Homework 1

Due 1/24

1. Gravity vs. E&M

Given the force of gravitational attraction

$$\mathbf{F}_g = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$$

and the force of electrostatic repulsion

$$\mathbf{F}_e = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}\hat{\mathbf{r}}$$

The ratio of the two forces between two electrons is

$$\frac{F_g}{F_e} = \frac{4\pi\epsilon_0 Gm_e^2}{q_e^2}$$

Using the values for the constants

$$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2/kg^2}, \quad \epsilon_0 = 8.85 \times 10^{-12}\,\mathrm{C^2/N\,m^2}$$
$$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}, \quad q_e = 1.60 \times 10^{-19}\,\mathrm{C}$$

we find

$$\boxed{\frac{F_g}{F_e} = 2.40 \times 10^{-43}}$$

this tells us that the denominator (the electrostatic force) is *much* larger than the numerator (the gravitational force), which convinces us that gravitaional forces are negligible for elementary particles.

2. Mesons and Baryons

(a) For mesons, you can have n possible quarks and n possible antiquarks, thus there are $\boxed{n^2}$ combinations.

For baryons order does not matter we have to make sure not to double count instances such as (uud) and (udu): This is essentially a combination problem with the solution

$$\binom{n}{3} = \frac{n!}{3!(n-3)!} = \boxed{\frac{n(n-1)(n-2)}{6}}$$

(b) Given the 6 flavors of quarks, we would expect $6^2 = 36$ mesons and $\binom{6}{3} = 20$ baryons.

(c) We haven't found all of them because of energy required to observe the heavier particles. In the Particle Data Group website, the heaviest baryon is in the order of 6000 MeV and the LHC has a beam energy of 6.5 TeV and the energy consumption is about 1.3 TWh per year compared to the global energy consumption of 20 000 TWh per year (source). In addition to the enourmous energy required to produce these particles, they are also very unstable and decay very quickly thus detecting them require us to measure at very small time scales.

3. Global Conservation Laws

(a) $n \rightarrow \bar{p} + e^+ + \nu_e$

not valid: violates Baryon number conservation

(b) $\nu_e + n \to p + e^-$

valid

(c) $\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$

valid

(d) $\mu^- \to e^- + \gamma$

not valid because it violates electron and muon lepton number conservation

(e) $e^+ + e^- \to \gamma$

valid

4. Nuclear β-decay

(a)

$$_Z^A X \to_{Z+1}^A Y + e^-$$

From the conservation of momentum

$$p_X^\mu = p_Y^\mu + p_e^\mu \quad \text{or} \quad p_Y^\mu = p_X^\mu - p_e^\mu$$

squaring both sides

$$p_Y^2 = p_X^2 + p_e^2 - 2p_X \cdot p_e$$

and since

$$p_X^2 = m_X^2 c^2, \quad p_Y^2 = m_Y^2 c^2, \quad p_e^2 = m_e^2 c^2$$

and

$$p_X p_e = \frac{E_X}{c} \frac{E_e}{c} - \mathbf{p}_X \cdot \mathbf{p}_e$$

but we know that particle X has momentum $\mathbf{p}_X = 0$ and rest mass $E_X = m_X c^2$ so

$$m_Y^2 c^2 = m_X^2 c^2 + m_e^2 c^2 - 2m_X E_e$$

solving for the Energy of the outgoing particle is

$$\boxed{E_e = \frac{m_X^2 + m_e^2 - m_Y^2}{2m_X} c^2}$$

to find the momentum of the outgoing electron we use energy-momentum relation

$$E_e^2 = |\mathbf{p}_e|^2 c^2 + m_e^2 c^4$$

or

$$|\mathbf{p}_e|^2 = \frac{E_e^2}{c^2} - m_e^2 c^2$$

using the energy of the outgoing electron we found earlier:

