the sync output of the waveform generator and saw a square wave. Which makes sense due to information rates being in binary format.

We eventually recorded the power spectrum of a monochromatic sinusodial wave of 1 V and 50 kHz (1.7), and the $V_{\text{rms}} = 0.707$ V as measured by the scope.

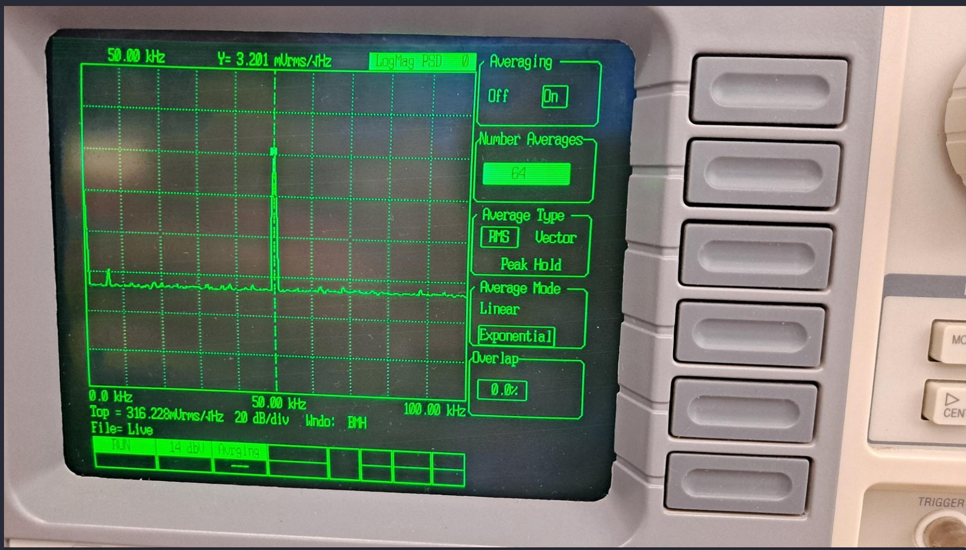

Figure 1.7: 1 V, 50 kHz sinusoidal wave on SR770

Measuring RMS Voltage of Noise For white noise, to find the mean square measure of voltage we must take the Power spectral density $PSD = VSD^2$ and integrate over the frequency space:

$$\int_0^{\infty} S(f)df = \langle V^2(t) \rangle$$

So in the case of white noise (Fig. 1.7)

$$\int_0^{100\,\text{kHz}} S(f)df = PSD * (\text{Frequency Range}) = \left(\frac{244\,\mu\text{V}}{\sqrt{\text{Hz}}}\right)^2 * 100\,\text{kHz} = 5.96 \times 10^{-3}\,\text{V}$$

Furthermore, we know the relation of mean-square voltage and the total power:

$$V_{\text{rms}} = \sqrt{\langle V^2(t) \rangle} = 0.0772V$$

Noise Plus Sine Wave

Experiment Part 1: Setup The Signal Pipeline TM:

BT Filtered Noise or Buried Signal → WIDE BAND AMP 25x → Low Pass 20 kHz, $Q = 0.71$ → Summer adding Sine 10kHz, 0.3 V (Output off initially).

770 Config

- SPAN: 195 Hz
- AVERAGES: 64 (2.048 s)
- MEASure: PSD (actually VSD but the 770 calls it PSD)
- WINDOW: Flattop (for peak amplitude)
- Display: Log Magnitude
- CENTER Freq. 10 kHz

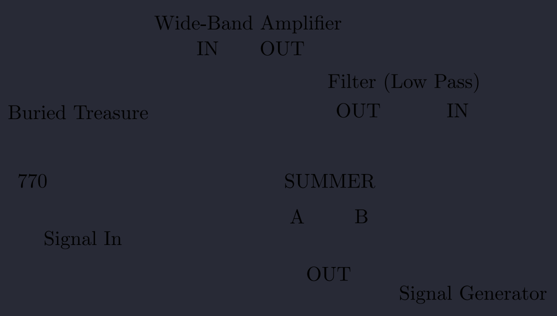

Figure 1.8: Control setup for Signal Under Noise

Pre Trial Observations Scope vs. 770

The scope obviously shows a noise signal where we can't see the sine wave, but the 770 clearly shows the sine wave as shown in Fig. 1.9.

