<Part 2> 데이터 사이언스 필수 논항

[Ch 01.] 딥러닝 네트워크의 연산

[CH 04.] 01, 02, 03

Lecture. 4

Loss functions

# <Lecture.4 Loss Functions>

## CH04_01. [이론강의]_Mean_Squared_Error.

<Cartesian Product for Predictions / Labels>

$\hat{y}$ → 실수이면 대부분 Regression 이라고 봄.

$\hat{y}$ → 0 → 강아지, 1 → 고양이, 2 → .. ⋮ ⇒ discrete → classification

class끼리 대소관계가 없음.

$\hat{y}$ ↑ Model ↑ $(\vec{x})^T$

- 목표 $\hat{y}$ (예측) ⟷ $y$ (실제값)

  $\hat{y}$ ↑ Model ↑ $(\vec{x})^T$

  ⇒ Loss 필요성.

Dataset

| 공부시간 | 성적 |
|---|---|
| 5 | 100 |
| 4 | 8 |
| x값 | y값 |

→ 비슷한값 $\hat{y}$ 만들기 : Model

- **Regression** ↓ $\hat{y} \in \mathbb{R}$ ↓ MSE

  **Classification** ↓ $\hat{y} \in C$ (class index) $\{0, 1, 2, \cdots, K\}$

  - Binomial ↓ Binary Cross Entropy
  - Multi ↓ Categorical Cross Entropy

- 집합

  $\mathbb{R}$ 실수집합

  $\mathbb{B} = \{0, 1\}$ Binary 집합

  $C = \{c_1, c_2, \cdots, c_K\}$ 멀티클래스집합

  $\mathbb{P} = \{x \mid 0 \le x \le 1\}$ 확률집합

  $\mathbb{B}_1^n = \{(b_1, b_2, \cdots, b_n)^T \mid \forall b \in \mathbb{B}, \sum_{i=1}^{n} b_i = 1\}$ : on-hot vector 집합.

  강아지 $= (0, 0, 1)$ 고양이 $= (0, 1, 0)$ 호랑이 $= (1, 0, 0)$

## <Mean Squared Error>

- Dataset for Regression

1번학생 : $(\vec{x}^{(1)}) = (x_1^{(1)}, x_2^{(1)}, x_3^{(1)}, \cdots, x_{l_I}^{(1)})$ $y^{(1)} \in R$

공부시간, 거주지, 학교

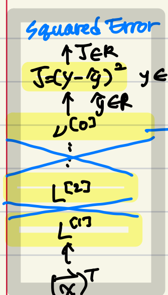
⋮

N번학생 $(\vec{x}^{(N)}) = (x_1^{(N)}, x_2^{(N)}, x_3^{(N)}, \cdots, x_{l_I}^{(N)})$ $y^{(N)} \in R$

$$\longrightarrow X^T = \begin{pmatrix} \leftarrow \vec{x}^{(1)} \rightarrow \\ \leftarrow \vec{x}^{(2)} \rightarrow \\ \vdots \\ \leftarrow \vec{x}^{(N)} \rightarrow \end{pmatrix} \in R^{N \times l_I}$$

$$Y = \begin{pmatrix} y^{(1)} \\ \vdots \\ y^{(N)} \end{pmatrix} \in R^{N \times 1}$$

- 단일데이터의 경우



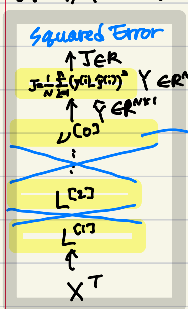

$J = (y - \hat{y})^2$

no activation, affine: $z = (\vec{a}^{[o-1]})^T \cdot \vec{w}^{[o]} + b^{[o]}$

$R$

- 여러데이터 → Matrix

$$J = \frac{1}{N}\sum_{i=1}^{N} (y^{(i)} - \hat{y}^{(i)})^2 : \text{MSE}$$

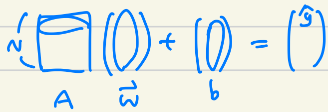



$$Y = \begin{pmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(N)} \end{pmatrix} \in R^{N \times 1}$$

$$\hat{Y} = \begin{pmatrix} \hat{y}^{(1)} \\ \hat{y}^{(2)} \\ \vdots \\ \hat{y}^{(N)} \end{pmatrix} \in R^{N \times 1}$$

affine: $Z = (A^{[o-1]})^T \cdot \vec{w}^{[o]} + b^{[o]}$

minibatch가 들어와도 그대로 한다.



