# Case Study: Bayesian Prediction of Bluffing in Poker

Jorge Chamorro Pedrosa

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## Objective

We illustrate how to use Bayesian inference and prediction to estimate the probability of bluffing behavior in Poker games.

We model bluffing as a Bernoulli random variable, and update our beliefs based on observed hands using a conjugate Beta prior.

Specifically, we aim to answer:

$$Pr(\text{Bluff} \mid \text{Observed hands})$$

## Data

Suppose we observe a professional poker player's behavior over 100 hands and record whether they bluffed (1) or did not bluff (0) in each hand.

For this case study, we simulate the data:

```r
set.seed(123)

# Simulate 100 hands: 1 = bluff, 0 = no bluff
n_hands <- 100
true_bluff_prob <- 0.2
hands <- rbinom(n_hands, size = 1, prob = true_bluff_prob)

# Summary of the data
table(hands)
```

```
## hands
##  0  1
## 82 18
```

```r
prop_bluff <- mean(hands)
prop_bluff
```

```
## [1] 0.18
```

## Bayesian Bernoulli Model

We model each hand as:

$$X_i \sim \text{Bernoulli}(\theta),$$

where $\theta$ is the probability of bluffing.

We assume a **Beta prior** for $\theta$:

$$\theta \sim \text{Beta}(\alpha, \beta).$$

First, we select a **informative prior**: $\text{Beta}(10, 90)$.

```
# Prior parameters
alpha <- 10
beta <- 90
```

Given observations $\{x_1, \dots, x_n\}$, the posterior distribution is:

$$\theta \mid \text{data} \sim \text{Beta}\left(\alpha + \sum_{i=1}^{n} x_i,\ \beta + n - \sum_{i=1}^{n} x_i\right).$$

Let's compute the posterior parameters:

```
sum_bluffs <- sum(hands)
n <- length(hands)

# Posterior parameters
alpha_post <- alpha + sum_bluffs
beta_post <- beta + n - sum_bluffs

c(alpha_post, beta_post)
```

```
## [1]  28 172
```

Thus, our updated belief about $\theta$ follows a $\text{Beta}(\alpha_{\text{post}}, \beta_{\text{post}})$ distribution.

## Prior vs Posterior Visualization

Let's visualize the prior and posterior distributions to see how the data updates our beliefs.

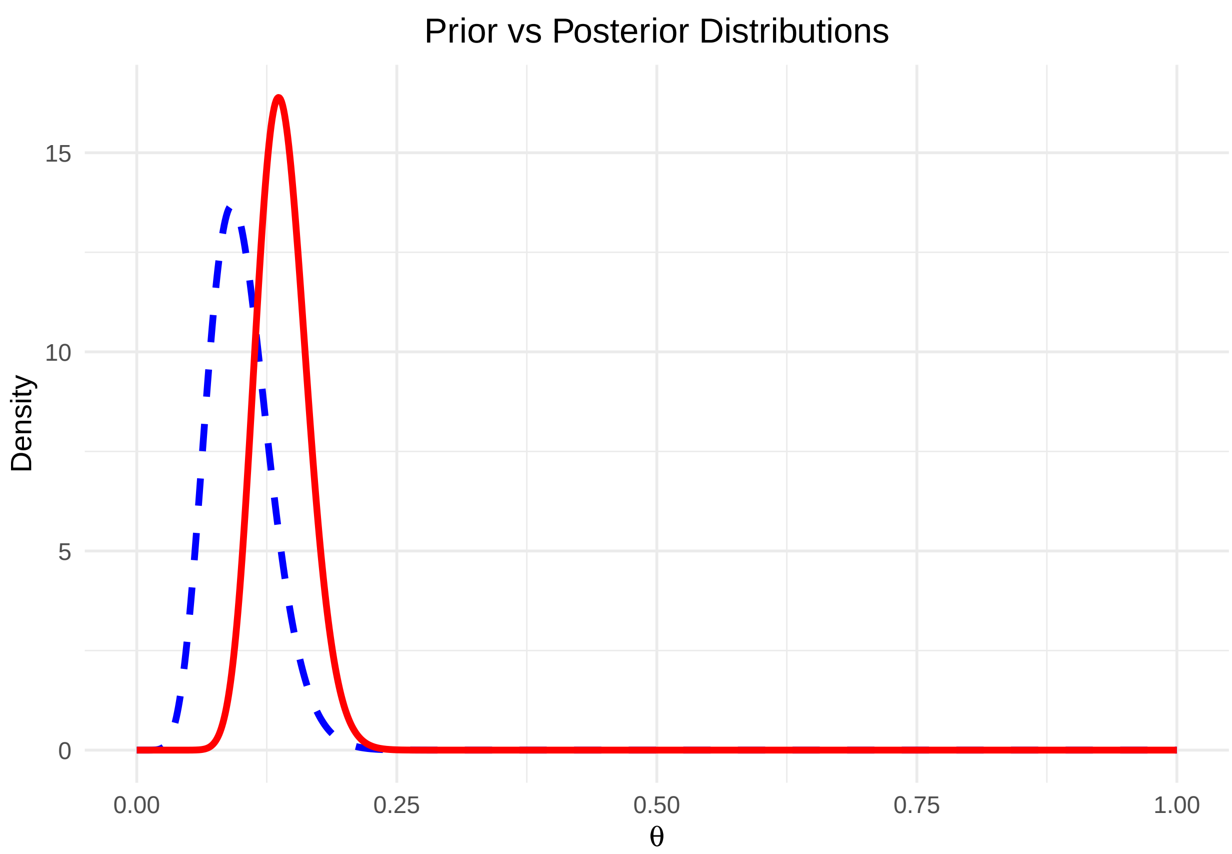
```
library(ggplot2)

# Create a grid of theta values
theta_grid <- seq(0, 1, length.out = 1000)

# Compute densities
prior_density <- dbeta(theta_grid, alpha, beta)
posterior_density <- dbeta(theta_grid, alpha_post, beta_post)
```

```r
# Combine into a data frame
density_data <- data.frame(
  theta = theta_grid,
  Prior = prior_density,
  Posterior = posterior_density
)

# Plot
ggplot(density_data, aes(x = theta)) +
  geom_line(aes(y = Prior), color = "blue", size = 1.2, linetype = "dashed") +
  geom_line(aes(y = Posterior), color = "red", size = 1.2) +
  labs(title = "Prior vs Posterior Distributions",
       y = "Density",
       x = expression(theta)) +
  theme_minimal() +
  theme(plot.title = element_text(hjust = 0.5))
```




