

1 Section

Paragraph

2 Module 8 Studio

2.1 Master Theorem

Set 1

1. $T(n) = 3T(n/2) + cn^2$

$$\begin{aligned} a &= 3, \quad b = 2, \quad f(n) = O(n^2) = O(n^c) \quad c = 2 \\ c_{\text{crit}} &= \log_b a = \log_2 3 \approx 1.58 < c \\ f(n) &= O(n^2) \end{aligned}$$

so we have $T(n) = \Theta(n^2)$.

2. $T(n) = 4T(n/2) + cn^2$

$$c_{\text{crit}} = \log_2 4 = 2, \quad c_{\text{crit}} = c$$

so we have $T(n) = \Theta(n^2 \log n)$.

3. $T(n) = T(n/2) + 2^n$

$$c_{\text{crit}} = \log_2 1 = 0, \quad c_{\text{crit}} < c$$

so we have $T(n) = \Theta(2^n)$.

4. $T(n) = 16T(n/4) + n$

$$c_{\text{crit}} = \log_4 16 = 2, \quad c = 1, \quad c_{\text{crit}} > c$$

so we have $T(n) = \Theta(n^2)$.

5. $T(n) = 2T(n/2) + n \log n$

$$\begin{aligned} c_{\text{crit}} &= \log_2 2 = 1, \quad c = 1, \quad c_{\text{crit}} = c \\ f(n) &= \Theta(n^{c_{\text{crit}}} \log n^k), \quad k = 1 \end{aligned}$$

so $T(n) = \Theta(n \log^2 n)$.

6. $T(n) = 2T(n/2) + n \log^2 n$

$$c_{\text{crit}} = 1, \quad f(n) = \Theta(n \log^2 n), \quad c = 1, k = 2$$

so $T(n) = \Theta(n \log^3 n)$.

7. $2T(n/2) + n/\log n$

$$c_{\text{crit}} = 1, \quad f(n) = \Theta(n/\log n), \quad c = 1, k = -1$$

so $T(n) = \Theta(n \log \log n)$.

8. $T(n) = \Theta(n!)$

9. $T(n) = \Theta(n^{\log_2 3} \log n)$

10. $T(n) = \Theta(n \log n)$