# CH04_02.[이론강의]_Binary_Cross_Entropy.

- Dataset for Binary Classification

$(\vec{x}^{(1)})^T = (x_1^{(1)} \quad x_2^{(1)} \quad x_3^{(1)} \dots x_{\ell_I}^{(1)})$ $y^{(1)} \in \mathbb{B}$ 1 합격

공부시간 사교육비 몸무게 ...

$(\vec{x}^{(2)})^T = (x_1^{(2)} \quad x_2^{(2)} \quad x_3^{(2)} \dots x_{\ell_I}^{(2)})$ $y^{(2)} \in \mathbb{B}$ 0 불합격

$(\vec{x}^{(N)})^T = (x_1^{(N)} \quad x_2^{(N)} \quad x_3^{(N)} \dots x_{\ell_I}^{(N)})$ $y^{(N)} \in \mathbb{B}$ 1 합격

$$X^T = \begin{pmatrix} \leftarrow (\vec{x}^{(1)})^T \rightarrow \\ \leftarrow (\vec{x}^{(2)})^T \rightarrow \\ \vdots \\ \leftarrow (\vec{x}^{(N)})^T \rightarrow \end{pmatrix} \in \mathbb{R}^{N \times \ell_I}$$

$$Y = \begin{pmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(N)} \end{pmatrix} \in \mathbb{B}^{N \times 1}$$

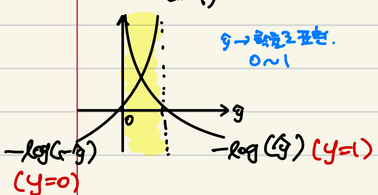
$$\Rightarrow \underset{\hat{y}}{\mathbb{R}} \sim \underset{y}{\mathbb{B}}$$

<Binary Cross Entropy>

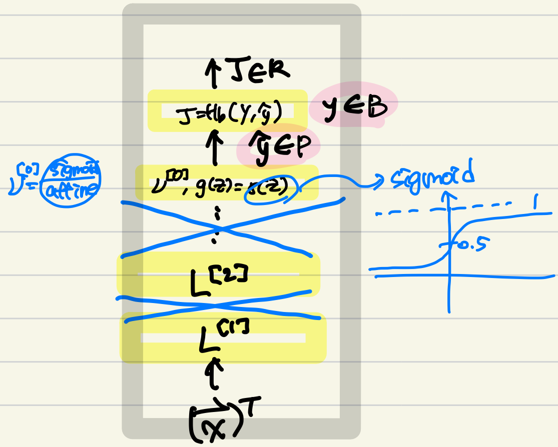
$$\begin{cases} y=1 \rightarrow \mathcal{L} = -\log(\hat{y}) \\ y=0 \rightarrow \mathcal{L} = -\log(1-\hat{y}) \end{cases}$$

$\Rightarrow$ 합치기 : $H_b(y, \hat{y}) = -[y\log(\hat{y}) + (1-y)\log(1-\hat{y})]$

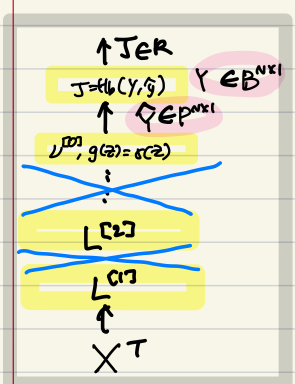
$H$ → Entropy, $b$ → binary




$\hat{y}$ → 확률로 표현.
0~1




- Minibatch




$$Y = \begin{pmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(N)} \end{pmatrix} \in \mathbb{B}^{N \times 1}$$

$$\hat{Y} = \begin{pmatrix} \hat{y}^{(1)} \\ \hat{y}^{(2)} \\ \vdots \\ \hat{y}^{(N)} \end{pmatrix} \in \mathbb{P}^{N \times 1}$$

$$J = H_b(Y, \hat{Y})$$

$$= \frac{1}{N}\sum_{i=1}^{N}[y^{(i)}\log(\hat{y}^{(i)}) + (1-y^{(i)})\log(1-\hat{y}^{(i)})]$$

# CH04_03. [이론정리]_Categorical_Cross_Entropy

- Dataset for Multi-class Classification

$\mathbb{C} = \{ \underset{\text{강아지}}{C_1}, \underset{\text{고양이}}{C_2}, \cdots, C_K \}$

$(\vec{x}^{(1)}) = (x_1^{(1)} \; x_2^{(1)} \cdots x_{\ell_I}^{(1)}) \quad y^{(1)} \in \mathbb{C}$

$(\vec{x}^{(2)}) = (x_1^{(2)} \; x_2^{(2)} \cdots x_{\ell_I}^{(2)}) \quad y^{(2)} \in \mathbb{C}$

