Homework 8

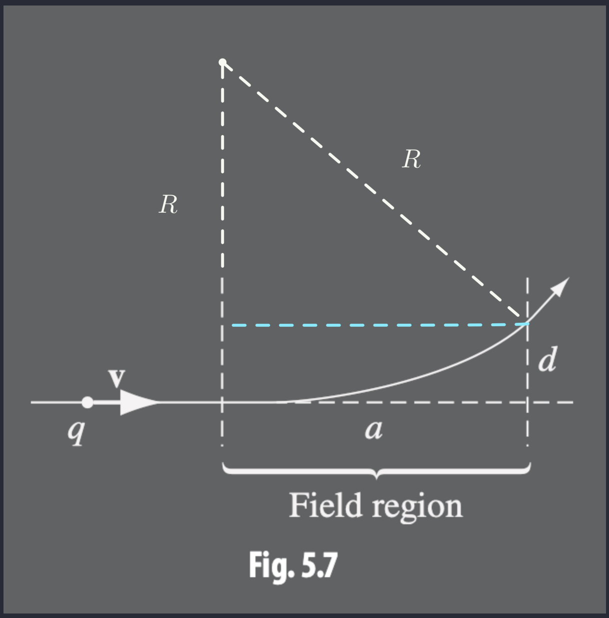

Figure 0.1: Extended geometry of particle path

5.1 From Griffiths, the momentum of the particle is given by the cyclotron formula

$$p = qBR \tag{5.3}$$

So from Fig. 0.1, the radius of the path is given by

$$\begin{aligned} R^2 &= (R-d)^2 + a^2 \\ R^2 &= R^2 - 2Rd + d^2 + a^2 \\ \implies R &= \frac{d^2 + a^2}{2d} \end{aligned}$$

Thus

$$\boxed{p = qB\frac{d^2 + a^2}{2d}}$$

5.3

(a) Adjusting the beam for zero deflection means that the Lorentz force is zero:

$$\begin{aligned} \mathbf{F} &= q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = 0 \\ \implies \mathbf{E} &= -\mathbf{v} \times \mathbf{B} \end{aligned}$$

or in terms of the magnitude

$$\boxed{v = \frac{E}{B}}$$

(b) Using 5.3 and the result from (a),

$$\begin{aligned} p = mv &= qBR \\ \implies \frac{q}{m} = \frac{v}{BR} &= \boxed{\frac{E}{B^2 R}} \end{aligned}$$

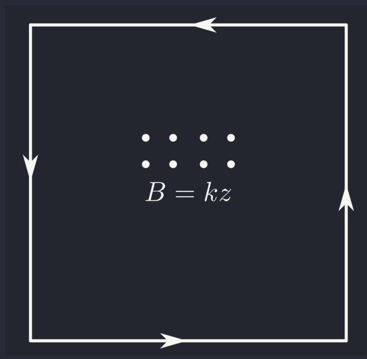

Figure 0.2: View from yz plane: Magnetic field points out of the page, on the

5.4 Given the magnetic field

$$\mathbf{B} = kz\hat{\mathbf{x}}$$

Using RHR:

- The left side of the loop has a force pointing to the left ($-\hat{\mathbf{y}}$) which cancels out with the
- Right side of the loop with force pointing to the right ($\hat{\mathbf{y}}$)
- The top ($z = a/2$) and bottom ($z = -a/2$) side of the loop have forces pointing in the same direction ($\hat{\mathbf{z}}$)

So the two forces are

$$F_{\text{top}} = 2IBa = I(ka/2)a = \frac{1}{2}Ika^2, \quad F_{\text{ bottom}} = -\frac{1}{2}Ika^2$$

and since they are in the same direction, the net force is

$$\mathbf{F} = Ika^2\hat{\mathbf{z}}$$

5.5 For a wire of radius a

(a) If the current is uniformly distributed over the surface, the current density is

$$K = \frac{\mathrm{d}I}{\mathrm{d}l_\perp} = \frac{I}{2\pi a}$$

since the width of the ribbon $\mathrm{d}l_\perp$ is the circumference of the wire cross-section.

