1. (a) From the Gell-Mann-Nishijima formula

$$Q = I_3 + \frac{1}{2}(A + S)$$

For the uds quarks, the isospin is $I_3 = \frac{1}{2}, -\frac{1}{2}, 0$, the baryon number is $A = \frac{1}{3}$ and the strangeness is $S = 0, 0, -1$ respectively. The charges are then

$$Q_u = \frac{1}{2} + \frac{1}{2}\left(\frac{1}{3} + 0\right) = \frac{2}{3}$$
$$Q_d = -\frac{1}{2} + \frac{1}{2}\left(\frac{1}{3} + 0\right) = -\frac{1}{3}$$
$$Q_s = 0 + \frac{1}{2}\left(\frac{1}{3} - 1\right) = -\frac{1}{3}$$

(b) The antiparticle will have the opposite charge $Q_{\bar{q}} = -Q_q$, baryon number $A_{\bar{q}} = -\frac{1}{3}$ and strangeness $S_{\bar{q}} = 0, 0, 1$, so the isospin states are

$$Q_{\bar{u}} = -\frac{2}{3} = I_3 + \frac{1}{2}\left(-\frac{1}{3} + 0\right) \implies I_3 = -\frac{1}{2}$$
$$Q_{\bar{d}} = \frac{1}{3} = I_3 + \frac{1}{2}\left(-\frac{1}{3} + 0\right) \implies I_3 = \frac{1}{2}$$
$$Q_{\bar{s}} = \frac{1}{3} = I_3 + \frac{1}{2}\left(-\frac{1}{3} + 1\right) \implies I_3 = 0$$

so the isospin assignments $|I, I_3\rangle$ are

$$\bar{u} = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle, \quad \bar{d} = \left|\frac{1}{2}, \frac{1}{2}\right\rangle, \quad \bar{s} = |0, 0\rangle$$

2. (a) For a the charged kaon

$$K^- \Leftrightarrow K^+$$

the charge is not conserved, so they cannot interconvert, so only the neutral mesons can mix. (b) We don't observe baryon-antibaryon interconversion because it violates baryon number conservation. (c) There is no mixing of neutral strange vector mesons because the K^{*0} and $\bar{K}^{*0}$ have different strangeness $S = +1, -1$, so they cannot mix due to strangeness conservation.

3. From the Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = H|\psi(t)\rangle$$

and the time reversal operator $|\psi(t)\rangle = T|\psi(-t)\rangle$, so the equation is

$$i\hbar\frac{\partial}{\partial(t)}T|\psi(-t)\rangle = HT|\psi(-t)\rangle$$

and the time derivative of the time-reversed state using chain rule

$$i\hbar\frac{\partial}{\partial(t)}|\psi(-t)\rangle = -i\hbar\frac{\partial}{\partial t}|\psi(-t)\rangle$$

for the right side, since T and H commute

$$-i\hbar\frac{\partial}{\partial t}|\psi(-t)\rangle = TH|\psi(-t)\rangle$$
$$-i\hbar\frac{\partial}{\partial t}T|\psi(t)\rangle = THT|\psi(t)\rangle = TTH|\psi(t)\rangle$$

and since $T^2 = 1$

$$-i\hbar\frac{\partial}{\partial t}T\left|\psi(t)\right\rangle = H\left|\psi(t)\right\rangle$$

or

$$Tc = c^*T$$

4. Given the Hamiltonian

$$H = -\frac{1}{|\mathbf{J}|}(\mu\mathbf{J}\cdot\mathbf{B} + d\mathbf{J}\cdot\mathbf{E})$$

(a) From Maxwells equations

$$\begin{aligned}\boldsymbol{\nabla}\cdot\mathbf{E} &= 4\pi\rho\\ \boldsymbol{\nabla}\times\mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t}\end{aligned}$$

under time reversal $t \to -t$ the electric field is T-even $E \to E$ and the magnetic field is T-odd $B \to -B$. From angular momentum

$$\mathbf{L} = \mathbf{r}\times\mathbf{p} = \mathbf{r}\times m\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$$

so under time reversal $t \to -t$ angular momentum is T-odd $\mathbf{L} \to -\mathbf{L}$, and since spin angular momentum is T-odd by the right hand rule, the total angular momentum is T-odd $\mathbf{J} \to -\mathbf{J}$.

For the parity, the magnetic field and angular momentum are even under parity since they are pseudovectors, and the electric field is odd under parity as a vector.

The charge conjugation of the electric field is $E \to -E$ and the magnetic field is $B \to -B$ since the antiparticle will have the opposite charge. The total angular momentum is invariant under charge conjugation $\mathbf{J} \to \mathbf{J}$ since the antiparticle will have the same spin.

$$\begin{aligned}C &: \mathbf{E}\to-\mathbf{E}, \quad \mathbf{B}\to-\mathbf{B}, \quad \mathbf{J}\to\mathbf{J}\\ P &: \mathbf{E}\to-\mathbf{E}, \quad \mathbf{B}\to\mathbf{B}, \quad \mathbf{J}\to\mathbf{J}\\ T &: \mathbf{E}\to\mathbf{E}, \quad \mathbf{B}\to-\mathbf{B}, \quad \mathbf{J}\to-\mathbf{J}\end{aligned}$$

(b) Since the Hamiltonian is invariant under time reversal so μ is T-even (odd times odd is even and even times even is even) and d is T-odd. The Hamiltonian is also invariant under parity so μ is P-even and d is P-odd. For charge conjugation, the Hamiltonian is invariant so μ is C-odd and d is C-odd.

$$\begin{aligned}C &: \mu\to\mu, \quad d\to-d\\ P &: \mu\to\mu, \quad d\to-d\\ T &: \mu\to-\mu, \quad d\to-d\end{aligned}$$