$\vdots$

$(\vec{x}^{(N)}) = (x_1^{(N)} \; x_2^{(N)} \cdots x_{\ell_I}^{(N)}) \quad y^{(N)} \in \mathbb{C}$

$\longrightarrow \quad X^T = \begin{pmatrix} \leftarrow \vec{x}^{(1)} \rightarrow \\ \leftarrow \vec{x}^{(2)} \rightarrow \\ \vdots \\ \leftarrow \vec{x}^{(N)} \rightarrow \end{pmatrix} \in \mathbb{R}^{N \times \ell_I}$

$Y = \begin{pmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(N)} \end{pmatrix} \in \mathbb{C}^{N \times 1}$

- One-hot Encoding

$(\vec{y}^{(1)})^T = (1 \; 0 \cdots 0), \; \alpha = 1$ 강아지

$(\vec{y}^{(2)})^T = (0 \; 1 \cdots 0), \; \alpha = 2$ 고양이

$\vdots$

$(\vec{y}^{(K)})^T = (0 \; 0 \cdots 1), \; \alpha = K$ 돼지

$(\vec{y}^{(2)})^T \in \mathbb{B}_1^{1 \times K}$

여기서는
one-hot encoding 이라는 의미로 사용

$(\vec{y}^{(1)})^T \in \mathbb{B}_1^{1 \times K}$

$(\vec{y}^{(2)})^T \in \mathbb{B}_1^{1 \times K}$

$\vdots$

$(\vec{y}^{(N)})^T \in \mathbb{B}_1^{1 \times K}$

$\longrightarrow \quad Y^T = \begin{pmatrix} \leftarrow (\vec{y}^{(1)})^T \rightarrow \\ \leftarrow (\vec{y}^{(2)})^T \rightarrow \\ \vdots \\ \leftarrow (\vec{y}^{(N)})^T \rightarrow \end{pmatrix} \in \mathbb{B}_1^{N \times K}$

- 계산과정

$$H(\vec{y}, \hat{\vec{y}}) = -\sum_{i=1}^{K} y_i \log(\hat{y}_i)$$

$$\uparrow J$$

$$\boxed{J = H(\vec{y}, \hat{\vec{y}})}$$

$(\vec{y})^T = (y_1 \; y_2 \cdots y_K)$

$y_i \in \{0, 1\}$

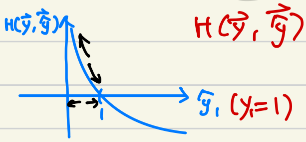
$(\hat{\vec{y}})^T = (P(C_1), P(C_2), \cdots, P(C_K))$

$= (\hat{y}_1 \; \hat{y}_2 \cdots \hat{y}_K)$

$0 \le \hat{y}_i \le 1$

근거
output layer에서
softmax를 거쳐서
0~1 사이의 값으로 변환함.

ex) $\vec{y} = (1 \; 0 \; 0 \; 0) \quad \hat{\vec{y}} = (0.8 \; 0.1 \; 0.1 \; 0)$




$H(\vec{y}, \hat{\vec{y}}) = -\log(\hat{y}_1)$

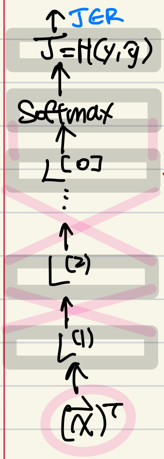
$= -\log(P(C_1))$

∴ $P(C_1)$가 0에 가까워질수록
loss값이 엄청 커짐.

⟷ 1에 가까워질수록 0에
가까워짐. → 목표!

- Model : 하나의 input vector 경우

Cross Entropy



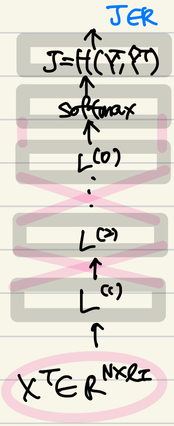

$(\vec{y})^T \in B_1^{1 \times K}$

$(\vec{\hat{y}})^T \in P^{1 \times K}$

→ No activation

Model : minibatch의 경우

Cross Entropy




$Y^T \in B_1^{N \times K}$

$\hat{Y}^T \in P^{N \times K}$

$$H(Y, \hat{Y}) = \frac{1}{N} \sum_{i=1}^{N} H(\vec{y}^{(i)}, \vec{\hat{y}}^{(i)})$$

$$= -\frac{1}{N} \sum_{i=1}^{N} \sum_{j=1}^{K} y_j^{(i)} \log(\hat{y}_j^{(i)})$$

↑

전체평균.