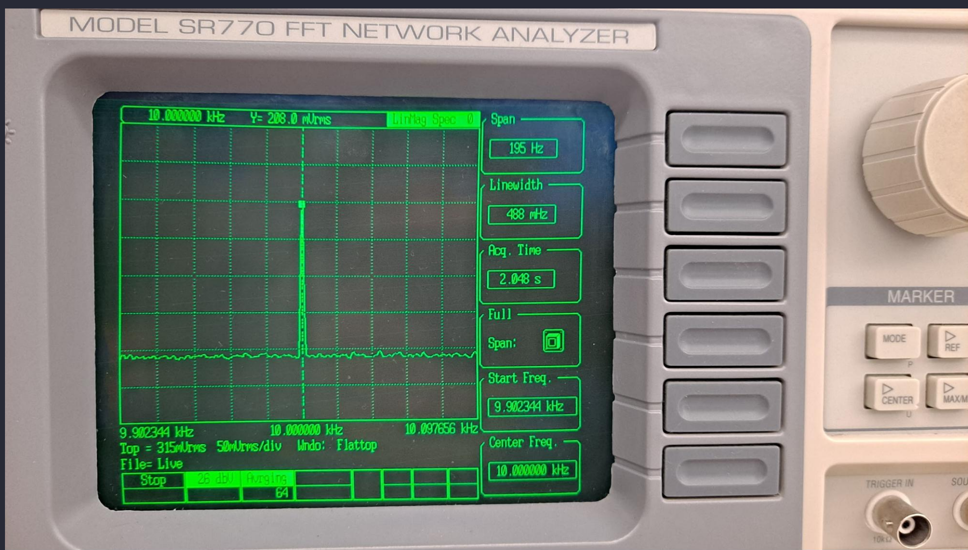

Figure 1.9: Sine wave on SR770 with noise

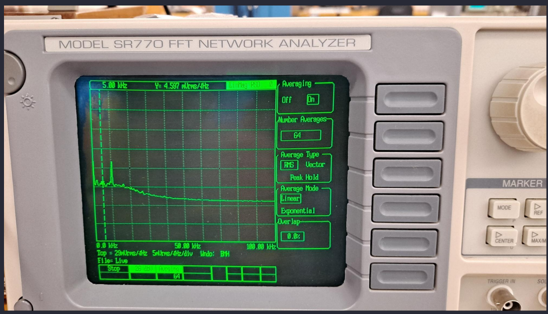

Figure 1.10: Filtered Noise with Sine wave on SR770

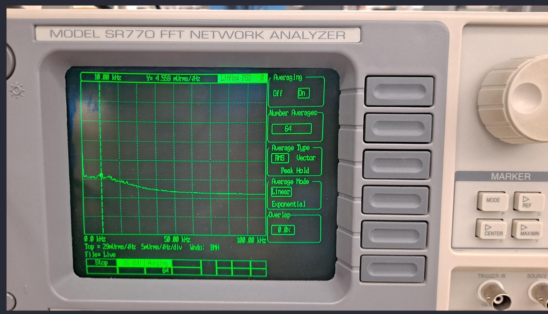

Figure 1.11: Filtered Noise without Sine wave on SR770

Procedure

- With the sine wave off (Fig 1.11), take a sample of the VSD at $5, 10, 15, 20$ kHz in order to calculate the average VSD of the noise floor. We have removed 0 Hz from the calculation as VSD goes to infinity as it approaches 0 Hz.
- Meausre $\langle V_n^2 \rangle$ using the equation for relating the spectral density to the mean-square voltage:

$$\langle V_n^2 \rangle = \int_0^{\infty} S(f)df = PSD * (\text{Frequency Range}) = VSD^2 * 20\,\text{kHz}$$

