Lab Notebook: Fourier Methods

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Contents

Intro

Low Temp 101 Using log temp scale to measure superductivity, electron-phonon and electron-defect scattering phenomena using vacuums and cryogenics.

0.1 Samples

- Copper (Cu) Wire
  - Diameter $D = 165 \pm 10\,\mu\text{m}$, Length $l = 482.0 \pm 0.5\,\text{cm}$: Area $A = \pi D^2/4$
  - Bloch-Güneisen formula

    $$[\rho(T) - \rho_0] = 4\rho_\Theta \left(\frac{T}{\Theta}\right)^5 \int_0^{\frac{\Theta}{T}} \frac{z^5 dz}{(e^z - 1)(1 - e^z)} \tag{0.1}$$

    where

    $$[\rho(T) - \rho_0] = \begin{cases} 124.4 & T \ll \Theta \\ \left(\frac{T}{\Theta}\right)\rho_\Theta & T \gg \Theta \end{cases}$$

    or for a free electron gas

    $$[\rho(T) - \rho_0] = 497.6\left(\frac{T}{\Theta}\right)^5 \rho_\Theta$$

- Dysprosium (Dy) foil
  - Width $w = (2.7 \pm 0.2)$ mm, Thickness $t = (0.11 \pm 0.01)$ mm, Length $l = (18.0 \pm 0.5)$ mm: Area $A = wt$
  - Ferromagnetic ordering (spins align) below Curie Temp
  - Phase transition to HCP (Hexagonal Close-Packed)
- Niobium Titanium (NbTi) wire
  - Diameter $D = 114 \pm 4$ μm, Length $l = 10.0 \pm 0.4$ cm: Area $A = \pi D^2/4$
  - Superconducting transition temperature at $T_c = 9.50$ K [2]
  - Electrical resisivity $\rho \to 0$ at $T < T_c$
  - Type II superconductor: Solenoid can generate 9 T magnetic field at 4 K
- Lead (Pb) foil
  - Width $w = 1.6 \pm 0.2$ mm, Thickness $t = 0.12 \pm 0.01$ mm, Length $l = 17.0 \pm 0.5$ mm: Area $A = wt$
  - Superconducting (Type I) transition temperature at $T_c = 7.19$ K [1]
  -

0.2 Measuring Resistivity

- 4 terminal remote sensing: to reduce voltage drop from force leads
- Resistivity to resistance formula

  $$\rho = R\frac{A}{l} = \frac{VA}{Il}$$

  where $I = 1$ mA for wire (Cu and NbTi) and $I = 10$ mA for foil (Dy and Pb)

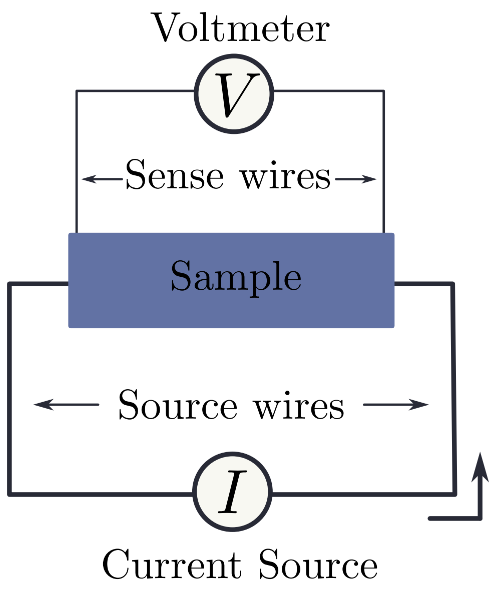

Figure 0.1: Four-point measurement: 1 & 4 are force leads, 2 & 3 are sense leads.

- fractional error for Cu and NbTi

$$\frac{\delta p}{p} = \sqrt{\left(2\frac{\delta D}{D}\right)^2 + \left(\frac{\delta l}{l}\right)^2 + \left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2}$$

and for Dy and Pb

$$\frac{\delta p}{p} = \sqrt{\left(\frac{\delta w}{w}\right)^2 + \left(\frac{\delta t}{t}\right)^2 + \left(\frac{\delta l}{l}\right)^2 + \left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2}$$

