Homework 2

Due 2/6 12pm

1.

$$P(r|\lambda) = \exp(-\lambda)\frac{\lambda^r}{r!}$$

(a) Taking the log of the likelihood function:

$$L(\lambda) = \ln P(r|\lambda) = -\lambda + r\ln\lambda - \ln r!$$

finding the maximum by taking the derivative with respect to λ and setting it to zero:

$$\frac{dL}{d\lambda} = -1 + \frac{r}{\lambda} = 0 \implies \hat{\lambda} = r$$

so the maximum likelihood estimate for λ is $\hat{\lambda} = r$.
(b) Given the derivative with respect to the function $\ln\lambda$:

$$\frac{\mathrm{d}}{\mathrm{d}(\ln\lambda)}u^n = nu^n, \qquad \frac{\mathrm{d}}{\mathrm{d}(\ln\lambda)}\ln\lambda = 1$$

we can find the curvature of the log likelihood function:

$$\frac{\mathrm{d}}{\mathrm{d}(\ln\lambda)}L(\lambda) = -\lambda + r = 0 \implies \hat{\lambda} = r$$
$$\frac{\mathrm{d}^2}{\mathrm{d}(\ln\lambda)^2}L(\lambda) = -\lambda = k$$

For a normal distribution with width σ, the curvature is $k = -1/\sigma^2$. So the width is approximately

$$\sigma \propto \frac{1}{\sqrt{-k}} = \frac{1}{\sqrt{\lambda}}$$

and the 95% confidence interval at the MLE is approximately

$$\hat{\lambda} \pm 2\sigma = r \pm \frac{2}{\sqrt{\hat{\lambda}}}$$

(c) Given the new Poisson distribution

$$P(r|\lambda) = \exp(-(\lambda + b))\frac{(\lambda+b)^r}{r!}$$

the log likelihood function is

$$L(\lambda) = -(\lambda + b) + r\ln(\lambda + b) - \ln r!$$

and the maximum likelihood estimate for λ is

$$\frac{dL}{d\lambda} = -1 + \frac{r}{\lambda + b} = 0 \implies \hat{\lambda} = r - b$$