- Measure the mean-square measure of the signal sine wave $\langle V_s^2 \rangle$ by connecting the sine signal directly to the 770 and measuring the V_{rms} using the Voltz rms UNITS menu & 64 Averages.
- Calculate the predict the measure of the signal (sine wave) V_s plus noise V_n using

$$\langle [V_s + V_n]^2 \rangle = \langle V_s^2 \rangle + \langle V_n^2 \rangle = \langle V_T^2 \rangle$$

  where the cross terms are zero since the signal and noise are independent of each other.
- Measure $\langle V_T^2 \rangle$ using the TDS1012 scope and calculate the average for 20 samples for the Signal plus noise input:
  - Finding rms voltage on scope: Go to SETTINGS; AQCUIRESE; SAMPLE; AVERAGE (16). Then take 20 samples and average them (refer to Fig. 1.12).
- Check if the measured value is within the predicted value. Then calculate how much of the signal plus waveform is due to the signal from a power basis i.e.

$$\%\ \text{Signal} = \frac{\langle V_s^2 \rangle}{\langle V_T^2 \rangle} * 100\%$$

- Repeat two more trial but with WIDEBAND AMP ($1 \times 1 \times 2.5 = 2.5$) and with Sine wave at 1 V (25x WIDEBAND AMP).

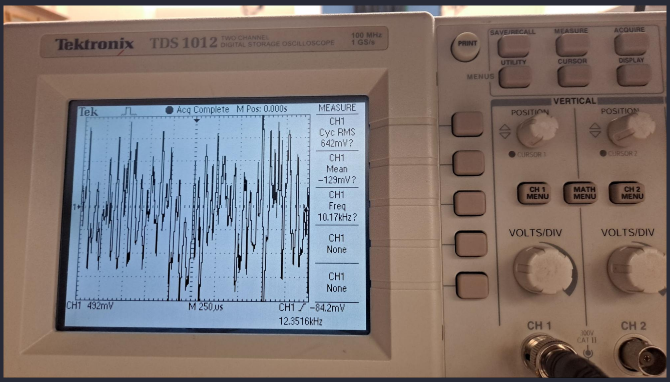

Figure 1.12: Sample measurement of $V_T = 642\,\mathrm{mV}$ on TDS1012

Data

Freq (Hz)	Trial 1 (mV/$\sqrt{\text{Hz}}$)	Trial 2 (mV/$\sqrt{\text{Hz}}$)	Trial 3 (mV/$\sqrt{\text{Hz}}$)
5	4.635	0.4759	4.268
10	4.52	0.4298	4.306
15	4.174	0.4145	3.853
20	3.193	0.307	3.201
Avg	4.1	0.41	3.9
STD (± mV)	0.7	0.07	0.5
Signal V_s (mV)	207.1	208.8	703.9

Table 1: 770 measurements for Average V_{rms} for Noise and Signal

Sample	Trial 1 (mV)	Trial 2 (mV)	Trial (V)
1	834	218	1.69
2	845	187	0.759
3	468	191	1.34
4	787	252	0.851
5	389	244	0.964
6	745	213	0.95
7	532	213	0.633
8	635	257	0.879
9	725	220	1.22
10	523	249	1.4
11	960	197	0.588
12	401	223	1.17
13	676	249	0.857
14	380	189	1.1
15	749	204	0.812
16	512	208	0.817
17	621	163	0.796
18	625	169	0.942
19	528	232	0.683
20	672	235	1.32
AVG (V)	0.6	0.22	1.0
STD (± V)	0.2	0.03	0.3

