

Physics 411: Mechanics

Homework: Junseo Shin

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Homework 1

Due 1/24 9pm

1. Given: the 2D Cartesian relation to polar coordinates

$$\hat{\mathbf{r}} = \hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi, \quad \hat{\phi} = -\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi \quad (1)$$

We can write \mathbf{v} as a linear combination of $\hat{\mathbf{r}}$ and $\hat{\phi}$

$$\begin{aligned} \mathbf{v} &= v_r \hat{\mathbf{r}} + v_\phi \hat{\phi} \\ &= v_r (\hat{\mathbf{x}} \cos \phi + \hat{\mathbf{y}} \sin \phi) + v_\phi (-\hat{\mathbf{x}} \sin \phi + \hat{\mathbf{y}} \cos \phi) \\ &= (v_r \cos \phi - v_\phi \sin \phi) \hat{\mathbf{x}} + (v_r \sin \phi + v_\phi \cos \phi) \hat{\mathbf{y}} \end{aligned}$$

and since we know the vector in Cartesian coordinates is

$$\mathbf{v} = v_x \hat{\mathbf{x}} + v_y \hat{\mathbf{y}}$$

we can equate the components to get

$$\begin{aligned} v_x &= v_r \cos \phi - v_\phi \sin \phi \\ v_y &= v_r \sin \phi + v_\phi \cos \phi \end{aligned}$$

multiplying the first equation by $\cos \phi$ and the second by $\sin \phi$ and adding them together

$$\begin{aligned} v_x \cos \phi &= v_r \cos^2 \phi - v_\phi \sin \phi \cos \phi \\ v_y \sin \phi &= v_r \sin^2 \phi + v_\phi \sin \phi \cos \phi \\ v_x \cos \phi + v_y \sin \phi &= v_r (\cos^2 \phi + \sin^2 \phi) \end{aligned}$$

or simply

$$v_r = v_x \cos \phi + v_y \sin \phi$$

Likewise,

$$\begin{aligned} v_y \cos \phi &= v_r \sin \phi \cos \phi + v_\phi \sin^2 \phi \\ v_x \sin \phi &= v_r \sin \phi \cos \phi - v_\phi \cos^2 \phi \end{aligned}$$

and subtracting the second equation from the first

$$v_y \cos \phi - v_x \sin \phi = v_\phi (\sin^2 \phi + \cos^2 \phi)$$

Therefore we get the components of \mathbf{v} in polar coordinates

$$\begin{aligned} v_r &= v_x \cos \phi + v_y \sin \phi \\ v_\phi &= -v_x \sin \phi + v_y \cos \phi \end{aligned}$$

Since $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are *constant*, the time derivatives of (1) are

$$\begin{aligned} \dot{\hat{\mathbf{r}}} &= \frac{d}{dt} \hat{\mathbf{r}} = \hat{\mathbf{x}} \frac{d}{dt} (\cos \phi) + \hat{\mathbf{y}} \frac{d}{dt} (\sin \phi) \\ &= (-\dot{\phi} \sin \phi) \hat{\mathbf{x}} + (\dot{\phi} \cos \phi) \hat{\mathbf{y}} = \dot{\phi} \hat{\phi} \end{aligned}$$

and

$$\begin{aligned} \dot{\hat{\phi}} &= \frac{d}{dt} \hat{\phi} = -\hat{\mathbf{x}} \frac{d}{dt} (\sin \phi) + \hat{\mathbf{y}} \frac{d}{dt} (\cos \phi) \\ &= (-\dot{\phi} \cos \phi) \hat{\mathbf{x}} + (-\dot{\phi} \sin \phi) \hat{\mathbf{y}} = -\dot{\phi} \hat{\mathbf{r}} \end{aligned}$$

Homework 2

Due 1/31

Check for the code. asdfasdf stuff here
why is this not wokring [here](#)

Homework 3

Due 2/7 9pm