0.3 Measuring Temperature

Using Chebyshev polynomial fit from Cernox manual to convert Resistance to temperature

$$\log_{10} T = \sum_i a_i \cos(iw \log_{10} R) + b_i \sin(iw \log_{10} R)$$

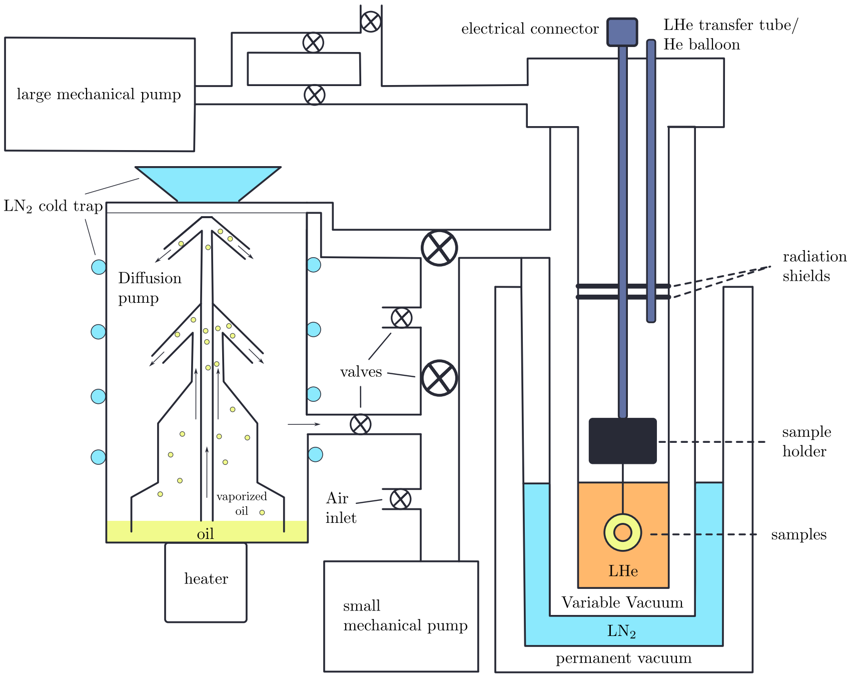

Figure 0.2: Main low temp apparatus: Drawn by Junseo! with help from Lab Manual

0.4 Cooling

- Cooling to 80 K
  - Variable Vacuum → 5-10 Torr (5-10 mbar); Check Pressure with G1 (Pirani Gauge)
  - Release V2 to ↓ P; V3 to ↑ P Until 5-10 Torr
  - Fill LN_2 into OUTER DEWAR
  - Wait roughly 4 hours to cool down to ∼ 80 K, refilling LN_2 as needed
  - Once we reach 80 K, let the sample warm back up to room temp

1 Data & Analysis

1.1 DATA COLLECTION DAY 1:

LAB CHART CONFIG

- Multiscan: Time between cycles = 30 s; Number of cycles = 4000 $\implies$ Total time 120000 s
- Timing: 200 ms Integration period; Range $-100mV \to 100mV$
- Variable Vacuum: 220; 215; 205

Notes

- We were able to cool down the sample to 80 K in preparation for the next experiment to cool the samples down to 4 K.
- Using the Multimeters and the lookup chart, we can quickly convert the voltage readings to temperature. But at room temperature, a voltage of 4.926mV, corresponded to a temperature of around 278K based off the manufacturer calibration settings which is quite cold. For room temp ($\sim 293.0K$) the predicted measured voltage is around 4.743mV. So there must be calibrations made to a reading to correct the temperature from the 15 K offset.

1.2 DATA COLLECTION DAY 2

LAB CHART CONFIG

- Multi Scan: 30s Time between cycles; 4000 cycles $\implies$ 120,000 s
- Timing: 200 ms Integration period; Range $-100mV \to 100mV$
- Variable Vacuum: $\to$ 80 K, 4.57 Torr; $\to 4K$, 0.39 Torr; Cool down to room temp 5.67 Torr at the end.

Cooling to 4 K

- Bringing Variable Vacuum to $< 10^{-4}$ Torr:
  - open V2
  - open v4
  - small pump on
  - Heater + fan + LN_2 on
  - wait 20 minutes to wamr up
  - open v1
  - close v2
- Adding liquid helium LHe (it is fragile in air):
  - Fill chamber with Helium gas (it comes out of rubber stop)
  - What the voltage wil read at 4.2 K: $400\Omega \to 4$ mV
  - Why do we take out the helium? As we take out the helium,
  - Superfluid transition below $20K \to 4K$ happens really fast.
  - Bfore adding LHe, remove rubber stop to vent inner dewar to 1 atm (if we don't do this, a sudden pressure build up will explode the glass)
  - Change current to 10 μ A for less than 40 K
  - Change current to 1 μ A for less than 2.57 K
- Measuring helium vapor Pressure
  - Replace balloon with tube to analog pressure gauge
  - Slowly release V7 (with rubber stopper in and V8 closed)
  - to reduce pressure when V7 is fully open, slowly open V6
  - Record Pressure and Temp at equilibrium

Notes

- Some obstacles in cooling down to 4 K:
  - The diffusion pump was left on since the last lab group, which led to some inconsistencies in getting the inner dewar to a low enough vacuum pressure before starting the liquid cool down.
  - We weren't able to get full data down to ~ 1 K as we ran out of LHe. Although we were able to get a few data points for the vapor pressure vs temperature.
  - Furthermore, we were unable to get precise measurements for slow temperature change via raising the samples above the LHe bath.