**Predictive Analysis**

We want to predict the probability that in the next hand, the opponent bluffs.

Given our Beta posterior Beta$(\alpha_{\text{post}}, \beta_{\text{post}})$,
the posterior predictive mean is:

$$\mathbb{E}[\theta \mid \text{data}] = \frac{\alpha_{\text{post}}}{\alpha_{\text{post}} + \beta_{\text{post}}}$$

Let's compute it:

```
# Predictive mean
predictive_mean <- alpha_post / (alpha_post + beta_post)
predictive_mean
```

```
## [1] 0.14
```

Thus, the estimated probability that the opponent bluffs in the next hand is approximately 0.14.

### Simulating Future Hands

We now simulate future hands based on our posterior belief:

- First, draw $\theta$ from the posterior Beta$(\alpha_{\text{post}}, \beta_{\text{post}})$,
- Then simulate new hands using this $\theta$.

```
# Set seed for reproducibility
set.seed(123)

# Number of simulations
M <- 1000

# Sample theta from posterior
theta_samples <- rbeta(M, alpha_post, beta_post)

# Simulate future hands: 1 = bluff, 0 = no bluff
future_hands <- rbinom(M, size = 1, prob = theta_samples)

# Proportion of bluffs
mean(future_hands)
```

```
## [1] 0.14
```