Table 2: Scope measurements for V_T

Example calculation for the rms measure of the signal plus noise in Trial 1:

$$\begin{aligned}\langle V_n^2\rangle &= PSD * \Delta f = \left(4.1\,\mathrm{mV}/\sqrt{\mathrm{Hz}}\right)^2 * 20\,\mathrm{kHz} = 0.34\,\mathrm{V}\\ \langle V_s^2\rangle &= 207.1\,\mathrm{mV}^2 = 0.04\,\mathrm{V}\\ \langle V_T^2\rangle &= 0.34\,\mathrm{V} + 0.04\,\mathrm{V} = 0.38\,\mathrm{mV}\\ V_T &= \sqrt{0.38\,\mathrm{V}} = 0.62\,\mathrm{V}\end{aligned}$$

and error

$$\delta V_n = 2 * 0.7 \times 10^{-3}\,\mathrm{V}/\sqrt{\mathrm{Hz}} * 20\,000\,\mathrm{Hz} = 30 \implies \delta V_T = \frac{1}{2} * 30 = 15V$$

The error bars on the 770 measurements are not very useful here since we are propogating the average spectral density based on only 4 frequencies (5, 10, 15, 20 Hz) onto 20,000 Hz. So we will rather use the error bars from the scope measurement and ignore the 770 error bars.

	Trial 1	Trial 2	Trial 3
V_n^2	0.341	0.0033	0.305
V_s^2	0.043	0.0436	0.495
V_T^2	0.384	0.0469	0.801
V_T	0.62	0.22	0.89
% Signal	11%	93%	62%

Table 3: Voltage measurements and Percent of Power due to Signal

Quick Analysis of Data As we compare the data from Table 2 and Table 3, all the measured values are within the predicted values, and the % signal is within the expected range stated in the Teach Spin manual for Trial 1 (11%).

Summary So we were able to successfully measure the mean-square voltage of the signal plus noise using the 770 and the scope. The weird unit of interest $V/\sqrt{\mathrm{Hz}}$, the voltage spectral density, becomes a useful tool to measure noise and delineate the signal from the noise. (9/5/24) Trial 2 clearly shows a smaller noise floor presents a clearer signal defined by the percent of signal power, and vice versa for Trial 3 and a larger sine wave amplitude.

Chapter 15: Signal Recovery Under Noise

Experiment Part 2: Signal Pipeline TM:
BT Noise (A, B, C, or D) $\to$ SR770

770 Config

- AVERAGE: 16 Exponential mode (for now)
- MEASure: PSD (Actually VSD but the 770 calls it PSD)
- WINDOW: Uniform for close peaks with similar amplitudes, Flattop for measurment

Notes from Teach Spin Inexact Calculation of

- 770 sorts frequencies into 400 bins of width δf, so any bin with just noise has expected value

  $$\langle V_n^2 \rangle = \int S(f)df \to S\delta f$$

  compared to an expected value of the signal (plus some noise) has expected value

  $$\langle \left[V_(t) + A\cos(\omega t - \phi) \right]^2 \rangle = S\delta f + \frac{A^2}{2} = \langle V_P^2 \rangle$$

  thus we need signal power in a single bin to be much larger than the STD of noise floor values in the bin.

- To find and N standard deviation detection of the signal with n acquisitions (or Averages) the noise of the bin must be smaller than

  $$\frac{A^2}{2} > N\frac{S\delta f}{\sqrt{n}}$$

  e.g. $N=$ 5-sigma detecti on of the signal with $n = 16$ Average.

Procedure

- Turn the Rotary switch to buried signal A and output the signal *directly* to the 770.
- Decrease the Frequncy span to the next smaller value and use exponential average to look for a peak that stands up atleast 20 dB above the noise floor.
- Repeatedly decrease the span and pan through the full range of the 770 until you find the peak.
- Once the peak is found, measure the peak voltage using the MEASure: Spectrum; Units: Voltz Pk.
- If the peak does not stand out as far, subtract the mean square power per bin (PSD per bin) between the peak V_P an an only-noise bin V_n i.e.

