Homework 6

Due 3/5

1. (a) The joint entropy is (where $\log = \log_2$)

$$\begin{split} H(V,T) &= \sum_{V,T} p(V,T) \log \left(\frac{1}{p(V,T)} \right) \\ &= \left[\frac{6}{16} \log(16) + \frac{4}{32} \log(32) + \frac{2}{8} \log(8) + \frac{1}{4} \log(4) \right] \\ &= 3.38 \text{ bits} \end{split}$$

(b) Given the marginal probability

$$p(V = \text{Sunny}) = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{4}{16} = \frac{1}{4}$$

$$p(V = \text{Cloudy \& dry}) = \frac{1}{16} + \frac{1}{8} + \frac{1}{32} + \frac{1}{32} = \frac{8}{32} = \frac{1}{4}$$

$$p(V = \text{Cloudy \& rain}) = \frac{1}{4}$$

$$p(V = \text{Cloudy \& snow}) = \frac{1}{4}$$

marginal entropy of V is

$$H(V) = \sum_{V} p(V) \log \left(\frac{1}{p(V)}\right)$$

$$= \frac{1}{4} \log(4) + \frac{1}{4} \log(4) + \frac{1}{4} \log(4) + \frac{1}{4} \log(4)$$

$$= 2 \text{bits}$$