

# 1 Statistical description of systems of particles

## Essential ingredients:

1. state of the system:
  - single spin-1/2 particle.  $\uparrow, \downarrow$
  - a bunch of spin-1/2 particles.  $\uparrow\uparrow\downarrow \dots$
  - a simple 1D Harmonic Oscillator:  $E = (n + 1/2)\hbar\omega$ , with states  $|n\rangle$
  - a bunch of 1D HO:  $|n_1, n_2, \dots, n_N\rangle$
2. Statistical ensemble: Instead of a simple experiments, we consider an ensemble of many exps.
3. Basic postulate about a priori probabilities (relative prob of finding the system in any of its accessible states)
4. Calculate probabilities

**Example:** 3 spin-1/2

State	Spin	Energy	$\Omega(E)$	$y_k = \uparrow, \downarrow$
$\uparrow\uparrow\uparrow$	3/2	$-3\mu H$	1	$\Omega(-\mu H, \uparrow)$
$\uparrow\uparrow\downarrow$	1/2	$-\mu H$	3	
$\uparrow\downarrow\uparrow$				
$\downarrow\uparrow\uparrow$				
$\uparrow\downarrow\downarrow$	-1/2	$\mu H$	3	
$\downarrow\uparrow\downarrow$				
$\downarrow\downarrow\uparrow$				
$\downarrow\downarrow\downarrow$	-3/2	$3\mu H$	1	

Table 1: Energy levels of 3 spin-1/2 particles

System: *isolated*: energy cannot change *equilibrium*: prob of finding the system in any one accessible state is constant in time

## A fundamental postulate:

An isolated system in equilibrium is equally likely to be in any of its accessible states

In calculating probabilities, e.g., isolated system with energy in range  $[E, E + \delta E]$

$\Omega(E)$ : total number of states of the system in this range

$\Omega(E, y_k)$ : in this energy range and some other property  $y_k$  where the probability of having this property is

$$P(y_k) = \frac{\Omega(E, y_k)}{\Omega(E)}$$

## Density of states (DOS)

$$\Omega(E) = w(E)dE, \quad w(E) \sim E$$

where  $w(E)$  is the density of states.