  $$\frac{A^2}{2} = \langle V_P^2 \rangle - \langle V_n^2 \rangle$$

BT Signal	A (freq)	V_P (mV)
A	2.875 kHz	15.62
B	70.26 kHz	6.466
C	33.57 kHz	1.46

Table 4: Measured Frequency and Peak Voltage for BT Signal

Figure 1.13: 770 View for BT Signal C

Summary From Fig. 1.13 We had to pan through the full range of the 770 at $100/2^6 = 1.5625$ kHz to find the peak at 33.57 kHz. This means that we had todecrease the span 6 times to find the peak, and comb through each $\tilde{1}.5$ kHz range and wait for the 770 to acquire n average each time. Since the 770 cannot analyze the full span at small frequency ranges all at once, we had to manually pan through the full range for each iteration. Finally, we chose not to look for Signal D as it was only limited by the time it would have taken to pan through the full range of the 770 at even smaller frequency ranges and larger averaging.

2 AM Radio Reception

2.1 Chapter 3: Modulated Waveforms - Amplitude Modulation

Notes AM Modulation in a Nutshell:

A high frequency "carrier" wave f_c transports a lower frequency "program" content f_p, i.e., for a simple sinusoidal carrier wave the modulated signal is given by

$$V(t) = [A(1 + \alpha \cos(2\pi f_p t))] \cos(2\pi f_c t)$$

where $\alpha < 1$ is the modulation index. We can pretend that $[\,] = A(\,)$ is equivalent to the amplitude of the modulated waveform, so this makes the amplitude vary over time between $A(1-\alpha)$ and $A(1+\alpha)$. With some trig identities, we can rewrite this as

$$V(t) = A\cos(2\pi f_c t) + \frac{A\alpha}{2}\cos(2\pi(f_c + f_p)t) + \frac{A\alpha}{2}\cos(2\pi(f_c - f_p)t)$$

Thus the sum of three sinusodial waves with frequencies f_c, $f_c + f_p$, and $f_c - f_p$—i.e., the information content of an AM signal is contained within the sidebands of the carrier wave in the frequency domain.

Exploration

- Carrier: 770 internal source—50 kHz, 1 V → Multipler module (MULT) input A
- Program: 33500B—2 kHz, 5 V → SUMMER module input B
- DC VOLTAGE module 5 V → SUMMER input A
- SUMMER output → MULT input B
- MULT output → scope & 770

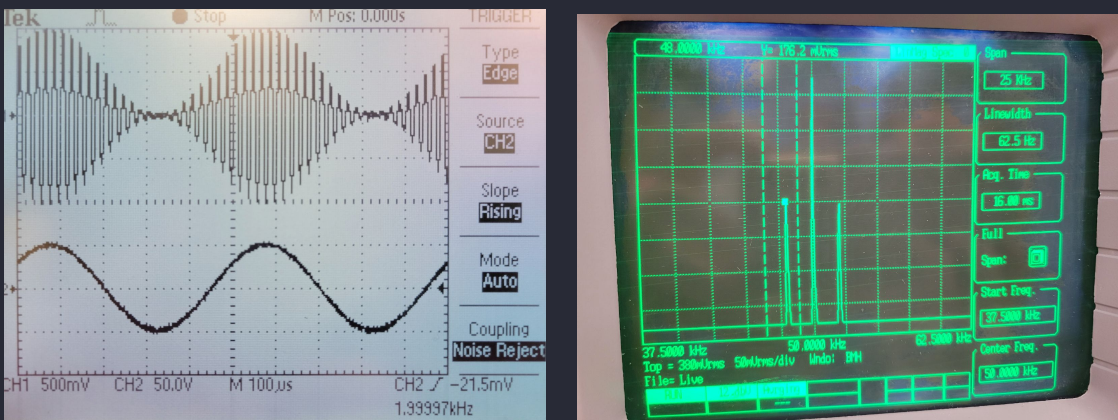

Figure 2.1: Scope (left) with bottom program content and 770 (right) view of AM Modulation of a 50 kHz carrier wave with a 2 kHz program wave.