### Analysis and Visualization of Future Hands

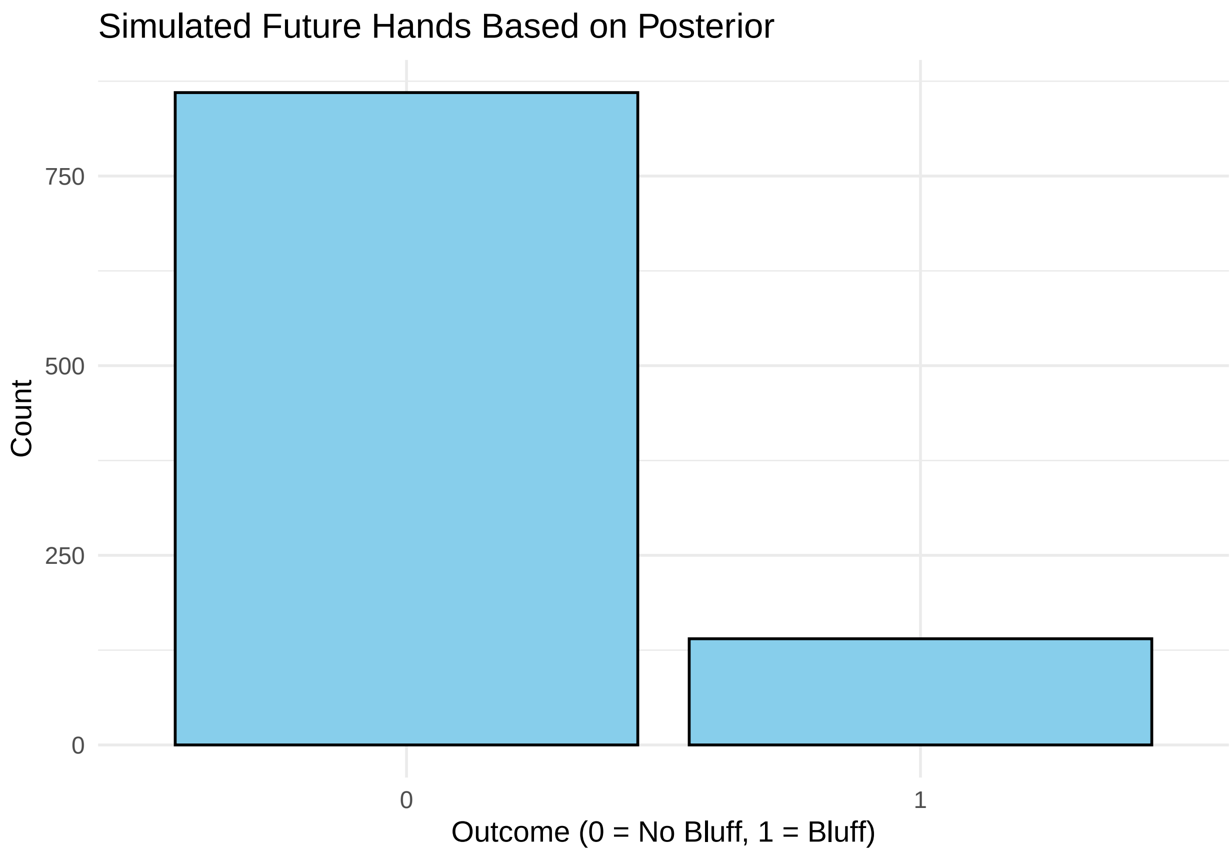
Let's analyze the outcomes of the simulated future hands.

```
# Plot histogram of future hand outcomes
library(ggplot2)

# Create a data frame
future_data <- data.frame(Outcome = future_hands)

# Plot
ggplot(future_data, aes(x = factor(Outcome))) +
  geom_bar(fill = "skyblue", color = "black") +
```

```r
  labs(x = "Outcome (0 = No Bluff, 1 = Bluff)",
       y = "Count",
       title = "Simulated Future Hands Based on Posterior") +
  theme_minimal()
```




We can also summarize the results:

```r
# Number of bluffs and no bluffs
table(future_hands)
```

```
## future_hands
##   0   1
## 860 140
```

This gives us an empirical estimate of how often we expect the opponent to bluff in future hands.

### Credible Interval for $\theta$

We can compute a 95% Bayesian credible interval for the bluffing probability $\theta$ based on the posterior distribution.

```r
# 95% credible interval
quantile(rbeta(10000, alpha_post, beta_post), c(0.025, 0.975))
```

```
##       2.5%      97.5%
## 0.09513089 0.19082236
```

Thus, there is a 95% probability that the true bluffing probability $\theta$ lies within this interval according to our posterior belief.

### Predicting Bluffing in Future Hands

Suppose we want to predict how many times the opponent will bluff in the next 5 hands.

Since the bluffing probability $\theta$ follows a Beta posterior, the number of bluffs in 5 hands follows a **Beta-Binomial** distribution.

