Homework 3

Due 2/7

1

(a) An electron has spin $s = \frac{1}{2}$ so

$$j = \frac{3}{2}, \frac{1}{2}$$

for the 6 possible states of $|j, j_z\rangle$, the $1 \otimes \frac{1}{2}$ C-B coefficients are

$$\begin{aligned}
\left|\frac{3}{2}, \frac{3}{2}\right\rangle &= 1\left|1, \frac{1}{2}\right\rangle = |m_l, m_s\rangle \\
\left|\frac{3}{2}, \frac{1}{2}\right\rangle &= \frac{1}{\sqrt{3}}\left|1, -\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|0, \frac{1}{2}\right\rangle \\
\left|\frac{3}{2}, -\frac{1}{2}\right\rangle &= \sqrt{\frac{2}{3}}\left|0, -\frac{1}{2}\right\rangle - \frac{1}{\sqrt{3}}\left|-1, \frac{1}{2}\right\rangle \\
\left|\frac{3}{2}, -\frac{3}{2}\right\rangle &= 1\left|-1, -\frac{1}{2}\right\rangle
\end{aligned}$$

and

$$\begin{aligned}
\left|\frac{1}{2}, \frac{1}{2}\right\rangle &= \sqrt{\frac{2}{3}}\left|1, -\frac{1}{2}\right\rangle - \frac{1}{\sqrt{3}}\left|0, \frac{1}{2}\right\rangle \\
\left|\frac{1}{2}, -\frac{1}{2}\right\rangle &= \frac{1}{\sqrt{3}}\left|0, -\frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|-1, \frac{1}{2}\right\rangle
\end{aligned}$$

(b) We can see for the $\left|j = \frac{3}{2}, j_z = \frac{1}{2}\right\rangle$ state, the probability of measuring a spin $s_z = \frac{1}{2}$ is proportional to the coefficient squared

$$P = \frac{2}{3}$$

2

(a) For the following processes

- The elastic processes are (from $a \to f$)

$$\begin{aligned}
\pi^+ + p &\to \pi^+ + p \quad (a) \\
\pi^0 + p &\to \pi^0 + p \quad (b) \\
\pi^- + p &\to \pi^- + p \quad (c) \\
\pi^+ + n &\to \pi^+ + n \quad (d) \\
\pi^0 + n &\to \pi^0 + n \quad (e) \\
\pi^- + n &\to \pi^- + n \quad (f)
\end{aligned}$$

- The inelastic processes are (from $g \to j$)

$$\begin{aligned}
\pi^+ + n &\to \pi^0 + p \quad (g) \\
\pi^0 + p &\to \pi^+ + n \quad (h) \\
\pi^- + p &\to \pi^0 + n \quad (i) \\
\pi^0 + n &\to \pi^- + p \quad (j)
\end{aligned}$$

The states are linear combinations of the states from Problem 1

$$(a) \to \left|1, \frac{1}{2}\right\rangle = \left|\frac{3}{2}, \frac{3}{2}\right\rangle$$
$$(b) \to \left|0, \frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}, \frac{1}{2}\right\rangle - \frac{1}{\sqrt{3}}\left|\frac{1}{2}, \frac{1}{2}\right\rangle$$
$$(c) \to \left|-1, \frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left|\frac{3}{2}, -\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2}, -\frac{1}{2}\right\rangle$$
$$(d) \to \left|1, -\frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}\left|\frac{3}{2}, \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2}, \frac{1}{2}\right\rangle$$
$$(e) \to \left|0, -\frac{1}{2}\right\rangle = \sqrt{\frac{2}{3}}\left|\frac{3}{2}, -\frac{1}{2}\right\rangle + \frac{1}{\sqrt{3}}\left|\frac{1}{2}, -\frac{1}{2}\right\rangle$$
$$(f) \to \left|-1, -\frac{1}{2}\right\rangle = \left|-1, -\frac{1}{2}\right\rangle = \left|\frac{3}{2}, -\frac{3}{2}\right\rangle$$

Looking at the coefficients and the Isospin states, we can see the amplitudes as

$$M_a = M_f = M_3$$
$$M_b = M_e = \frac{2}{3}M_3 + \frac{1}{3}M_1$$
$$M_c = M_d = \frac{1}{3}M_3 + \frac{2}{3}M_1$$
$$M_g = M_h = M_i = M_j = \frac{\sqrt{2}}{3}(M_3 - M_1)$$

and the cross sections are proportional to the square of the amplitudes (coefficient square) or $\sigma \propto |M|^2$, but...

$$\sigma_a : \sigma_c = |M_3|^2 : \left|\frac{1}{3}M_3 + \frac{2}{3}M_1\right|^2$$
$$9|M_3|^2 : |M_3 + 2M_1|^2$$

so the total ratios are

$$\sigma_a : \sigma_b : \sigma_c : \sigma_d : \sigma_e : \sigma_f : \sigma_g : \sigma_h : \sigma_i : \sigma_j =$$
$$9|M_3|^2 : |2M_3 + M_1|^2 : |M_3 + 2M_1|^2 : |M_3 + 2M_1|^2 : |2M_3 + M_1|^2$$
$$: 9|M_3|^2 : 2|M_3 - M_1|^2 : 2|M_3 - M_1|^2 : 2|M_3 - M_1|^2 : 2|M_3 - M_1|^2$$

and for $M_3 \gg M_1$ the ratios are

$$\boxed{9 : 4 : 1 : 1 : 4 : 9 : 2 : 2 : 2 : 2}$$

(b) For $M_3 \ll M_1$ the ratios are

$$\boxed{0 : 1 : 4 : 4 : 1 : 0 : 2 : 2 : 2 : 2}$$

where (a) and (f) are very, very small cross sections in comparison.

3

(a) for protons $I = \frac{1}{2}$ and neutrons, $I = -\frac{1}{2}$ so the isospin of the α particle is $I = 0$.
(b) On the LHS the isospin of the deuteron is $I = 0$, and on the RHS the isospin of the α particle is $I = 0$ so the isospin of the pion is $I = 1$. Since the isospin is not conserved $0 \not\to 1$ the reaction is not allowed.
(c) The 4-proton state has isospin $I = 2$ and this is *does not exist* since the isospin $I = 1$ of the ^{4}Li does not exist. The 4-neutron state with isospin $I = -2$ *does not exist* as well due since the ^{4}H isotope of $I = -1$ does not exist. There can only be one possible 4-nucleon state: ^{4}He with isospin $I = 0$.