In Figure 2.1, we see the scope and 770 view of the AM modulation of a 50 kHz carrier wave with a 2 kHz program wave. The scope view shows familiar looking AM modulated waveform, while the 770 clearly shows the carrier frequency at the center and the two sidebands which dictate the frequency of the program content.

Changing content of the two waves

- Increase frequency of program content:
  - The sidebands move further away from the carrier frequency just as the theory predicts $f_c \pm f_p$.

- Change carrier frequency:
  - Moves the 3 peak structure left or right (low freq and high freq respectively) on the 770 (Figure 2.2).
  - The envelope of the modulated waveform remains the same, but the inside oscillations increase as f_c increases.

Figure 2.2: Increasing (right) and decreasing (left) program frequency.

- Program amplitude:
  - Increasing the program amplitude increases the amplitude of the sidebands independent of the carrier amplitude (Fig. 2.3).

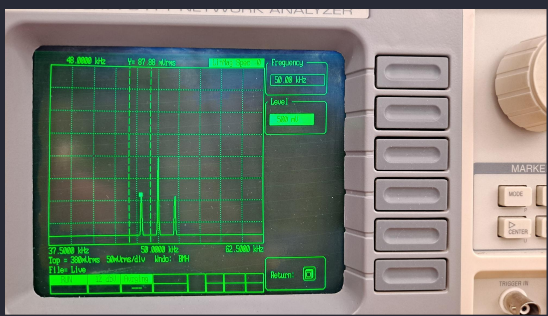

Figure 2.3: Decreasing program amplitude.

- Carrier amplitude:
  - Increasing the carrier amplitude increases the amplitude of the carrier wave independent of the program amplitude (Fig. 2.4).

Figure 2.4: Increasing (right) and decreasing (left) carrier frequency.

- Program content to square wave:
  - There are multiple sidebands at odd multiples of the program frequency.

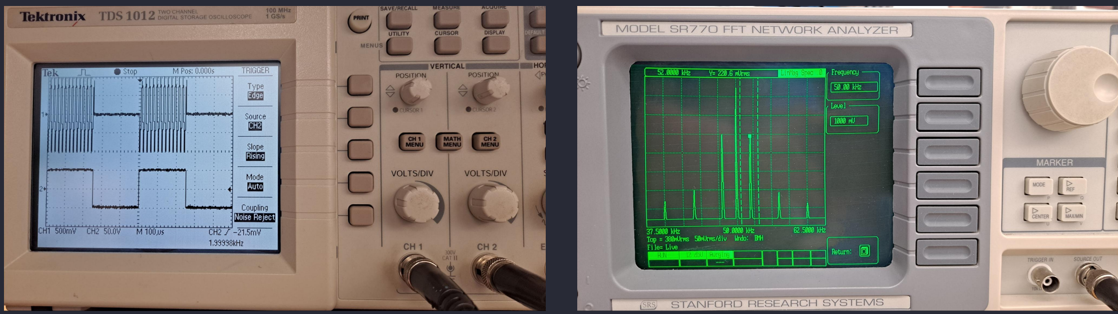

Figure 2.5: Scope (left) with program content in the bottom and 770 (right) view of AM Modulation of a 50 kHz carrier wave with a 2 kHz square wave program wave.

Why $\alpha < 1$? The multiplier ouputs a scaled product of the two input signals

$$V_{\text{out}} = \frac{V_A V_B}{10}$$

for inputs with voltage ±10 V, and frequency lower than 1 MHz. Thus for 5 V DC input from the summer with the program signal gives

$$\begin{aligned} V_{\text{out}} &= (5V + P\cos(2\pi f_p t))(A\cos(2\pi f_c t))/10 \\ &= \frac{5A}{10}V\left(1 + \frac{P}{5V}\cos(2\pi f_p t)()\right)\cos(2\pi f_c t) \end{aligned}$$

So the waveform has a modulation index $\alpha = \frac{P}{5V}$. In our first case (Fig. 2.1), $P = 5V$ so $\alpha = 1$, Here the scope clearly shows the two distinct frequencies that make up the AM waveform—i.e., the envelope matches the program content shown simultaneously below, and the carrier frequency is resolved in the small oscillations within the envelope.