$$|\mathbf{p}_e|^2 = c^2 \left(\frac{m_X^2 + m_e^2 - m_Y^2}{2m_X} \right)^2 - m_e^2 c^2$$

$$= \frac{c^2}{4m_X^2} (m_X^2 + m_e^2 - m_Y^2)^2 - \left(\frac{c^2}{4m_X^2} \right) 4m_X^2 m_e^2$$

$$= \frac{c^2}{4m_X^2} (m_X^4 + m_e^4 + m_Y^4 + 2m_X^2 m_e^2 - 2m_X^2 m_Y^2 - 2m_e^2 m_Y^2 - 4m_X^2 m_e^2)$$

$$= \frac{c^2}{4m_X^2} (m_X^4 + m_e^4 + m_Y^4 - 2m_X^2 m_e^2 - 2m_X^2 m_Y^2 - 2m_e^2 m_Y^2)$$

and therefore the momentum of the outgoing electron is

$$\left|\mathbf{p}_e\right| = \frac{c}{2m_X}\sqrt{m_X^4 + m_e^4 + m_Y^4 - 2m_X^2 m_e^2 - 2m_X^2 m_Y^2 - 2m_e^2 m_Y^2}$$

(b) For the decay including an anti-neutrino

$$^A_Z X \rightarrow ^A_{Z+1} Y + e^- + \bar{\nu}_e$$

For the massless neutrino, the energy is

$$E_\nu = \left|\mathbf{p}_\nu\right|c$$

or from planck's relation

$$E_\nu = h\nu = \frac{hc}{\lambda}$$

so the energy of the neutrino is

$$E_\nu = \frac{6.63 \times 10^{-34}\,\mathrm{J\,s} \cdot 3 \times 10^8\,\mathrm{m\,s^{-1}}}{10^{-15}\,\mathrm{m}} = 1.99 \times 10^{-10}\,\mathrm{J} = 1240\,\mathrm{MeV}$$

and the momentum of the neutrino is

$$\left|\mathbf{p}_\nu\right| = \frac{E_\nu}{c} = \frac{1.99 \times 10^{-10}\,\mathrm{J}}{3 \times 10^8\,\mathrm{m\,s^{-1}}} = 6.63 \times 10^{-19}\,\mathrm{kg\,m\,s^{-1}} \quad \text{or} \quad 1240\,\frac{\mathrm{MeV}}{c}$$

This is much larger compared to the typical neutrino energy (keV). This means that the neutrino could not have come from inside the nucleus.

Using the Heisenberg uncertainty principle

$$\Delta p \Delta x \geq \frac{h}{4\pi} \quad \text{or} \quad \Delta p \geq \frac{h}{4\pi \Delta x}$$

and the typical size of a nucleus is $\Delta x \approx 1 \times 10^{-15}\,\mathrm{m}$ so

$$\Delta p \geq \frac{6.63 \times 10^{-34}\,\mathrm{J\,s}}{4\pi \cdot 10^{-15}\,\mathrm{m}} = 0.53\,\frac{\mathrm{J}}{\mathrm{m/s}}$$

or in more convenient units $1\,\mathrm{eV} = 1.6 \times 10^{-19}\,\mathrm{J}$ and $c = 3 \times 10^8\,\mathrm{m/s}$:

$$\Delta p \geq 0.53\,\frac{\mathrm{J}}{\mathrm{m/s}}\frac{1\,\mathrm{eV}}{1.6 \times 10^{-19}\,\mathrm{J}}\frac{3 \times 10^8\,\mathrm{m/s}}{c}$$

$$\boxed{\Delta p \geq 99\,\frac{\mathrm{MeV}}{c}}$$

and the energy of the neutrino (a massless particle) is

$$\boxed{E_\nu = \left|\mathbf{p}_\nu\right|c \geq 99\,\mathrm{MeV}}$$

Compared to the typical neutrino energy of $1\,\mathrm{keV}$, this is much larger and thus the neutrino could not have come from inside the nucleus.