〈Part 2〉 데이터 사이언스 필수수학

[Ch 02.] Jacobian Matrix & Backpropagation

[CH 02.] 06, 07, 08

Lecture.2

Basic Differentiation

YHJ

CH02_06. Constant_Multiple_and_Sum_Rules

<Differentiation Rules>

- Constant Multiple Rule

$$\frac{d}{dx}[c\cdot f(x)] = c\cdot\frac{d}{dx}[f(x)]$$

ex1) $f(x)=2x \quad g(x)=x,\ g'(x)=1$

① $f'(x)=2\cdot 1\cdot \cancel{x^{1-1}}=2$

② $f'(x)=2\cdot g'(x)=2$

ex2) $f(x)=-3\cdot e^{x},\ g(x)=e^{x},\ g'(x)=e^{x}$

$f'(x)=-3\cdot e^{x}$

- Sum Rule

$$\frac{d}{dx}[f(x)+g(x)] = \frac{df}{dx}+\frac{dg}{dx}$$

ex1) $f(x)=2x^2-x+1$

$\rightarrow f'(x)=4x-1$

ex2) $f(x)=\sin(x)-\frac{1}{\sqrt{x}}$

$=\sin x - x^{-\frac{1}{2}}$

$f'(x)=\cos x+\frac{1}{2}\cdot x^{-\frac{3}{2}}$

$=\cos x+\frac{1}{2x\sqrt{x}}$

ex3) $f(x)=\sinh(x)$

$=\frac{e^{x}-e^{-x}}{2}$

$=\frac{1}{2}e^{x}-\frac{1}{2}e^{-x}$

$y=f(g(x))$

$y'=f'(g(x))g'(x)$

$f'(x)=\frac{1}{2}e^{x}+\frac{1}{2}e^{-x}$

ex4) $f(x)=\cosh(x)$

$=\frac{e^{x}+e^{-x}}{2}$

$f'(x)=\frac{e^{x}-e^{-x}}{2}$

$\frac{d}{dx}$

CH02_07. LTI_System_and_Differentiation

- Linearity of Diff.

$Sys\{\alpha \cdot f(t)\} = \alpha \cdot Sys\{f(t)\}$ → Homogeneity

$\frac{d}{dx}[\alpha f(x)] = \alpha \cdot \frac{d}{dx}[f(x)]$

∴ 미분은 Homogeneity를 갖는다.

$Sys\{f(t)+g(t)\} = Sys\{f(t)\} + Sys\{g(t)\}$ → Additivity

$\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}[f(x)] + \frac{d}{dx}[g(x)]$

∴ 미분은 Additivity를 갖는다.

Additivity + Homogeneity ⇒ Linearity를 갖는다.

$Sys\{\alpha \cdot f(t) + \beta \cdot g(t)\} = \alpha Sys\{f(t)\} + \beta Sys\{g(t)\}$

$\frac{d}{dx}[\alpha \cdot f(x) + \beta g(x)] = \alpha \cdot \frac{d}{dx}[f(x)] + \beta \cdot \frac{d}{dx}[g(x)]$

∴ 미분은 Linearity를 갖는다.

- Time-Invariance of Diff

$$Sys\{f(t)\} = f'(t) \Rightarrow Sys\{f(t-\tau)\} = f'(t-\tau)$$

$$\frac{df(t)}{dt} = f'(t) \Rightarrow \frac{df(t-\tau)}{dt} = f'(t-\tau)$$

- LTI (Linearity 만족 + Time-Invariance 만족) systems and Diff.

$$\frac{d}{dx}[\alpha \cdot f(t-\tau) + \beta \cdot g(t-\tau)] = \alpha \cdot f'(t-\tau) + \beta$$

∴ 미분은 LTI 만족한다.

CH02_08. Product_and_ Quotient_ Rules

- product Rule

$$\frac{d}{dx}[f(x)\cdot g(x)] = \frac{d}{dx}[f(x)]\cdot g(x) + f(x)\cdot\frac{d}{dx}[g(x)]$$
$$= f'(x)g(x) + f(x)g'(x)$$

ex1) $f(x) = x^3$

$f'(x) = 3x^2$

$f(x) = x \cdot x^2 = x^2 \cdot x = 1 \cdot x^3$

ex2) $f(x) = e^x \cdot \ln x$

$f'(x) = e^x \cdot \ln x + e^x \cdot \frac{1}{x}$

$= e^x[\ln x + \frac{1}{x}]$

- Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{\frac{d}{dx}[f(x)]\cdot g(x) - f(x)\cdot\frac{d}{dx}[g(x)]}{(g(x))^2}$$

ex) $f(x) = \tanh(x) = \frac{\sinh(x)}{\cosh(x)}$ ※ $\frac{d}{dx}[\sinh(x)] = \cosh(x)$

$\frac{d}{dx}[\cosh(x)] = \sinh(x)$

★ $f'(x) = \frac{\cosh^2(x) - \sinh^2(x)}{\cosh^2(x)} = 1 - \left(\frac{\sinh(x)}{\cosh(x)}\right)^2$

$= 1 - (\tanh(x))^2 = (1 + \tanh(x))(1 - \tanh(x))$