1 Stat Mech Results and Methods

Our Return to th stat mech part...with systems A and heat reservoir A' where

$$A \ll A'$$

What is the prob of finding system A in a ny particular microstate r with energy E_r?

$$E_r + E' = E^{(0)}, \implies E' = E^{(0)} - E_r$$

And from the DoS the number of states in A' is

$$\Omega'(E^{(0)} - E_r)$$

or the Multiplicity of A' given E_r. The prob P_r has a proportionality

$$P_r \propto C'\Omega'(E^{(0)} - E_r)$$

Since $A \ll A'$ and $E_r \ll E^{(0)}$ we can take the log and Taylor expand

$$\ln \Omega'(E^{(0)} - E_r) = \ln \Omega'(E^{(0)}) - \left.\frac{\partial \ln \Omega'}{\partial E'}\right|_{E^{(0)}} E_r$$

where the derivative is the thermodynamic beta

$$\left.\frac{\partial \ln \Omega'}{\partial E'}\right|_{E^{(0)}} = \frac{1}{kT} = \beta$$

which is independent of E_r. So taking the exponential again...

$$\Omega'(E^{(0)} - E_r) = \Omega'(E^{(0)})e^{-\beta E_r} = Ce^{-\beta E_r}$$

where the $\Omega'(E^{(0)})$ is a constant, i.e.

$$P_r = Ce^{-\beta E_r}$$

This must be normalized by

$$\sum P_r = 1$$

or

$$C = \frac{1}{\sum e^{-\beta E_r}}$$

where the “Partition Function” is

$$Z \equiv \sum_r e^{-\beta E_r}$$

coined by Planck (1920) as “Zustandsumme” or “Sum over all states”.
The probability is

$$P_r = \frac{e^{-\beta E_r}}{Z}$$

Where we have a “Boltzmann funtion” $e^{-\beta E_r}$ and P_r is the cannonincal distribution.