1.3 Data Analysis

- Resistivity Error Analysis: Example for Copper Wire

$$\frac{\delta p}{p} = \sqrt{\left(2\frac{\delta D}{D}\right)^2 + \left(\frac{\delta l}{l}\right)^2 + \left(\frac{\delta V}{V}\right)^2 + \left(\frac{\delta I}{I}\right)^2}$$
$$= \sqrt{\left(2\frac{10}{165}\right)^2 + \left(\frac{0.5}{482}\right)^2} = 0.121 \implies 10\%$$

with negligible error in V and I since they are measured with high precision.

Sample	$\delta p/p$
Cu	10 %
NbTi	8 %
Pb	20 %
DY	10 %

Table 1: Fractional uncertainty in resistivity for each sample

- Resistance → Temperature conversion using Chebyshev polynomial fit python code:

```python
import numpy as np

def temp_K(ohm):
    a = [.8992, -.5026, .1524, -.306, .3517, -.204, 0.03868, 0.01532, -0.006606]
    b = [0, -1.226, .5757, -.3768, .1169, .09066, -.09729, .03033, -.001984]
    w = 2.299
    logT = 0;
    for i in range(0, len(a)):
    logT += a[i] * np.cos(i * np.log10(ohm) * w) + b[i] * np.sin(i * np.log10(ohm) * w)
    return 10**logT

```

e.g. `print(temp_K(1023.57))` prints out `1.19937` which matches the expected value from the lookup table.

Preliminary Data Analysis

- From log resistivity vs temperature, we can find the critical temperature T_c for each sample: ~ 10 K which is visually close to the expected values.

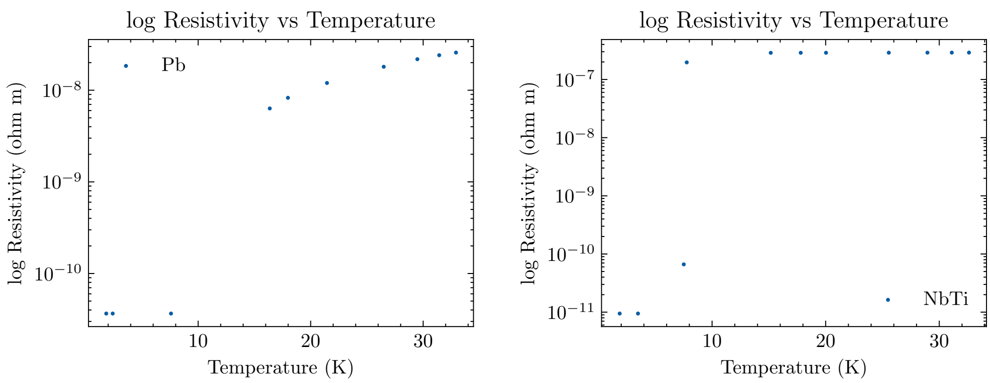

Figure 1.1: log Resistivity vs Temperature for finding critical temperature T_c

- Vapor Pressure Temperature vs Voltage Temperature readings

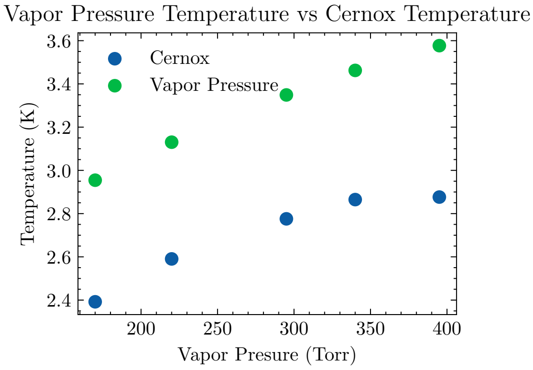

Figure 1.2: Cernox comes from Voltage reading to Temperature, while Vapor Pressure comes from analog pressure gauge

- RRR Calculation

Material	ρ_0 ($n\Omega \cdot m$)	ρ_{293} ($n\Omega \cdot m$)	RRR
NbTi	298	356	1.19 ± 0.14
Cu	0.135	17.0	126 ± 22
Dy	62.7	1.13×10^3	18.8 ± 3.3

- Bloch-Güneisen Formula Fit for Cu wire using numerical integration:

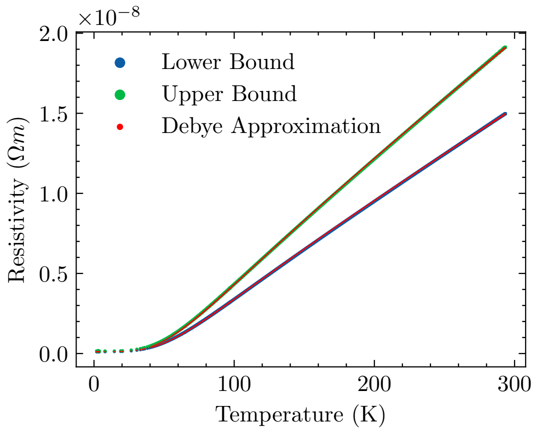

Figure 1.3: Bloch-Güneisen Formula Fit for Cu wire

We are unsure why the lower and upper bounds have such a large discrepancy, but the fit is quite reasonable for the data.

- Resistivity vs Temperature graphs for paper

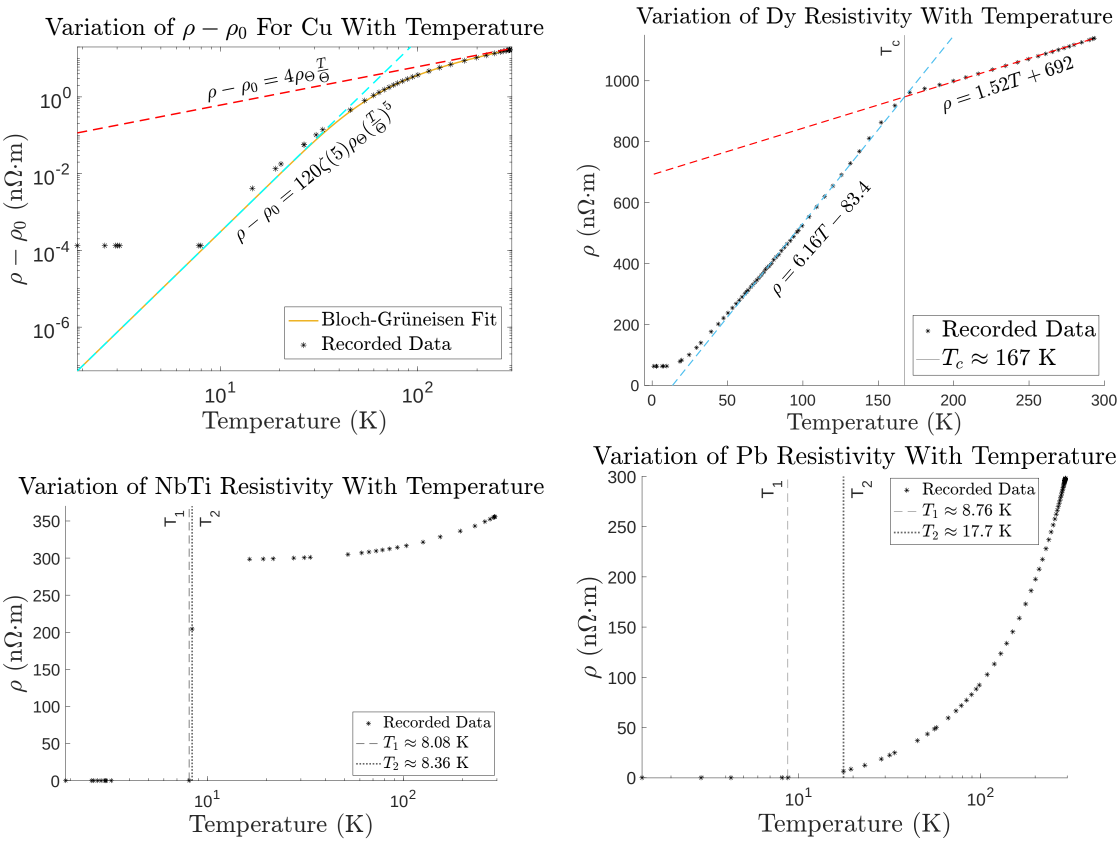

Figure 1.4: Resistivity vs Temperature for different materials

Notes

- We were able to get the data for the resistivity vs temperature for all the samples, and we were able to get the RRR values for each sample.
- Upon further analysis, we had to account for the offset in the temperature reading for both high and low temperatures i.e. we sclaed the resistance by a factor of 0.9665 to get a slightly more accurate temperature reading.
- The RRR make sense for the samples: Cu has the highest RRR as it is a good conductor vs NbTi which only gets low resistivity after it becomes superconducting.

References

[1] B. T. Matthias, T. H. Geballe, and V. B. Compton. Superconductivity. *Reviews of Modern Physics*, 35(1):1–22, January 1963.

[2] Advanced Lab Washington University. *Low Temperature Experiment Manual*, 2023.