(b) If the volume current density is inversely proportional to the distance from the axis, we can use the result from (a):

$$J(s) = \frac{k}{s}$$

and integrating to find k:

$$\begin{aligned} I &= \int J(s)\,\mathrm{d}A = k\int \frac{1}{s}(s\,\mathrm{d}s\,\mathrm{d}\phi) \\ &= k(2\pi)\int_0^a \mathrm{d}s = 2\pi k a \\ \implies k &= \frac{I}{2\pi a} \end{aligned}$$

or the same result as (a). Thus

$$\boxed{J(s) = \frac{I}{2\pi a s}}$$

5.6

(a) A phonograph with static electricity σ rotating at ω. The surface current density K at a distance r from the center is (using $v = r\omega$)

$$K = \sigma v = \boxed{\sigma r \omega}$$

(b) A uniform sphere of radius R with charge Q spins around the z axis at a constnat angular velocity ω. The volume current density $\mathbf{J}$ at any point in the sphere is (using $v = r\omega \sin\phi$)

$$\mathbf{J} = \rho \mathbf{v} = \rho r \omega \sin\phi \hat{\phi}$$

where $\rho = \frac{Q}{4\pi R^3/3}$ is the charge per volume of the sphere, so

$$\boxed{\mathbf{J} = \frac{Q}{4\pi R^3} r\omega \sin\phi \hat{\phi}}$$

5.11 For a tightly wound solenoid with n turns per unit length and radius a, we start with a single loop (From Griffiths Example 5.6)

$$\begin{aligned} B_{\text{loop}} &= \frac{\mu_0 I}{4\pi}\int \frac{\mathrm{d}l'}{\mathscr{r}^2}\cos\theta = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{\mathscr{r}}\right)2\pi a \\ &= \frac{\mu_0 I}{2}\frac{a^2}{(a^2+z^2)^{3/2}} \end{aligned}$$

so for n turns per unit length we multiply by n and integrate over the length of the solenoid

$$B = \frac{\mu_0 n I}{2}\int \frac{a^2}{(a^2+z^2)^{3/2}}\,\mathrm{d}z$$

Using $\tan\theta = a/z \implies z = a/\tan\theta$ and $\mathrm{d}z = -\frac{a}{\sin^2\theta}\,\mathrm{d}\theta$, the integral becomes

$$\begin{aligned} \int \frac{a^2}{(a^2+(a/\tan\theta)^2)^{3/2}}\left(-\frac{a}{\sin^2\theta}\right)\mathrm{d}\theta) &= -\int \frac{a^2}{(a^2)^{3/2}\left(1+\frac{\cos^2\theta}{\sin^2\theta}\right)^{3/2}}\left(\frac{a}{\sin^2\theta}\right)\mathrm{d}\theta \\ &= -\int \frac{1}{\frac{1}{(\sin^2\theta)^{3/2}}(\sin^2\theta+\cos^2\theta)^{3/2}\sin^2\theta}\,\mathrm{d}\theta \\ &= -\int \frac{\sin^3\theta}{\sin^2\theta}\,\mathrm{d}\theta \\ &= -\int \sin\theta\,\mathrm{d}\theta = \cos\theta\Big|_{\theta_1}^{\theta_2} = \cos\theta_2 - \cos\theta_1 \end{aligned}$$

So

$$\boxed{B = \frac{\mu_0 n I}{2}(\cos\theta_2 - \cos\theta_1)}$$

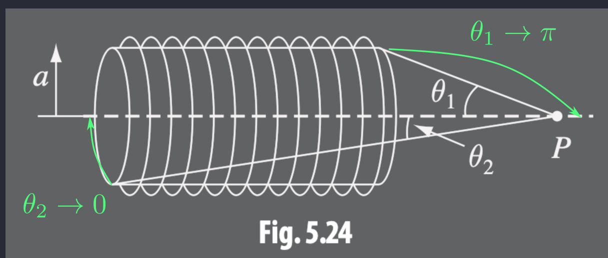

Figure 0.3: As solenoid becomes infinitely long, $\theta_2 \to 0$ and $\theta_1 \to \pi$

As the solenoid becomes infinitely long (Fig.0.3), then $\theta_2 \to 0$ and $\theta_1 \to \pi$ so

$$B = \frac{\mu_0 n I}{2}(1-(-1)) = \boxed{\mu_0 n I}$$