When we increase $\alpha \to 2$ (Fig. ref here), the program content is overmodulated or *distorted* which makes it hard to extract out the program content from the modulated waveform.

Chapter 11: AM Radio Reception

Notes

- Since AM radio signals are roughly 540-1600 kHz, i.e., 500-200 m wavelength, the EM waves are pretty uniform and can be received by our simple electrical wire antenna connected to an LC-resonant circuit.
- The LC-resonant slighly tunes the frequency range into a narrow band of frequencies, which can be changed by adjusting the number of inductors we put in series (Fig. 2.7).
- Downcoversion: Before we narrow the frequency search range, we can first use downconversion to bring the high frequency AM signals to a lower frequency range provided by the High-Frequency (HF) Mixer module.
  - Local-oscillator (LO) source: Using the 33500B, we set the LO frequency so that the difference between the LO and the AM signal is in the range of our 770 (100 kHz). e.g., for target radio station (RF) 850 kHz, setting the LO to 770 or 930 kHz will output a 80 kHz difference frequency from the HF mixer. It will also ouput and sum & difference frequencies from other radio stations which we must filter with the IF output.
  - IF Filter: To create a fixed pass band that only allows the a narrow range of difference frequencies to pass through.

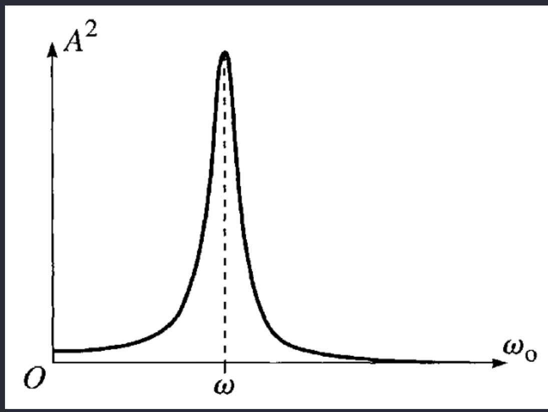

Figure 2.6: Resonant frequency of RLC circuit (Taylor, Classical Mechanics). Our circuit has a broad resonance (rather than sharp) which will receive a band full of AM stations.

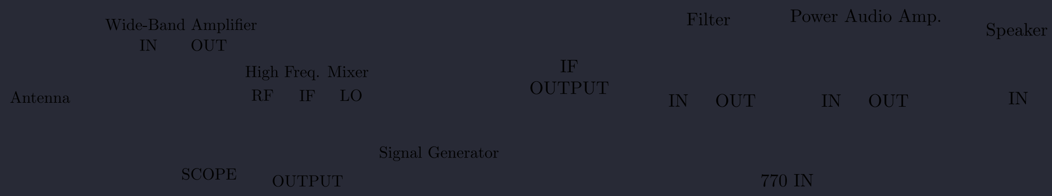

Figure 2.7: Experimental setup for AM radio reception.

Experiment: Finding Radio Stations

- First note a near by radio station we can pick up from St. Louis (Radio Station List). e.g. KFUO 850 kHz.
- Set three inductors in series on the radio antenna circuit
- Radio antenna circuit OUPUT → Wideband Amp (10x) or until the signal is visible on the scope
- Wideband Amp OUTPUT → HF Mixer INPUT RF
- 33500B 0.7 to 1.1 V (1 V Choosen) with difference frequency of 80 kHz OUTPUT → HF Mixer INPUT LO
- IF OUT → Filter Module 80 kHz; 8 Gain
- Filter OUT → scope & 770 and note the visualized signal
- Tune the LO frequency to get a strong signal
- De-modulate the signal by tuning the LO frequency *exactly* to the Radio station frequency (carrier freq), i.e., the zero beat frequency, e.g.,

3 The Fluxgate Magnometer