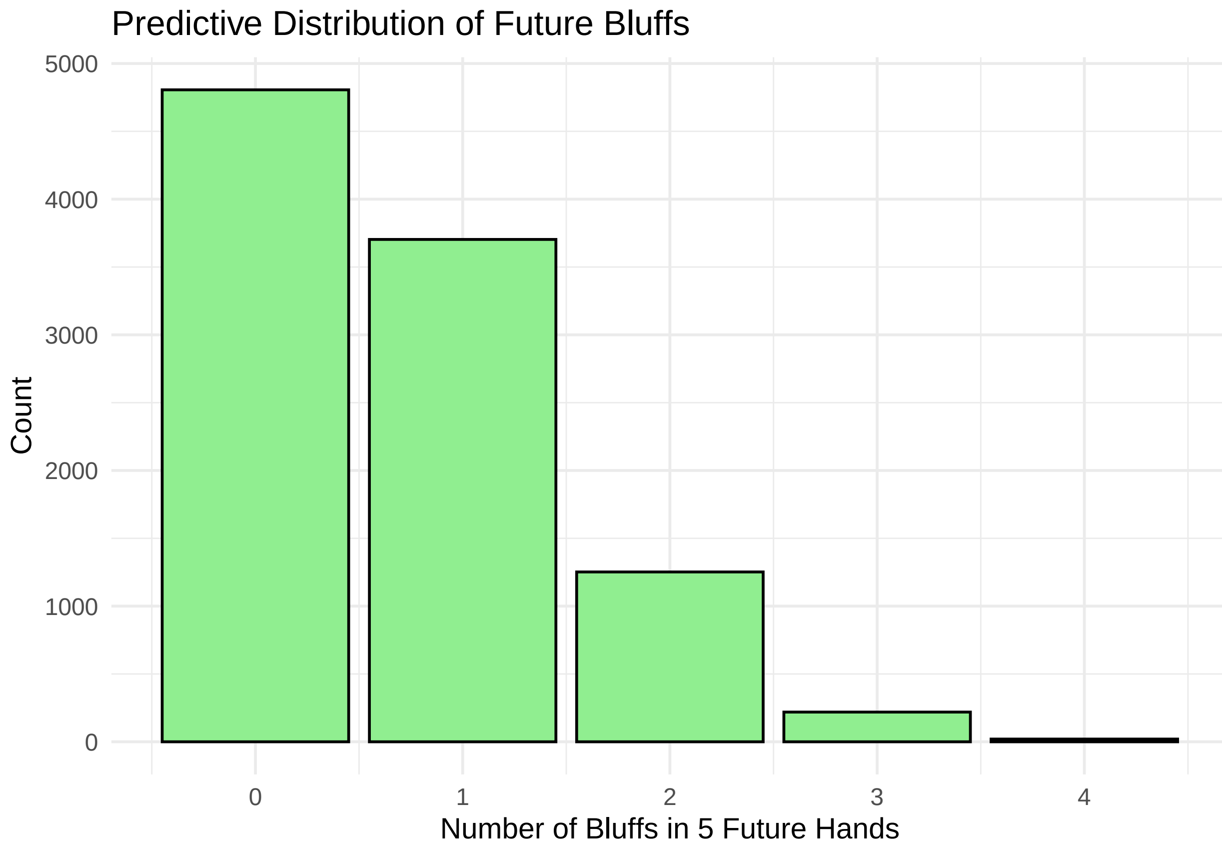
We can simulate this:

```r
# Simulate future bluff counts
M <- 10000
theta_samples <- rbeta(M, alpha_post, beta_post)
future_bluffs <- rbinom(M, size = 5, prob = theta_samples)

# Plot the distribution
library(ggplot2)

future_bluffs_df <- data.frame(Bluffs = future_bluffs)

ggplot(future_bluffs_df, aes(x = Bluffs)) +
  geom_bar(fill = "lightgreen", color = "black") +
  labs(x = "Number of Bluffs in 5 Future Hands",
       y = "Count",
       title = "Predictive Distribution of Future Bluffs") +
  theme_minimal()
```




We can also compute probabilities like:

- Probability opponent bluffs at least 3 out of 5 hands:

```
mean(future_bluffs >= 3)
```

```
## [1] 0.0239
```

**Conclusion**

In this case study, we used Bayesian data analysis to model an opponent's bluffing probability in poker.

Starting from a prior belief (Beta(10, 90)), we updated our knowledge based on observed hands (18 bluffs out of 100).
We computed the posterior distribution, made predictions about future hands, and built credible intervals to quantify uncertainty.

- The Bayesian approach **naturally balances** prior beliefs with observed data.
- In our case, the data had strong influence because we observed many hands.
- The final estimated bluff probability shifted slightly, showing how evidence dominates when enough data is available.

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**Real-World Use in Poker**

In real poker scenarios, such analysis can be extremely useful:

- **Opponent Modeling**: Continuously update your belief about how often a player bluffs.
- **Decision Making**: If you believe an opponent bluffs often, you might call more often. If rarely, fold more often.
- **Risk Management**: Quantifying uncertainty helps make **more robust** decisions under pressure.

Thus, Bayesian thinking gives players a huge edge: decisions are **evidence-based**, **adaptive**, and **mathematically sound**.